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DURELL'S ARITHMETIC

BOOK TWO

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DURELL'S ARITHMETIC

BOOK TWO

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BY

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NEW YORK
CHARLES E. MERRILL COMPANY
1911

~~Math 458.1~~
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JUL 20

DURELL'S ARITHMETIC

BOOK ONE
For Grades I–IV. 332 pages
With Notes to Teachers. 364 pages

BOOK TWO
For Grades V, VI. 400 pages

BOOK THREE
For Grades VII, VIII

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PREFACE

The main purpose in writing this series of Arithmetics has been to give the subject a broader and more vital practical meaning. By special treatment of the concrete applications of arithmetic, the series develops bread-winning power; and by cultivating a grasp of the functions of number processes, it develops a broad, cultural power.

Among the features which contribute to these ends are the *Industry Reviews*, which treat of the arithmetical element in representative industries, as agriculture, mining, and forestry. The *permanent number facts* in geography, history, physics, and other departments have been collected in tables and used repeatedly in examples. These features give a *correlation* of arithmetic with other studies. Furthermore, the examples, especially those devised by the pupil, are often shaped toward some definite, useful end. Such examples add to the broad, practical spirit of the books.

The systematic development of principles is a primary aim of the series. All informational matter has been introduced in such a way as not to break up the regular formal development of principles. For instance, the Industry Reviews are each placed at the end of a chapter.

Attention is called to the various special *drills* which, besides their direct value, give variety to the work. In the earlier grades, many of these drills take the form of games. They are described in Exercises throughout the book, and also in detail on pages 391–395.

The all-important subject of *review* work has received special attention. In many places the examples call for both written and oral review of important parts of the preceding year's work. This repeated review of preceding work, not only gives a more thorough mastery of processes, but in large measure relieves the teacher of the labor of dictating supplementary examples. If the prescribed drills and reviews are followed, practically all the pupils using these books should become *rapid and accurate computers.*

Book Two is divided into two parts, Part I being designed for Grade V under ordinary circumstances, and Part II for Grade VI. Schools, however, may readily vary from this arrangement, if they choose.

In Book Two, emphasis is laid on *diagramming* (as on pages 122, 134, 135) and particularly on the use of squared paper, as an aid in visualizing numbers, drawing to scale, and in making clear the meaning and properties of common and decimal fractions.

Special mention should be made also of those Exercises which develop a grasp of principles of *efficiency and economy,* and thus prepare pupils, not only for efficient business management, but for efficient life in general (as in Exercises 1, 6, etc.).

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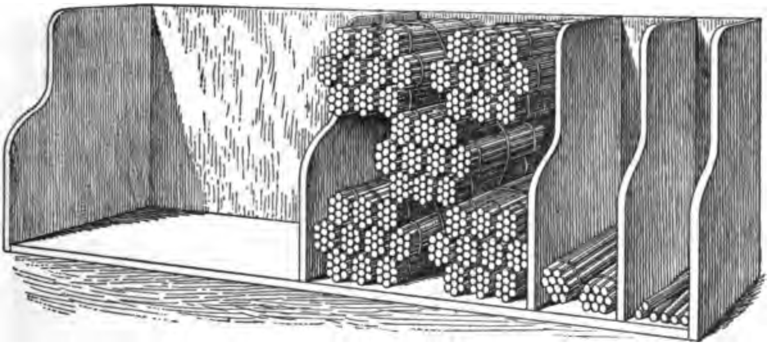
PART ONE

CHAPTER I

NUMERATION AND NOTATION THROUGH MILLIONS

COMMON NOTATION



No thousands 5 hundreds 2 tens 6 units

EXERCISE 1

REVIEW OF MEANINGS AND USES

1. Write: five hundreds two tens six units. How many letters have you written? Write 526. How many number signs have you written?

2. In 3526, what does the 2 stand for? 5? 6? 3?

3. In 3526, the figure 6 expresses 6 units. It also makes 2 express tens. (Take away the 6 and show what the 2 then expresses.) It also helps 5 express hundreds and 3 express thousands. Hence, how many items of work does the figure 6 do in all?

4. In 3526, how many items of work does the figure 2 do? The figure 5? 3?

5. In 3858, the value of the left-hand figure 8 is how many times as great as that of the right-hand 8?

6. Write the largest number you can with the figures 6, 7, 0, 2, 8.

7. Write the largest number you can containing four figures.

In order to get the greatest possible benefit from the short way of writing numbers, we now make certain definitions and rules.

1. A **unit** is a single object of any given kind.

Thus, each of the following is a unit: one apple, one marble, one cow.

A **number** is a unit or a collection of units of the same kind.

Thus, *one apple* is a number; also *5 apples* is a number.

A **unit of higher order** is a unit formed by combining several smaller like units.

Thus, ten cents are ten like units which are combined to make a higher unit, the dime.

2. **Advantages in the Use of Units and Numbers.** If we treat things as made up of units, and group like units together, objects and materials can be handled in groups in many easy ways, giving both direct and indirect advantages.

Thus, instead of always dealing with chickens indefinitely by flocks, we often deal with them by units and tens; instead of dealing with apples and wheat by heaps, we often deal with them by bushels, and thus save much time and labor.

3. **The Number Words and Signs** for the small numbers are as follows:

one	two	three	four	five	six	seven	eight	nine	ten
1	2	3	4	5	6	7	8	9	10

The sign 0 is called *naught,* or *zero,* and means the absence of number.

The first nine number signs are called *figures* or *digits.*

4. The Ten or Decimal System of Grouping Units. In numbering objects, it is the custom to group objects by tens, and to make each unit of a higher order equal ten units of the next lower order.

Our system of numbering objects is called the *decimal system,* from the Latin word *decem,* which means *ten.*

We group by tens because our savage ancestors did their counting on their ten fingers or ten toes, and thus came to regard ten as the *primary group* in numbering objects.

5. Words and Symbols for Larger Numbers. In grouping units by tens, words are used for each higher unit as shown in the following table:

NUMBER OF PERIOD	3d	2d	1st
NAME OF PERIOD	Millions	Thousands	Units
ORDER OF UNITS	Hundred-millions Ten-millions Millions	Hundred-thousands Ten-thousands Thousands	Hundreds Tens Units
NUMBER	3 2 7,	5 6 1,	4 8 2

Figures are combined to express numbers larger than nine by the use of the following principle:

In a number expressed in figures, each figure has a value ten times as great as if it were in the next place to the right.

What is the meaning of the number words: eleven, twelve, thirteen, twenty?

6. Periods. The reading of large numbers is aided by marking off the figures of a large number, by commas, in groups of three figures each, beginning at the right. These groups are called *periods*.

Mark off the following numbers into periods: 6710328; 15916724.

7. **Numeration** is reading a number in words.

Numeration includes also the grouping by tens, hundreds, etc., which is necessary for the naming of number.

To read a number given in figures,

Begin at the right and separate the given figures into periods of three figures each;

Then begin at the left and read each group, giving it the name of the period to which it belongs;

Omit the name of the units' period in reading.

Do not use the word *and* in reading a number composed of whole units.

Ex. Read 26542308.

Separating the given number into periods of three figures each, beginning at the right, we have 26,542,308.

Reading these periods in order from the left, we have "26 million 542 thousand 3 hundred 8."

8. **Notation** is writing a number by the use of figures, or other number signs, as when we change "two hundred sixty-five" into 265.

The method of representing numbers which has been explained in the preceding pages is called the *Arabic Notation*, because the people of Europe first learned it from the Arabs. The real inventors of it, however, were the Hindoos or inhabitants of India.

EXERCISE 2

Mark off the following in periods and read:

1. 19876
2. 75207
3. 235196
4. 910076
5. 900030
6. 911008
7. 36758911
8. 98070104
9. 971030047

Read:

10. The area of the United States and its dependencies is 3,742,155 square miles.

11. The distance from the earth to the sun is about 92,800,000 miles.

12. It is estimated that the water power of Niagara Falls is equal to 7,000,000 horse power.

Express in figures:

13. 7 thousand 5 hundred eleven.

14. Nine thousand seventy-one.

15. 70 thousand 9 hundred one.

16. One million six hundred twenty-seven thousand.

17. Three hundred six thousand five hundred forty.

18. Three million ninety-six thousand four hundred ten.

19. Two hundred sixty-five million forty-eight thousand six hundred eleven.

20. Eighty-eight thousand eighty.

21. 478 million 923 thousand 17.

Rewrite, putting figures in place of number words:

22. The moon is about two hundred forty thousand miles from the earth.

23. The area of Texas is two hundred sixty-five thousand seven hundred eighty square miles.

24. The area of the United States is approximately three million square miles.

25. Write first in words, and then in figures, the population of the state in which you live.

You can find this population in the list of Important Numerical Facts on p. 386 or in the index of your geography.

26. The area of the United States is 3,025,600 square miles. Write this number in both words and figures. Compare the length of this number when written in words with its length when written in figures.

To *compare* is to estimate how many times as long the one expression is as the other.

27. If numbers are much shorter when expressed in figures, why do we sometimes express them in words?

EXERCISE 3

SIGHT WORK

To the Teacher. Paper and pencil are not to be used in Sight and Oral Work. The books of the pupils should be closed, and the teacher should read each example aloud and, when necessary, write on the blackboard the numbers occurring in an example.

1. In 7,654,327, how many items of work does each figure do?

2. In 6,092, what does the zero show? Of what use is the zero?

3. In 7,654,327, the left-hand figure 7 is how many times as great in value as the right-hand 7?

4. If I annex a zero to the right of 523 and make it 5230, how is the value of the 3 changed? Of the 2? Of 5? Hence, what change is made in the value of the entire number?

5. If I annex two zeros to the right of 523, how is the value of each figure in 523 changed? How is the value of the entire number changed?

6. Make up and work an example similar to Ex. 5.

7. Annexing one zero to the right of a number multiplies the number by what? Annexing two zeros?

8. In 58976154, give the name of each place (or order).
How many periods are there? Name each of these periods.

9. How many places does a period contain?

10. Write a number containing three full periods.

11. Write a number containing seven places.

12. Write the number which is ten times greater than 762.

13. Write the number which is 100 times greater than 762. The number 1000 times greater than 762.

14. Count by 2's, 3's, and 4's from 0, 1, 2, 3, and 4, with 100 as a limit. Count downward from 100 by 2's, 3's, and 4's.

ROMAN NOTATION

9. **Uses of the Roman Notation.** When we use two sets of numbers together, as in using one set for the pages of a book and another for the chapters, to prevent confusion we sometimes use the Roman notation for one set of the numbers. This method of writing numbers was invented by the ancient Roman people.

10. **Single Symbols.** Seven of the capital letters are used to denote numbers in the Roman notation, thus:

I = 1 V = 5 X = 10 L = 50 C = 100 D = 500 M = 1000

11. **Symbols Combined.** These letters, when combined according to the following principles, will express any number:

1. *Each repetition of a letter repeats its value.*

Thus, XXX denotes 30. CC denotes 200.

2. *When a letter is placed* **after** *another of greater value, the* **sum** *of the letters is expressed.*

Thus, VI represents 6. XVI represents 16. LXXXI, 81.

3. *When a letter is placed* **before** *another of greater value, the* **difference** *of the letters is expressed.*

Thus, IV represents 4. XL, 40. XC, 90.

A letter placed *between* two letters, both of greater value than itself, is regarded as preceding the last letter (not as following the first).

Thus, XIV represents 14.

4. *A dash placed over a letter increases the value of the letter a thousandfold.*

Thus, $\overline{\mathrm{V}}$ = 5000.

I = 1
II = 2
III = 3
IV = 4
V = 5
VI = 6
VII = 7
VIII = 8
IX = 9
X = 10

XI = 11
XII = 12
XIII = 13
XIV = 14
XV = 15
XIX = 19
XX = 20
XXI = 21
XXIX = 29
XXX = 30

XL = 40
L = 50
LX = 60
XC = 90
CC = 200
CD = 400
D = 500
DC = 600
M = 1000
$\overline{\mathrm{X}}$ = 10,000

State which of the principles of the Roman notation given in Art. 11 are used in expressing each of the preceding numbers.

EXERCISE 4

Express by Roman notation:

1. 13
2. 24
3. 18
4. 29
5. 34
6. 38
7. 45
8. 49
9. 57
10. 66
11. 77
12. 89
13. 91
14. 98
15. 99
16. 117
17. 165
18. 368
19. 786
20. 2697

Read the following:

21. XXXVI
22. XLIV
23. LXIX
24. LXXXIX
25. XCVI
26. CXIV
27. CLXIX
28. CCCXXXIII
29. CCXCVII
30. DCXL
31. MDLXVI
32. MDXLIV

33. State which of the principles 1, 2, 3, and 4 of Art. 11 are used in Ex. 21. In Ex. 22. Ex. 29. Ex. 31.

34. In 3777, what does the left-hand 7 denote? The middle 7? The right-hand 7?

35. In XXX, what does the left-hand X denote? The middle X? The right-hand X?

36. Write ten X's in a row. Write one C. About how much more labor is there in writing the one than the other?

37. Draw a picture of a clock face, using the Roman system to denote the numbers which indicate the hours.

38. Make up and work an example similar to Ex. 20. To Ex. 34. To Ex. 35.

39. Count by 5's, 6's, 7's, 8's, and 9's from 0, 1, 2, and 3, with 100 as a limit. Count downward from 100 by 5's, 6's, 7's, 8's, and 9's.

UNITED STATES MONEY

12.

TABLE

10 mills = 1 cent.
10 cents = 1 dime.
10 dimes = 1 dollar.
10 dollars = 1 eagle.

The *mill* is not coined and is used only as a help in numerical work.

13. In **Writing United States Money,** a period called the *decimal point* is placed between the number of dollars and the number of cents, and the *dollar mark* is placed before the entire number.

In *reading* a number in United States money, the number of dollars is read first, then the number of cents (reading dimes as cents), then the number of mills. The word *and* is used where the decimal point occurs.

Thus, $756.283 is read "756 dollars and 28 cents 3 mills." Since each unit in United States money equals ten units of a lower order, the system of notation used is the decimal system, aided by the use of the decimal point.

EXERCISE 5

1. Read the following:

$67.23 $.76 $1.005 $592.376 $.007
$190.308 $.05 $3765.50 $700.08 $1.01

Write each of the following in figures:

2. Three hundred seventy-two dollars and fifty cents.

3. Eighty dollars and seventy-three cents six mills.

4. Five thousand six hundred seventy dollars and twenty cents five mills.

5. Eight mills; one dollar and seven mills; six cents three mills.

6. In Ex. 2, about how much shorter is the number when expressed in figures?

7. If it is shorter to express United States money in figures, why do we ever express it in words?

8. Rewrite the following, using the dollar mark:

672¢ 1072¢ 37¢ 622¢ 1065¢

9. Make up and work an example similar to Ex. 1. To Ex. 2. Ex. 6.

10. Repeat the tables for dry measure. For liquid measure. For linear measure. Give rapidly the different facts in these tables as the teacher calls for them.

CHAPTER II

FUNDAMENTAL OPERATIONS

ADDITION

EXERCISE 6

REVIEW OF MEANINGS AND USES

1. Take 4 splints; count 7 splints on to the 4 splints. How many splints have you now?

2. Compare the work of counting 7 on to the 4, with that of remembering the result of a former counting and saying "4 and 7 are 11"?

3. Which is easier: to count 18 on to 11, or to add the tens and then the units and say "18 and 11 are 29"? About how much easier?

4. Compare the work of writing 27 chickens + 12 chickens + 23 chickens, with that of writing 62 chickens. That of writing $12 and $16 and $42, with writing $70.

5. Write on the blackboard the following arrangement of numbers. As the teacher points, state quickly the sum of each number in the lower row and each number in the upper row.

7	1	5	9	3	8	4	2	6
3	5	8	2	1	7	9	6	4

Let the teacher or a pupil write other arrangements of the nine digits on the blackboard. As one pupil points, let others add pairs of numbers.

6. Practice adding by decades, as 7 + 8, 17 + 8, 27 + 8, etc. Practice the rapid addition of single columns of figures. Use the various drills until the 45 combinations in addition are fixed in mind. (See 45 Combinations, p. 395.)

To help us in getting the greatest benefit from the short method of combining two or more numbers into one number, we now make definitions and rules, and signs or symbols.

14. The **sum** of two or more numbers is the number which they equal when taken together.

Addition is a short way of finding the sum of two or more numbers.

The **addends** are the numbers added.

The **sign of addition** is the erect cross, +, which is read "plus" or "and."

The **sign of equality** is =, or two short horizontal lines placed one above the other. It is read "equals."

Thus, in 7 + 6 = 13, 7 and 6 are the addends, and 13 is the sum.

Compare the work of writing "numbers to be added," with that of writing "addends."

15. Uses of Abstract Words in Adding Numbers.

Only like numbers can be added.

Thus, 7 cows + 8 horses cannot be added.

By using a more general word instead of particular words, we are often able to add numbers which we cannot otherwise add.

Thus, if we use "animals" instead of "horses" and "cows," we may write 7 animals + 8 animals = 15 animals, and thus add the two numbers.

This illustrates one use of abstract or general number. Later we shall find that abstract number has other uses.

16. Method for Addition. In order to add two or more numbers to the best advantage, we

Write the given numbers so that units shall be under units, tens under tens, hundreds under hundreds, and so on;

Add each column separately, beginning at the right;

If the sum of any column exceeds 9, *set down the right-hand figure of the sum and add the other figure to the next column.*

Ex. Add 674, 892, 476, 514, and 962.

$$\begin{array}{rl} 674 & \\ 892 & \\ 476 & \\ 514 & \\ 962 & \\ \hline 3518 & \textit{Sum} \end{array}$$

The sum of the right-hand column is 18. Write the **8** (units) under this column (of units) and add the 1 (ten) to the next column (of tens), etc.

In stating orally the work of adding a column, give successive results only. Thus, if you begin at the bottom of the right-hand column, say 2, 6, 12, 14, 18. In adding the next column, say 1, 7, 8, 15, 24, 31.

17. Checks. The accuracy of an addition may be checked, or tested, in several ways. The most common method is to

Check by adding each column in the opposite direction from that in which it was originally added. Thus, add a column first up and then down.

In working an example in addition, it is often an advantage to write on a separate sheet of paper the sums of the columns and to add these results. Thus, for the example worked in Art. 16 write:

$$\begin{array}{rrrr} & & 1 & 8 \\ & 3 & 0 & \\ 3 & 2 & & \\ \hline 3 & 5 & 1 & 8 \end{array} \left.\vphantom{\begin{array}{c}1\\3\\3\end{array}}\right\} \textit{Sums of Columns}$$

By this means, in case of interruption, you can continue the addition without going over the work previously done. This method of writing the sums of the columns separately is also an aid in testing the accuracy of the work.

18. Special Check by Adding by Rows and Columns. Sometimes it is possible to add an array of numbers both by rows and by columns, and to test the accuracy of the work by comparing the final sums obtained in the two ways.

Ex. Add the following both by rows and by columns:

$$\begin{array}{rcrcrcrcrcr}
6 & + & 8 & + & 17 & + & 9 & + & 3 & = & 43 \\
10 & + & 12 & + & 13 & + & 7 & + & 5 & = & 47 \\
2 & + & 1 & + & 4 & + & 9 & + & 3 & = & 19 \\
\underline{6} & + & \underline{7} & + & \underline{4} & + & \underline{3} & + & \underline{7} & = & \underline{27} \\
24 & + & 28 & + & 38 & + & 28 & + & 18 & = & 136 \ \textit{Sum}
\end{array}$$

The work of addition can be done rapidly and accurately on an *adding machine* if one can use such a machine. Do you know whether any business man in your neighborhood has such a machine?

EXERCISE 7

Add the following and check your work in each example.

Make a record of the time it takes you to work Exs. 6–10; also Exs. 11–15.

1. $7215	2. 4967 in.	3. 4275 ft.	4. $8.75	5. $100.32
5927	5923 in.	3519 ft.	93.82	476.05
3764	801 in.	8012 ft.	14.72	521.13

6. 8439	7. $5246	8. 18,792 lb.	9. $912.62	10. $1054.37
2706	3897	90,723 lb.	817.42	392.42
384	4213	874 lb.	109.72	8654.72
5123	2092	1,516 lb.	3761.75	437.05
7092	17962	37,924 lb.	17362.50	6923.82

11.	**12.**	**13.**	**14.**	**15.**
47,155	7,912	1,066	91,718	$36.48
23,076	2,162	1,776	4,376	35.62
76,389	33,000	1,620	432	57.42
17,138	5,280	1,783	10,205	34.73
3,765	1,760	2,240	9,116	18.98
40,708	1,094	6,288	37,346	4.14
6,407	1,728	5,175	853	92.95
975	4,840	49,170	9,943	47.39
71,779	43,560	7,815	82,705	96.25
8,543	31,416	29,002	85,432	39.74
90,763	1,492	14,147	17,765	72.86

16. A man owns in one town three houses worth $760, $5290, and $3750; in another town he owns properties worth $7528, $32,000, and $567. What is the value of all his property?

17. If a miller has 3289 bu. of oats, 4765 bu. of corn, 2763 bu. of rye, and 8576 bu. of wheat, how many bushels of grain has he in all?

18. Four men have respectively $9763.41, $7169.73, $8563.27, and $4763.19. How much have they all together?

19. In one trainload of coal there are 4763 tons, in another 8755 tons, in a third 4958 tons, and in a fourth 3908 tons. How many tons are there in all the trains?

20. The companies of a certain regiment contained the following numbers of soldiers respectively: 62, 73, 59, 66, 49, 72, 69, 61, 73, 65, 62, 74. How many men were in the regiment?

21. The sides of an irregular field are 512 ft., 482 ft., 763 ft., 724 ft., and 642 ft. long respectively. Find the length of the fence around the field.

22. In the year 1800 the area of the United States was 827,844 square miles. By the Louisiana Purchase 875,625 square miles were annexed. What was the area after the purchase?

What is a square mile? Can you name a part of the city or neighborhood where you live which contains a square mile? What was the Louisiana Purchase?

To the Teacher. Questions like these should be asked whenever the opportunity offers, in order to recall to the pupils facts already learned and to connect arithmetic work with other studies.

23. The value of a farm and its buildings is $8750, of the crops $2650, the livestock $1520, the furniture and implements $2170. What are these worth taken together?

24. Reduce to the simplest form sixty-two thousand four hundred seventy-two plus five hundred ninety-six thousand eight hundred seven plus three million twenty-six plus three hundred thousand two hundred twelve.

25. Find the total area of the New England states. (See p. 386.)

26. What is the total area of the Middle states?

27. Find the total area of the states which border on the Atlantic Ocean. On the Pacific Ocean.

28. Find the total area of the states bordering on the Mississippi River. On the Gulf of Mexico. On the Great Lakes.

29. Measure the length of the schoolroom. Also its width. Find the distance around the schoolroom. Can you make a diagram of the schoolroom? (Show, if possible, the position of doors, windows, and teacher's and pupils' desks.)

30. Make up and work a problem in addition similar to Ex. 16. To Ex. 17. Ex. 19.

31. Have a rapid oral review of the tables for weight, area, and time.

EXERCISE 8

1. The tickets of admission sold at a county fair were as follows. Find in two ways the total number of tickets sold.

Tickets	Tues.	Wed.	Thur.	Fri.	Sat.
Children	217	1615	2953	1817	542
Adults	462	2713	3602	2013	617
Single Carriage	23	117	242	116	42
Double Carriage	21	105	202	91	23

Make up and work similar examples concerning

2. The attendance in the different rooms in the school for a week.

3. The receipts of a dealer in coal.

4. The number of cans of milk sent to a creamery by different farmers.

5. Any similar subjects.

The teacher should supply the numbers in Exs. 3 and 4, if the pupils find it difficult to get probable numbers.

Add each of the following:

6. $75,340	7. $68,901	8. $64,268	9. $346,768
6,731	50,345	405	21,380
748	75,005	1,708	4,075
68,451	29,450	43,671	126,849
396	80,063	72,049	257
7,503	91,700	492	1,305
46,075	43,621	1,760	24,350
1,290	47,834	25,357	439,871
25,738	83,276	1,434	40,306
46,803	25,327	84,162	601,734

19. Oral Work in Two-Column Addition.

Ex. At sight find the sum of 31 and 27.

In adding two-figured numbers orally, it is best first to add the tens of one number to the whole of the other number, and then to add the units of the first number to the sum thus obtained.

Thus, for the above example we say 27, 57, 58. The reason for adding the tens first, is that we read numbers from left to right and hence can more readily remember the successive sums in the above process by adding the tens first.

EXERCISE 9

SIGHT WORK

Add the following at sight:

1. 27	**4.** 37	**7.** $31	**10.** 314	**13.** $12	**14.** 22 in.
32	24	49	120	7	10 in.
				5	4 in.
2. 18	**5.** 43	**8.** 27	**11.** 613	11	3 in.
41	28	14	121	10	21 in.
				5	13 in.
3. 76	**6.** 51	**9.** 121	**12.** $7.12		
12	16	304	1.52		

15. Let the teacher dictate examples similar to Ex. 1–14 and let the pupils answer them orally, no written figures being used.

16. Walter had 23 marbles and bought 14 more. How many did he then have?

17. Mr. Anderson had 62 apple trees in his orchard and planted 24. How many apple trees did he then have?

18. In the Panama Canal, the Gatun Lake is 23 miles long. (Do you know some distance which is 23 miles long?) The rest of the canal is 26 miles long. How long is the entire canal?

19. Repeat the definition of a unit given on p. 10. About how much shorter is the word *unit* than the group of words it stands for?

20. Add 5 sheep and 7 cows by substituting a more general word for both "sheep" and "cows." Make up for yourself a similar example.

21. Have a rapid oral review of the tables of dry measure, liquid measure, and cubic measure.

SUBTRACTION

EXERCISE 10

REVIEW OF MEANINGS AND USES

1. Take 11 splints and count off 7 splints. How many splints are left?

2. Compare the work of counting 7 from 11, with that of remembering the result of a former counting and saying "11 less 7 are 4."

3. Which is easier: to count 27 from 39, or to subtract the tens from the tens and the units from the units and say "39 less 27 are 12"? About how much easier?

4. Compare the work of writing 39 – 27, with that of writing 12.

5. Write on the blackboard the following arrangement of numbers. As the teacher points at a pair of the numbers, state quickly the difference between the pair.

6. Drill on the 45 combinations in subtraction until they are fixed in your mind. Practice subtraction by decades, as 13 – 7, 23 – 7, 33 – 7, etc.

9	5	12	4	10
1	14	7	13	2
15	3	16	20	17
11	19	6	18	8

To help us in getting the greatest benefit from the short way of finding the difference between two numbers, we now make certain definitions and rules.

20. Subtraction is a short way of finding what number is left when one number is taken from another.

The **subtrahend** is the number which is subtracted.

The **minuend** is the number from which the subtrahend is taken.

The **difference** is the result of the subtraction. It is the number which must be added to the less number in order to make it equal the greater. The difference is also called the *remainder*.

The **sign of subtraction** is the horizontal dash, −, which is read "minus" or "less." It means that the number on the right is to be subtracted from the number on the left.

In 7 − 3 = 4, point out the minuend, subtrahend, and difference.

21. Method for Subtraction.

Ex. Subtract 2475 from 5638.

5638 *Minuend*
2475 *Subtrahend*
3163 *Difference*

5 units from 8 units leaves **3** units.

We cannot subtract 7 tens from 3 tens. Hence, we borrow 1 hundred, or 10 tens, from the 6 hundreds. 7 tens from 13 tens leaves **6** tens.

4 hundreds from 5 hundreds leaves **1** hundred.

2 thousands from 5 thousands leaves **3** thousands.

When the method is understood, it is enough to say, "5 from 8, 3; 7 from 13, 6; 4 from 5, 1; 2 from 5, 3."

To check the accuracy of a subtraction, *add the difference to the subtrahend*. The sum should equal the minuend.

Check the result obtained in the example which has just been worked.

EXERCISE 11

Subtract and check each result. Make a record of the time which it takes you to work Exs. 6–20.

1.	**2.**	**3.**	**4.**	**5.**
765	$8.92	873	727	3447
423	7.61	549	549	1235

6. 8127 5043	**9.** 63,360 43,560	**12.** 29,002 14,147	**15.** $27.75 15.62	**18.** 215,042 31,416
7. 2240 1728	**10.** $76.53 15.98	**13.** 4000 yd. 1760 yd.	**16.** $700.02 511.09	**19.** $8400.00 3926.72
8. 4840 2162	**11.** $37.23 15.85	**14.** 5280 ft. 3145 ft.	**17.** 100.00 in. 39.37 in.	**20.** 63,360 in. 28,912 in.

21. Find the difference between 785,632 and 597,543.

22. How many dollars must be added to $3892 to make $5000?

23. What is the subtrahend in Exs. 20, 21, 22? What is the minuend in each case?

24. Gertrude noticed that her mileage book was punched at 876 when she began her journey. She has traveled 108 miles. At what number should the conductor punch the book?

25. Denver is 5175 feet above the sea level. How much less than a mile is this?

26. Mt. Washington is 6290 feet high. How much more than a mile is this?

27. A certain field, when planted with selected seed corn, yielded 2976 bushels. The year before, when planted with seed corn taken at random (unselected), it yielded 2187 bushels. What was the gain?

28. The distance from New York to San Francisco by way of Cape Horn is 13,090 miles. By way of Panama it is 5278 miles. How many miles will a canal at Panama save in going from New York to San Francisco by water?

29. Mr. Brown bought a horse for $265 and sold it for $300. What was his gain?

30. The distance of the moon from the earth varies, the greatest distance at any time being 252,830 miles and the least 221,520 miles. Find the difference.

31. The area of the United States proper is 3,025,600 square miles. The area of the United States together with its outlying possessions is 3,742,155 square miles. What is the area of the outlying possessions? (Can you name these possessions?)

32. The area of Massachusetts is 8,315 square miles, of New York 49,170 square miles, and of Texas 265,780 square miles. Find the difference in area between each pair of these states.

33. What is the area of the state in which you live? Find the difference between that and the area of each state given in Ex. 32.

Subtract the following and check each result:

34. 4165
2340

35. 5600
3000

36. 8670
7364

37. 17,265
13,167

38. 21,480
20,372

39. 30,671
26,140

40. 45,723
31,203

41. 81,647
57,025

42. 265,328
140,300

43. 170,643
106,340

44. 465,746
241,680

45. 694,270
590,395

46. Make up and work a problem in subtraction similar to Ex. 24. To Ex. 27. Ex. 29.

47. Practice the rapid addition of columns of figures.

48. Have a Time Drill with Exs. 34–45. (See Games and Drills, p. 392.)

22. Oral Work in Two-Column Subtraction.

Ex. At sight subtract 27 from 53.

First subtract the tens of the subtrahend from the whole minuend, and then subtract the units of the subtrahend from the remainder.

Thus, 20 from 53 leaves 33. 7 from 33 leaves 26.

EXERCISE 12

SIGHT WORK

Subtract at sight:

1. $\begin{array}{r} 54 \\ \underline{31} \end{array}$
2. $\begin{array}{r} 87 \\ \underline{51} \end{array}$
3. $\begin{array}{r} 60 \\ \underline{21} \end{array}$
4. $\begin{array}{r} 90 \\ \underline{37} \end{array}$
5. $\begin{array}{r} 50 \\ \underline{28} \end{array}$
6. $\begin{array}{r} 75 \\ \underline{59} \end{array}$
7. $\begin{array}{r} 41\text{¢} \\ \underline{17\text{¢}} \end{array}$
8. $\begin{array}{r} 75 \\ \underline{29} \end{array}$
9. $\begin{array}{r} 100 \\ \underline{72} \end{array}$
10. $\begin{array}{r} 150 \\ \underline{120} \end{array}$
11. $\begin{array}{r} 532 \\ \underline{311} \end{array}$
12. $\begin{array}{r} 246 \\ \underline{115} \end{array}$
13. $\begin{array}{r} 300 \\ \underline{209} \end{array}$
14. $\begin{array}{r} 300 \\ \underline{219} \end{array}$
15. $\begin{array}{r} 470 \\ \underline{235} \end{array}$

16. A man has 32 teeth in all, and 12 are molars. How many teeth has he besides the molars?

17. Maud had a yard of ribbon and cut from it 19 inches. How can she tell how many inches she has left without measuring?

18. 25 ounces taken from 2 pounds leaves how many ounces?

19. If our school roll has 57 names on it and 38 pupils are present, how many are absent?

20. If another school roll has 312 names on it and 287 pupils are present, how many are absent?

21. Have a Time Drill with Exs. 1–16 on pp. 28, 29, the teacher dictating the numbers.

23. Making Change; Cash Register. In making change, subtraction is performed in the form of addition. Thus, if at a store you should buy groceries costing $1.37 and should hand the salesman a two-dollar bill, he would pay you the difference between \$1.37 and \$2, speaking as follows: "\$1.37 and 3¢ are \$1.40 (he pays you 3¢); \$1.40 and 10¢ are \$1.50 (he pays you 10¢); \$1.50 and 50¢ are \$2 (he pays you 50¢)."

If the store contains a cash register, before making change for the \$1.37, the salesman will press in succession the keys marked \$1, 30, and 7. The action of these keys pushes up cards showing the amount of the purchase; it also prints the amount of the sale on a slip hidden within, and opens the cash drawer so that change can be made. To prevent mistakes, it is well, in making a purchase, to observe that the sum shown on the register is correct.

The addition method of making change is used because addition is more direct and more frequently used than subtraction, and is therefore easier.

Some persons prefer to do all subtraction in the form of addition. Thus, to subtract 2475 from 5238 we might say

5 and **3** are 8 (write 3)
7 and **6** are 13 (write 6)
5 and **7** are 12 (write 7)
3 and **2** are 5 (write 2)

5238
2475
2763 *Difference*

EXERCISE 13

Show how to use the cash register for the following purchases (*i. e.*, tell what keys you would press and what sums you would pay out):

Purchase	Sum Paid	Purchase	Sum Paid
1. $1.27	$2.00	9. $3.27	$5.00
2. $1.62	$2.00	10. $4.16	$5.00
3. $ .72	$1.00	11. $6.93	$10.00
4. $ .63	$ .75	12. $1.16	$5.00
5. $ .16	$ .50	13. $7.02	$10.00
6. $ .12	$1.00	14. $ .72	$5.00
7. $ .37	$2.00	15. $3.16	$4.00
8. $ .75	$1.00	16. $10.12	$15.00

17. If you buy $3.12 worth of groceries and offer in payment a five-dollar bill, what change will you receive?

18. If you buy $1.87 worth of groceries and $2.12 worth of dry-goods and offer a five-dollar bill, what change will you receive?

19. Make up and work three examples similar to Ex. 18.

20. Let one boy or girl in the class act as cashier till a mistake is made. Let each of the other pupils make a supposed purchase, naming the amount and offering a supposed sum of money in payment. Let the cashier make change accordingly.

21. Work Exs. 1–10 of Exercise 12 by the addition method of subtracting numbers.

EXERCISE 14

1. Find the difference between the sum of $312 + $272 + $316 and the sum of $215 + $305 + $157.

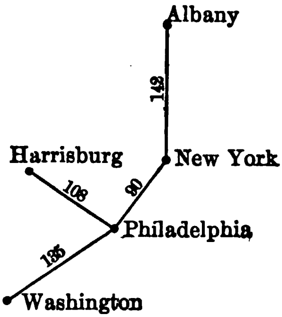
2. A dealer in grain had 3172 bushels of wheat and received at different times during the week 217 bu., 318 bu., 219 bu., and 150 bu. He sold during the week 127 bu., 50 bu., 627 bu., 140 bu., 322 bu., and 156 bu. How many bushels did he have at the end of the week?

3. A steamer in crossing the Atlantic Ocean had 2891 miles to go. After the steamer had gone 501 mi., 472 mi., 512 mi., and 496 mi. on four successive days, how many miles did it still have to go?

4. The population of a certain city on January 1 was 78,563. During one year 942 new people came to the city to live and 329 left. There were 1964 births and 1428 deaths. What was the population at the end of the year?

5. A man having $30,000 to invest bought a house for $6000 and sold it for $7500. He then bought another house for $3000, spent $1500 in repairing it, and sold it for $6000. How much money did he then have?

6. Mr. Walters had $672.90 in the bank on Monday morning. During the week on different occasions he put into the bank $127.30, took out $78.42, put in $200, took out $312, took out $37.12, put in $26.42, and put in $18.40. How much did he have in the bank at the end of the week?



7. The distance from New York to Albany by railroad is 142 miles; that from New York to Philadelphia 90 miles; from New York to Washington 228 miles; and from Philadelphia to Harrisburg 108 miles. Find the distance from

Albany to Washington by railroad. From Philadelphia to Washington. From Harrisburg to New York.

8. Draw the diagram of Ex. 7 on paper; also add a line showing the distance from Albany to Buffalo (298 mi.). Add another line showing the distance from New York to Boston (234 mi.). Make up and work five examples in addition and subtraction concerning the diagram.

On the diagram a line 1 inch long represents 200 miles. Hence the diagram is said to be *drawn to the scale* of 200 miles to the inch.

9. A man bought at a store sugar worth 36¢, coffee worth 32¢, rice worth 17¢, and cranberries worth 25¢. What change would he receive for a two-dollar bill?

10. Find the difference between the total area of the New England states and that of the Middle states.

11. What is the difference between the total area of the states which border on the Atlantic and that of the states which border on the Pacific Ocean?

12. Find the difference between the total area of the states bordering on the Great Lakes and that of the states bordering on the Gulf of Mexico.

13. Make up and work an example similar to Ex. 3. To Ex. 4. To three other examples indicated by the teacher.

14. Make up and work two examples entirely your own involving both addition and subtraction.

15. Practice rapid addition and subtraction by the use of cards containing examples prepared by the teacher. The pupils should draw these cards from a box. (See p. 391.)

Multiplication

EXERCISE 15

Review of Meanings and Uses

1. Count by 4's from 4 to 64. By 9's from 9 to 108.

Review various other countings at the discretion of the teacher.

2. Compare the work of finding the sum of $9 + 9 + 9 + 9 + 9 + 9 + 9$, with that of remembering the result of a former counting and saying "7 times 9 are 63."

3. Compare the work of writing 7 times 9 are 63, with that of writing $7 \times 9 = 63$.

4. Make up an illustration of the advantage of multiplication over addition, similar to Ex. 2.

5. Write the following arrangement of numbers on the blackboard. As the teacher points, give the product of each number in the upper row by each number in the lower.

7	1	5	9	3	8	4	2	6
3	5	8	2	1	7	9	6	4

6. Repeat the multiplication tables up to 9×9.

7. Count by 11's from 1 to 132. Recite the multiplication table for 11.

8. Count by 12's from 1 to 144. Recite the multiplication table for 12.

9. Add the following columns, shortening the work by the aid of multiplication wherever possible:

(*1*)	(*2*)	(*3*)	(*4*)	(*5*)	(*6*)
4	4	1	8	11 in.	$42
3	8	9	3	10 in.	42
8	4	2	3	9 in.	40
8	8	9	3	11 in.	42
8	4	3	8	10 in.	8
8	8	9	3	19 in.	48
8	4	9	8	19 in.	42
8	8	9	3	9 in.	40

To get the greatest benefit from the short method of adding equal numbers, we now make certain definitions and rules.

24. **Multiplication** is a short way of getting the sum of a set of equal numbers.

The **multiplicand** is one of the equal numbers to be added.

The **multiplier** is the number which indicates how many equal numbers are to be added.

The **product** is the result of the multiplication.

Factor is a short name for both multiplicand and multiplier.

The **sign of multiplication** is × and is read "times" or "multiplied by."

In 8 times 7 = 56, point out the product, multiplicand, multiplier, and factors.

Looked at in another way, multiplication is the process of taking one number as many times as there are units in another number. Then the number taken is the multiplicand, and the number which indicates how many times the given number is to be taken is the multiplier.

EXERCISE 16

1. Write 543. What does each figure in 543 stand for? Now annex a zero to the right of 543. What does each figure stand for now?

2. Annexing one zero to the right of 543 multiplies 543 by what? Annexing two zeros? Three zeros?

3. Make a rule for multiplying any number by 10. By 100. By 1000.

4. Write a two-figured number and multiply it by 10. By 100. By 1000. Treat some three-figured number in the same way.

5. Since $20 = 2 \times 10$, we have $7 \times 20 = 7 \times 2 \times 10 = 14 \times 10 = 140$. Find the value of 9×20 in like manner.

6. Multiply each of the following by 20: 6, 5, 23, 46, 112. Also by 30. By 200. By 300.

7. Make a rule for multiplying a given number by a two-figured number ending in zero.

8. Also make a rule for multiplying a given number by a three-figured number ending in two zeros.

25. Method for Multiplication.

Ex. 1. Multiply 1546 by 372.

	Work in Full		Short Form of Work	
Multiplicand	1546		1546	
Multiplier	372		372	
1st Partial Product	3092	= 2 × 1546	3092	
2d Partial Product	108220	= 70 × 1546	10822	
3d Partial Product	463800	= 300 × 1546	4638	
Entire Product	575112	= 372 × 1546	575112	*Product*

In multiplying, where do you place the multiplier? With what figure do you multiply first? Where do you set down the right-hand figure of each partial product?

To check a multiplication, *divide the product by the multiplier.* The quotient should equal the multiplicand.

The preceding form of multiplication is generally used, but often the following form is preferable. It is called *reversed multiplication.*

$$\begin{array}{rl} 1546 & \\ \underline{372} & \\ 463800 & = 300 \times 1546 \\ 108220 & = 70 \times 1546 \\ \underline{3092} & = 2 \times 1546 \\ 575112 & \textit{Product} \end{array}$$

This method of multiplication has the advantage of putting the largest partial product first; and hence of teaching us to estimate the size of a product before making the complete multiplication.

Ex. 2. Multiply 672 by 403.

$$\begin{array}{r} (1) \\ 672 \\ \underline{403} \\ 2016 \\ 000 \\ \underline{2688} \\ 270816 \end{array} \qquad \begin{array}{rl} (2) & \\ 672 & \\ \underline{403} & \\ 2016 & \\ \underline{2688} & \\ 270816 & \textit{Product} \end{array}$$

We see that since there is a zero in the tens' place of the multiplier, the product for that place is zero and should be left vacant.

When multiplying by 4, we set down 8 (the first figure of the next partial product) in the hundreds' place and proceed as usual.

EXERCISE 17

Multiply the following and check each result:

1. 58 ft. 23	**4.** 571 47	**7.** 4321 243	**10.** 9076 289	**13.** 8876 708
2. 47 in. 36	**5.** 387 52	**8.** 5413 217	**11.** 8761 305	**14.** $97.15 485
3. $254 36	**6.** 416 324	**9.** 8517 346	**12.** 7649 507	**15.** $31.37 318

16. 476 by 123

17. 834 by 309

18. 7654 by 417

19. 4328 by 876

20. 9643 by 754

21. 8845 by 675

22. 42,785 by 76

23. 92,543 by 68

24. 76,514 by 318

Multiply the following and check each result:

25. 850,706 by 94
26. 76,543 by 289
27. 60,435 by 704
28. 54,865 by 647
29. 43,672 by 564
30. 5967 by 504
31. 65,780 by 467
32. 48,030 by 325
33. 27,340 by 234
34. 74,321 by 839
35. 543,267 by 156
36. 684,039 by 783

37. If 1 mile of fence costs $478, what will 108 miles cost?

38. If 1 train carries 4328 tons, how many tons can 49 trains of the same size carry? 87 trains? 325 trains?

39. There are 5280 feet in 1 mile. How many feet are there in 37 miles? 258 miles?

40. Time yourself in working Ex. 11 on p. 39. Can you estimate about how long it would take you to write 8761 three hundred five times and add? About how many times as long would it take to work Ex. 11 by addition as by multiplication?

41. Mt. Everest is 29,003 feet high. How much more is this than 5 miles?

42. A man's salary is $225 a month. Find his salary for 2 years and 3 months.

43. If sound travels 1090 ft. per second, how far will it travel in 6 sec.? In 2 min.? 12 sec.?

44. How much are 5 loads of flour worth, if each load contains 12 barrels worth $6 a barrel?

45. Multiply 327 by 30. By 50. By 100. By 720. By 3800. By 4600.

46. Multiply 8760 by 20. By 80. By 900. By 460. By 5300. By 7100.

47. A farmer took to market 26 dozen eggs worth 18¢ a dozen, and 37 pounds of butter worth 30¢ a pound. How much money should he receive?

48. What is the cost of 43 yards of carpet at 75¢ a yard, and 62 yards of matting at 48¢ a yard?

49. A drover bought 36 calves and 44 sheep for $375, and sold the calves at $6 each, and the sheep at $5 each. What was his gain?

50. What is the value of a drove of 87 cows worth $64 each, 65 calves worth $8 each, and 79 sheep worth $5 each?

51. A miller bought wheat for $2197. From this wheat he made 476 barrels of flour, paying $622 for the grinding and $192 for the barrels. If he sold the flour at $7 a barrel, what did he make?

52. Make up and work an example similar to Ex. 38. Ex. 44.

53. Make up two examples entirely your own in which you use multiplication.

54. Practice the rapid addition of columns of figures written on the blackboard.

55. Play the Tally Game with examples in multiplication. (See p. 393.)

26. Concrete and Abstract Number in Multiplication.

Concrete number is number of some particular kind; as 5 apples, 3 hats.

General or **abstract number** is number which includes several kinds of concrete number.

Thus, "quadrupeds" includes horses, dogs, cats, but not birds or fishes. "Animals" is more general than "quadrupeds," and includes birds and fishes, as well as horses and dogs. A number such as 7 units, or 7, includes any like units, and hence is more general than "animals."

One of the uses of abstract number is that it enables us to change certain numbers so that they can be added. (See Ex. 20, p. 26.) Also by the use of abstract numbers the writing of certain expressions may be shortened.

Thus, instead of "7 units + 4 like units = 11 units of the given kind," we may write 7 + 4 = 11.

Likewise, in "7 groups of 5 apples each," *groups* is an abstract term (since we may have groups of many different kinds of objects, as of peaches, cents, etc.) and may be dropped or replaced by a symbol. Hence, the above expression may be written 7 × 5 apples.

In using abstract numbers and the shortened forms of expression, we need the following rules to prevent our making mistakes:

1. In a multiplication, *the multiplier is always abstract* (since it is a number of groups).

2. *If the number to be multiplied* (the multiplicand) *is concrete, the product is concrete number of the same kind.*

Thus, 7 × 8 apples = 56 apples.

The sign × must always be read so as to make the multiplier abstract. Thus, 7 × 8 apples is read "7 times 8 apples." But 8 apples × 7 is read "8 apples multiplied by 7."

EXERCISE 18

SIGHT WORK

Read Exs. 1–9 after the teacher has written them on the blackboard:

1. 18 × $4
2. $4 × 18
3. 9 × 5 marbles
4. 5 marbles × 9
5. 5 yd. × 320
6. $31.46 × 520
7. 1672 × 5¢
8. 8 gloves × 420
9. 3192 × 5 in.

10. When you read × as "multiplied by," does the multiplier precede or follow? When you read it as "times," where is the multiplier placed? Write a rule telling how to read ×.

11. Write in the shortest way: 8 units less 5 like units = 3 units of the given kind.

12. Write 6 groups of 8 boys each, in the shortest way.

13. Can you add 7 roses and 5 pinks? Can you add 7 flowers and 5 flowers?

14. Change 8 horses and 6 cows so that you can add them.

15. Make up and work an example similar to Ex. 14.

16. What are some of the uses of abstract number?

17. Have a Time Test with drill cards on the 45 combinations in multiplication.

27. Shortening the Work by letting the multiplier and the multiplicand change places.

Ex. 1. Multiply 9 by 824. Also 824 by 9. Count the number of figures used in each process.

$$\begin{array}{rl}
9 & \\
\times\ \underline{824} & \\
36 & \text{(14 figures} \\
18 & \quad \text{are used.)} \\
\underline{72} & \\
7416 & \textit{Product}
\end{array}
\qquad\qquad
\begin{array}{rl}
824 & \text{(8 figures} \\
\underline{9} & \quad \text{are used.)} \\
7416 & \textit{Product}
\end{array}$$

Hence,

In multiplying together a large and a small number, it saves labor to use the small number as the multiplier.

Ex. 2. What is the cost of 735 lb. of sugar at 4¢ a pound?

$$\text{Write:}\quad \begin{array}{rl}
735 & \\
\underline{4} & (¢) \\
\$29.40 & \textit{Cost}
\end{array}
\qquad\qquad \text{Read: 735 times 4¢}$$

EXERCISE 19

1. Multiply 762 × 8 in both the long and the short way and count the number of figures used in each process.

In the shortest way, find the value of:

2. 1214 × 9

3. 27 × 3762

4. $416.28 × 24

5. 762 times 2¢

6. 5 in. × 9163

7. 862 × 12 qt.

8. If the price of sugar is 5¢ a pound, find the cost of 756 pounds.

9. If the price of flour is 4¢ a pound, find the cost of 1272 pounds.

10. If 1 acre of wheat yields 23 bushels, how many bushels will 672 acres yield?

11. What is the cost of a tract of land containing 3724 acres, at $4 per acre? At $11? At $60?

12. Practice the oral addition of two-figured numbers as in Exercise 9 (p. 26).

28. Multiplication Shortened by Simple Division. By remembering simple cases of division and of fractions, multiplication by certain numbers can be much shortened.

Ex. Multiply $324 by 25.

$$25 = \frac{100}{4}. \quad \text{Hence, } \$324 \times 25 = \$324 \times \frac{100}{4} = \frac{\$32400}{4} = \$8100.$$

Hence,

To multiply a number by 25, *annex two zeros to the number and divide by* 4.

Find a similar method of multiplying by 50.

EXERCISE 20

Multiply each of the following in the shortest way:

1. $212 × 25
2. 416 in. × 25
3. 272 ft. × 25
4. $192 × 25
5. 25 × 7624
6. $812 × 50
7. 9172 × 50
8. 1612 × 50
9. 892 × 25
10. 7642 × 50
11. 25 × 7240
12. 50 × 8106

13. Find the cost of 300 suits of clothes at $25 per suit.

14. If 1 lady's coat costs $50, what will 250 coats cost? 316? 450?

15. If 1 acre of land costs $50, what will 112 acres cost at the same rate? 350 acres? 17 acres?

16. If 1 city lot costs $250, what will 32 lots cost at the same rate? 52 lots? 23 lots?

17. Find the difference between the area of New York State and 25 times the area of Rhode Island. (See p. 386.)

18. Find the difference between the area of Texas and 50 times the area of Texas.

19. Find the difference between 50 times $1268 and 25 times $2364.

20. Find the difference between 25 times $648 and 128 times $216.

21. Make up and work an example similar to Ex. 15. To Ex. 17, concerning Delaware and Alabama.

22. How many of the examples in this Exercise can you work at sight?

23. Practice oral subtraction with two-figured numbers as in Exercise 12 (p. 31).

EXERCISE 21

SIGHT WORK

Multiply at sight:

1. $\begin{array}{r} 31 \\ 4 \\ \hline \end{array}$

2. $\begin{array}{r} 27 \text{ in.} \\ 3 \quad \\ \hline \end{array}$

3. $\begin{array}{r} 113 \\ 5 \\ \hline \end{array}$

4. $\begin{array}{r} 223 \\ 2 \\ \hline \end{array}$

5. $\begin{array}{r} 1023 \\ 4 \\ \hline \end{array}$

6. $\begin{array}{r} \$21.15 \\ 4 \\ \hline \end{array}$

7. $\begin{array}{r} 1012 \\ 8 \\ \hline \end{array}$

8. $\begin{array}{r} 206 \\ 9 \\ \hline \end{array}$

9. $\begin{array}{r} \$13.12 \\ 7 \\ \hline \end{array}$

10. $\begin{array}{r} \$20.42 \\ 6 \\ \hline \end{array}$

11. If 1 yard of muslin costs 11¢, what will 8 yards cost? 5 yd.? 9 yd.? What change would be received for $1 in each case?

12. If 1 can of corn costs 13¢, what will 4 cans cost? 7 cans? 6 cans? What change for $1 in each case?

13. If 1 yard of cloth costs $1.12, what will 4 yards cost? What change for $5?

Multiply at sight:

14.	**15.**	**16.**	**17.**	**18.**
112	203	12	131	113
30	40	200	300	700

19. 106 by 20. By 300. By 60. By 120. By 900.

20. 213 by 30. By 500. By 6000. By 70. By 11.

21. Find the cost of 36 acres of land at $20 per acre. At $100 per acre. At $30.

22. If 1 watermelon costs 30 cents, what will 23 watermelons cost? 41? 112?

23. If 3 large ears of corn will make a quart of shelled corn, how many ears are needed to make a bushel? 1½ bushels? 2 bushels?

24. If 4 small ears of corn will make a quart of shelled corn, how many ears are needed to make 1¼ bushels?

25. Add in the shortest way:

(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*
4	8	7	9	32	28	64
4	8	5	5	37	78	63
4	8	5	9	57	38	62
4	3	5	5	57	35	65
5	3	5	9	32	38	60
5	8	7	9	52	55	68
5	8	7	9	37	68	69
5	8	7	5	27	35	67

26. Add at sight:

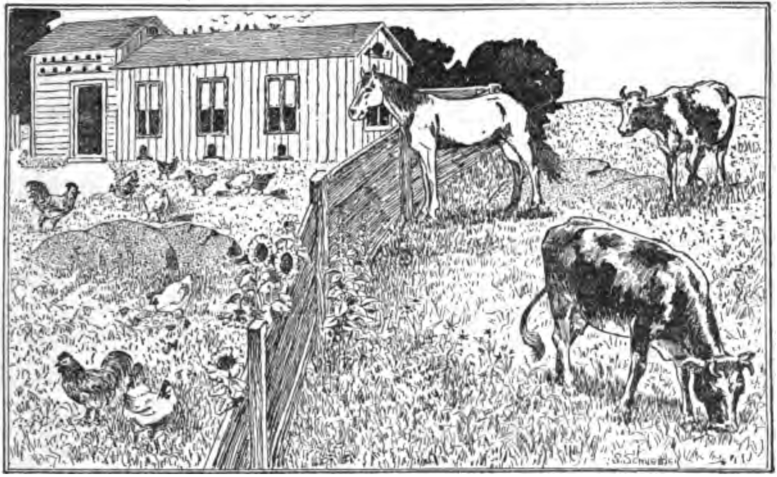
(1)	*(2)*	*(3)*	*(4)*	*(5)*
32	68	93	105	227
29	57	24	67	46

27. Subtract at sight:

(1)	*(2)*	*(3)*	*(4)*	*(5)*
48	58	97	208	372
39	37	49	23	86

EXERCISE 22

INDUSTRY REVIEW: FARM ANIMALS



1. Alice wanted to make some money with which to buy books for her library. Her mother let her have the use of the chicken house free. Alice paid for chickens at different times during the year $4.25, $8.75, $3.17, and $6.72. She paid for chicken feed at different times $1.20, $3.15, $2.14, $1.08, and $2.72. How much did she pay out all together?

2. She received for eggs sold at different times during the year 20¢, $1.12, 37¢, $2.40, 80¢, $3.60, $2.15, $1.80, $3.16, and $4.02. How much did she receive in all for eggs?

3. She received for chickens which she sold at different times $2.14, $4.08, $2.18, $1.12, 94¢, $4.18. How much did she receive for chickens sold during the year?

4. If Alice sold the chickens which she had left at the end of the year for $24.60, how much did she make during the year?

5. With what she had earned, would she be able to buy a copy of Longfellow's poems costing $1.50, a set of Walter Scott's novels costing $8.90, and a set of Hawthorne's works costing $12?

6. At the beginning of one week Alice had on hand 22 eggs. During the week at different times she found 17, found 12, sold 18, found 23, sold 12, found 6, and sold 30. How many dozen eggs did she have on hand at the end of the week?

7. If a white Leghorn hen laid 210 eggs during a year while a common hen laid only 120 eggs, how many more eggs would Alice get from 30 white Leghorn hens than from 30 common hens? What would these extra eggs be worth at 28 ¢ a dozen?

8. What would 425 pounds of poultry be worth, if sold near home at 16¢ a pound? What would it bring, if sold in the city at 20¢ a pound? If the cost of sending it to the city is $1.25, how much would be gained by sending it?

9. Let each girl in the class make a similar set of examples about keeping ducks; and each boy, about keeping pigeons. Let each tell how he would prefer to spend what he makes.

10. In six successive months a cow gave milk as follows: 850 lb., 916 lb., 923 lb., 872 lb., 724 lb., and 625 lb. How many pounds did the cow give in six months?

11. A farmer had 23 cows on January 1. During the year he bought 12 cows at one time and 8 at another, and sold 17 cows. How many cows did he have at the end of the year?

12. During one year a cow gave 26,825 pounds of milk. During the first 6 months of the year she gave 17,008 pounds. How many pounds did she give during the last half of the year?

13. If one horse weighs 1550 pounds and another weighs 1312 pounds, find the difference in their weights.

CHAPTER III

FUNDAMENTAL OPERATIONS (*Continued*)

Division

EXERCISE 23

Review of Meanings and Uses

1. Count by 7's from 63 down to 0. Review similar countings.

2. Compare the work of subtracting by 7's from 63 down to 0 and counting the number of subtractions, with that of remembering the result of a former counting and saying "there are nine 7's in 63."

3. Make up for yourself a similar illustration of the advantage of division.

4. In the multiplication table, 7 × ? = 63. Hence 7 is contained how many times in 63?

5. How many days can a man who has $24 for hotel expenses stay at a hotel charging $3 a day? Find this out (1) by successive subtractions; (2) by division. Compare the work in the two processes.

6. Recite the division tables up to 144 ÷ 12.

7. Why do we now make definitions and rules?

29. Division is a short way of finding how many times one number is contained in another; or of separating a number into equal parts.

The **dividend** is the number to be divided.

The **divisor** is the number by which we divide.

The **quotient** is the number which shows how many times the divisor is contained in the dividend.

If the division is not exact, the part of the dividend left over is called the **remainder**.

The **sign of division** is ÷ and is read "divided by." Placed between two numbers, it means that the first number is to be divided by the second.

Thus, $42 \div 7$ means that 42 is to be divided by 7.

Division is also indicated sometimes by writing the divisor under the dividend with a line between. Thus, $\frac{42}{7}$ means 42 divided by 7. The division of 42 by 7 may also be indicated thus: $7\overline{)42}$.

30. Short Division. When the divisor is not greater than 12, the process called short division is used.

Ex. Divide 882 by 6.

$$\begin{array}{r} 6\overline{)882} \\ 147 \end{array}\ \textit{Quotient}$$

Where do you place the divisor? 6 is contained in 8 (hundreds) 1 (hundred) times with a remainder of 2 (hundreds). Write 1 in the hundreds' place of the quotient. 2 hundreds + 8 tens = 28 tens.

Let the pupil complete the explanation.

To check a division, *multiply the quotient by the divisor.* The product plus the remainder, if there is any, should equal the dividend.

EXERCISE 24

Divide and check each result:

1. $4\overline{)\$768}$
2. $3\overline{)135}$ in.
3. $6\overline{)538}$ qt.
4. $9\overline{)486}$
5. $2\overline{)2198}$ yd.
6. $5\overline{)\$8065}$
7. $8\overline{)8968}$ in.
8. $7\overline{)9457}$
9. $3\overline{)\$123.45}$
10. $9\overline{)34569}$
11. $4\overline{)47128}$
12. $7\overline{)93765}$
13. $8\overline{)75800}$
14. $9\overline{)\$876.51}$
15. $11\overline{)78903}$
16. $12\overline{)85140}$

17. 716 by 4

18. 875 by 5

19. $896 by 8

20. 723 ft. by 3

21. $78.47 by 7

22. $41.74 by 2

23. 75,942 by 6

24. 6781 by 5

25. 4442 by 9

26. 45,888 by 12

27. 76,453 by 5

28. 82,354 by 4

29. 52,387 by 6

30. 63,874 by 7

31. 75,327 by 9

32. 56,230 by 10

33. 18,396 by 12

34. 65,924 by 8

35. A man died leaving $3275 to be divided equally among his 5 children. How much would each receive?

36. If 3 building lots cost $825, what is the price of one lot?

37. If an aeroplane goes 5256 yd. in 3 minutes, how far does it go in 1 minute? In 2 minutes? In 10 minutes?

38. The profits from a certain mine are $18,200 in one year. If this is divided equally among the 8 owners of the mine, how much does each receive?

39. Find the average area of the New England states.

To get the average, find the sum of the areas of these states and divide this sum by the number of the states. How would you find the average area if there were 9 states in this group?

40. How many times can $7 be subtracted from $1000? From $32,000?

41. If a man has $224 for hotel expenses, how many days can he stay at a hotel at $4 a day? How many weeks?

42. How many times can 4 be taken from 215? About how much shorter is it, to find this out by division than by successive subtractions?

43. Make up and work an example similar to Ex. 35. To Ex. 37. To Ex. 39.

44. How many of the examples in this Exercise can you work at sight?

45. Have a Time Test with drill cards on the 45 combinations in division. (See p. 392.)

31. Approximate Divisions. It is often useful to be able to find mentally the first figure of a quotient. This is also an important aid in long division.

Ex. Determine the approximate quotient of 600 ÷ 19.

$\frac{600}{20} = \frac{60 \text{ tens}}{2 \text{ tens}} = 30.$ But 19 is a little less than 20; hence, 19 will be contained in 600 a little more than 30 times. Hence, $\frac{600}{19} = 3$ tens +.

EXERCISE 25

SIGHT WORK

Find at sight the exact quotient where possible, and in other cases find the first figure of the quotient:

1 $20\overline{)600}$
2. $19\overline{)600}$
3. $21\overline{)600}$
4. $32\overline{)900}$
5. $50\overline{)1500}$
6. $51\overline{)1500}$
7. $52\overline{)1500}$
8. $70\overline{)21000}$
9. $71\overline{)21000}$
10. $69\overline{)21000}$
11. $59\overline{)16000}$
12. $300\overline{)18000}$
13. $296\overline{)18000}$
14. $412\overline{)20000}$
15. $400\overline{)2400}$
16. $403\overline{)2400}$
17. $391\overline{)2400}$
18. $700\overline{)3600}$
19. $800\overline{)63000}$
20. $798\overline{)63000}$

21. Give at sight the first figure in the quotient of each example on pp. 53, 54, from Ex. 3 to Ex. 12. From Ex. 23 to Ex. 30.

22. Work again Exercise 3 (p. 14).

32. Long Division; Divisors Containing Two Figures.

Ex. 1. Divide 3276 by 52.

$$\begin{array}{rll} & \underline{63} & \textit{Quotient} \\ \textit{Divisor}\ 52) & 3276 & \textit{Dividend} \\ & \underline{312} & (= 60 \text{ times } 52) \\ & 156 & \\ & \underline{156} & (= 3 \text{ times } 52) \end{array}$$

52 is contained in 327 (tens) 6 (tens) times with a remainder of 15 (tens). We write the **6** over the tens of the dividend.

15 tens and 6 units are 156 units.

52 is contained in 156 units 3 times with no remainder. We write **3** in the units' place of the quotient.

Observe that in long division the following five steps are used over and over:

1. *Divide* (as 52 in 327).
2. *Set down* (as 6 in quotient).
3. *Multiply* (as 52 times 6).
4. *Subtract* (as 327 − 312).
5. *Bring down* (as 6 after 15).

If in the above work we had tried 7 as the first figure of the quotient, how should we have found out our mistake? How, if we had tried 5?

Ex. 2. Divide 7898 by 19.

DIVISION	CHECK
415 *Quotient*	415 *Quotient*
Divisor 19)7898 *Dividend*	19 *Divisor*
76	3735
29	415
19	7885
108	13 *Remainder*
95	7898 *Dividend*
13 *Remainder*	

Let the pupil supply an explanation of the method of checking a division.

EXERCISE 26

Divide and check each result:

1. 2668 in. by 23
2. 738 by 21
3. 4128 by 32
4. $7092 by 59
5. 17,625 by 81
6. 28,123 by 68
7. 8756 by 62
8. 71,659 by 69
9. 90,087 by 73
10. 95,632 by 99
11. 29,602 by 82
12. 21,268 by 52
13. 8712 by 74
14. 37,654 by 26
15. 77,777 by 93
16. 85,632 by 25
17. 96,753 by 37
18. 87,009 by 87

Divide and check each result:

19. $46,725 by 15
20. $5684 by 18
21. 7863 by 21
22. 4827 by 33
23. 6972 in. by 35
24. 8253 by 47
25. 6893 ft. by 82
26. 9721 by 65
27. 23,456 by 28
28. 72,350 by 45
29. 80,854 by 84
30. 92,635 by 92

31. If 39 acres of land cost $8654, what will 1 acre cost?

32. There are 32 quarts in a bushel. How many bushels are there in 1216 quarts?

33. A man paid $6900 for a farm of 92 acres. How much did he pay per acre?

34. If 35 bushels of potatoes cost $18.45, what is the price of 1 bushel?

35. If a field of 27 acres yields 1806 bushels of corn, what is the yield per acre?

36. If an automobile travels 522 miles in 15 hours, how far will it go in 1 hour?

37. How many beeves, at $35 each, can a dealer buy with $2380? $6550?

38. The area of the United States is 3,025,600 square miles. Subtract from this the area of the District of Columbia, and find the average area of the remaining 48 states. (See p. 386.)

39. Make up and work an example similar to Ex. 32.

40. Also one similar to Ex. 36 concerning an aeroplane.

41. Practice the multiplication and division tables through 12 × 12 and 144 ÷ 12.

33. Division by Three-Figured Numbers.

Ex. 1. Divide 73,320 by 312.

```
       235
312)73320
     624
     1092
      936
      1560
      1560
```

312 is contained in 733 (hundreds) 2 (hundreds) times with 109 (hundreds) as a remainder. Hence the figure **2** is set down in the hundreds' place of the quotient. 109 hundreds + 2 tens = 1092 tens. 312 is contained in 1092 (tens) 3 (tens) times with 156 (tens) as a remainder. Set down **3** in the tens' place of the quotient.

Let the pupil complete the explanation.

Ex. 2. Divide 151,032 by 372.

```
        406
372)151032
    1488
      2232
      2232
```

The first remainder is 223. This will not contain 372, the divisor. Hence we set zero in the second place of the quotient, bring down another figure, and continue the division as usual.

EXERCISE 27

Divide and check each result:

1. $4452 by 106
2. $11,890 by 205
3. 25,232 in. by 304
4. 245,548 by 391
5. 62,543 by 587
6. 854,321 by 796
7. 246,447 by 417
8. 87,979 by 907
9. 802,392 by 998
10. 14,892 by 73
11. 51,689 by 127
12. 166,952 by 328
13. 472,131 by 627
14. 357,738 by 547
15. 204,688 by 336
16. 658,296 by 738
17. 62,542 by 827
18. 7915 by 853
19. 396,372 by 493
20. 477,442 by 938
21. 2,135,328 by 354
22. 18,146 by 683
23. 62,346 by 254
24. 70,893 by 532
25. 294,763 by 306
26. 375,426 by 521
27. 2,445,224 by 812
28. 3,568,325 by 904
29. 4,256,348 by 638
30. 5,437,502 by 743

31. There are 231 cubic inches in a gallon. How many gallons are there in 1728 cubic inches? In 10,000 cu. in.?

32. At $117 a share, how many shares of stock can be purchased for $8541?

33. The Susquehanna River is 617 miles long and the Mississippi River is 3160 miles long. The Mississippi River is how many times as long as the Susquehanna?

34. If a locomotive runs 744 miles in 1 day, how many days will it take to run 10,000 miles?

35. Given the number of acres in a tract of land and the cost of the tract, how would you find the cost of one acre? Make a rule stating how to solve such an example. Give an example and solve it by your rule.

36. Given the cost of all the bushels and the number of bushels, form a rule by which to find the cost of one bushel. Give an example.

37. Find the value of $\frac{232}{29} \times 15$. (Divide 232 by 29 and multiply the result by 15.)

Find the value of $\frac{232 \times 15}{29}$. (Multiply 232 by 15 and divide the result by 29.)

Does $\frac{232}{29} \times 15$ equal $\frac{232 \times 15}{29}$ in value?

38. Make up and work an example similar to Ex. 31. To Ex 33. To Ex. 34.

39. Play the Tally Game with examples in division.

34. Divisors Ending in Zero. In case a divisor ends in one or more zeros, the work of division may be shortened by methods shown in the following examples:

Ex. 1. Divide 8749 by 40.

$$\frac{8749}{40} = \frac{8740+9}{40} = 218\tfrac{29}{40} \quad \text{Or, more briefly:} \quad \begin{array}{l} 4\cancel{0})874|9 \\ \hline 218\tfrac{29}{40}\ \textit{Quotient} \end{array}$$

Since 8749 = 8740 + 9, we set aside 9 as part of the remainder. We then divide 8740 by 40, and add 9 to the remainder 20.

Ex. 2. Divide 8659 by 270.

$$\begin{array}{r} 32\tfrac{19}{27}\ \textit{Quotient} \\ 27\cancel{0})\overline{865|9} \\ \underline{81} \\ 55 \\ \underline{54} \\ 1 \end{array}$$

Ex. 3. Divide 7917 by 200.

$$\begin{array}{l} 2\cancel{0}\cancel{0})79|17 \\ \hline 39\tfrac{117}{200}\ \textit{Quotient} \end{array}$$

EXERCISE 28

Divide and check each result:

1. $21,468 by 60
2. 51,912 in. by 90
3. 41,237 by 40
4. 74,625 by 50
5. 31,248 by 800
6. 27,139 by 700
7. 92,367 by 110
8. 63,782 by 250
9. 676,215 by 120
10. 128,823 by 280
11. 86,400 by 2400
12. 32,517 by 1300
13. 1,331,489 by 4200
14. 2,448,700 by 4700
15. 3,924,600 by 5200

16. How many hours are there in 17,287 minutes?

17. If the distance from New York to Philadelphia is 90 miles, how many single trips between the two places can be made with a 1000-mile mileage book?

18. If 30 acres of land produce 527 bushels of wheat, how many bushels does 1 acre produce?

19. At 40 miles an hour, how long will it take a train to go from New York to Buffalo (440 miles)? From New York to Pittsburg (443 miles)? From New York to Chicago (912 miles)?

20. If 70 acres of land are worth $8470, what is 1 acre worth?

21. The Mississippi River (3160 miles) is how many times as long as the Hudson River (280 miles)?

22. Make up and work an example in division concerning the yield of 40 acres of wheat.

23. Practice making change rapidly.

The teacher should dictate examples similar to those given on p. 33.

35. Uses of Division. Division has two uses, called partition and measurement.

In **partition** we know the dividend and the number of parts or groups, and by division we determine the size of each part.

Ex. 1. If a man divides 36 cents equally among 9 boys, how many cents does each boy receive?

$$\begin{array}{r} 9\overline{)36¢} \\ 4¢ \end{array} = \text{one share}$$

In **measurement** we know the dividend and the size of one part or group, and by division we determine the number of groups.

Ex. 2. To how many boys can a man give 4 cents each, if he distributes 36 cents in all?

$$\begin{array}{r} 4¢\overline{)36¢} \\ 9 \end{array}$$

Hence, the dividend being unchanged, we have

	Given	*Required*
Partition	No. of groups	Size of 1 group
Measurement	Size of 1 group	No. of groups

If a division can be expressed as the finding of a fractional part of a number, it is partition. If not, it is a case of measurement.

In measurement, if the dividend is concrete, the divisor must be a concrete number of the same kind. In partition the quotient must be a concrete number of the same kind as the dividend.

In Exs. 17 and 18 of Exercise 28, determine whether each example is a case of measurement or partition. Also in Exs. 31–34 of Exercise 27.

EXERCISE 29

Sight Work

Divide at sight:

1. 4)$84
2. 7)$714
3. 8)1608 in.
4. 5)$225
5. 6)1326
6. 3)2181
7. 11)1122
8. 12)2436
9. 20)220
10. 30)630
11. 300)6300
12. 50)2400
13. 7)$7.14
14. 40)1640
15. 8)$33.68
16. 600)3600

17. If 3 large ears of corn will make a quart of shelled corn, how many quarts will 30 ears make? 42 ears?

18. If 3 gallons of milk are needed to make 1 pound of butter, how many pounds of butter will 12 gallons of milk make?

19. If the Panama Canal is 50 miles long and the Suez Canal 100 miles long, the Suez Canal is how many times as long as the Panama Canal?

20. About how many times is 203 contained in 4200? In 6300? In 1284?

21. About how many times is 198 contained in 4000? In 6000? In 1200?

22. About how many times is 47,000 contained in 100,000? In 250,000? In 500,000?

23. If the area of the Philippine Islands is 115,000 square miles, about how many times as large as New York State are they? As your own state? Compare the area of the Philippines with that of Texas. (See p. 386.)

24. If the Simplon Tunnel is 12 miles long, the Panama Canal is about how many times as long as the Simplon Tunnel? The Suez Canal is how many times as long as the Tunnel? (See Ex. 19.)

COMBINATIONS OF OPERATIONS; SHORTENING THE WORK

36. Order of Operations. In computing the value of a series of numbers connected by the signs +, −, ×, and ÷, it is customary to perform the operations indicated by the × and ÷ signs before those indicated by the + and − signs.

Ex. Find the value of $36 + 24 \div 12 - 4 \times 2$.

Perform the operations of multiplication and division first, thus:

$$36 + 24 \div 12 - 4 \times 2 = 36 + 2 - 8 = 38 - 8 = 30 \textit{ Ans.}$$

EXERCISE 30

In each of the following, first state the order of operations and then compute:

1.	$416 \div 52 - 4$	4.	$1000 \div 25 - 17 \times 16$
2.	$7 \times 19 \times 8 - 5$	5.	$246 + 312 - 240 \div 16$
3.	$400 - 60 \div 12$	6.	$56 \times 32 - 18 \div 6$

7. Practice the addition of columns of figures.

37. The Parenthesis sign is ( ). It is used to indicate that the numbers within the sign are to be treated like a single quantity.

Ex. 1. By aid of the parenthesis, write $374 \times 16 - 372 \times 16$ in a shorter way.

We may write it thus: $(374 - 372) \times 16$

Ex. 2. Compute $374 \times 16 - 372 \times 16$ in the long way and the short way.

LONG COMPUTATION		SHORT COMPUTATION
$\begin{array}{r}374\\ \underline{16}\\ 2244\\ \underline{374}\\ 5984\\ \underline{5952}\\ 32 \textit{ Difference}\end{array}$	$\begin{array}{r}372\\ \underline{16}\\ 2232\\ \underline{372}\\ 5952\end{array}$	$\begin{array}{r}374\\ \underline{372}\\ 2\\ \underline{16}\\ 32 \textit{ Difference}\end{array}$

Notice that the second computation requires less than one fourth the labor in the first, and that in it we are much less likely to make a mistake.

38. The Law of the Parenthesis. In solving an example containing a parenthesis, *the operations within the parenthesis are to be performed first.*

Ex. 3. Compute $200 - (63 + 28)$.

Adding 63 and 28 first, since they are inside the parenthesis, we have $200 - (63 + 28) = 200 - 91$, or 109 *Ans.*

EXERCISE 31

1. On squared paper mark out a rectangle 5 spaces long and 3 spaces wide. Next to this, mark out another rectangle 2 spaces long and 3 spaces wide. By uniting these rectangles, show that $5 \times 3 + 2 \times 3 = (5 + 2) \times 3$.

For a picture of squared paper see p. 134.

2. Also show that $7 \times 5 - 3 \times 5 = (7 - 3) \times 5 = 4 \times 5$.

3. Make up and work an example similar to Ex. 1. Also an example similar to Ex. 2.

Write the following in a briefer form by aid of the parenthesis. Notice how you save time by using the short way of writing each expression.

4. $7 \times 5 + 12 \times 5$

5. $\$482 \times 18 - \$470 \times 18$

6. $672 \div 24 - 648 \div 24$

7. $\frac{672}{24} - \frac{648}{24}$

8. $926 \times 5 + 842 \times 5 - 1727 \times 5$

9. 100 less the sum of 27 and 32

Write in a briefer form by aid of the parenthesis:

10. The sum of $27 and $146 less the sum of $22 and $14.

11. The sum of 327 and 116 multiplied by their difference.

12. The sum of 327 and 116 divided by their difference.

13. Compute Ex. 5 both in the long way and in the short way. State about what fraction of the work is saved by using the short method. Is there any other advantage in using the short method?

14. Treat Ex. 6 in the same way:

15. Compute Exs. 7–12.

In the following examples indicate the work by use of the parenthesis and then compute in the short way:

16. A dealer bought 3726 bushels of wheat at 83¢ per bushel and sold it at 85¢ per bushel. How much did he make on the transaction?

17. A field of 12 acres in a certain year yielded 270 pounds of cotton per acre. The next year it was planted with selected cotton seed and yielded 412 pounds per acre. Find the increased yield on the whole field.

18. Using abbreviations, see if you can write the tables for liquid, dry, weight, time, square, and cubic measures in 10 minutes.

EXERCISE 32

Oral Work

Compute at sight:

1. $7 \times 15\text{¢} + 3 \times 15\text{¢}$

2. $4 \times 15\text{¢} + 16 \times 15\text{¢}$

3. $7 \times 15\text{¢} - 3 \times 15\text{¢}$

4. $18 \times 12\text{¢} - 8 \times 12\text{¢}$

5. $17 \times 27\text{¢} + 13 \times 27\text{¢}$

6. $17 \times 27\text{¢} - 13 \times 27\text{¢}$

7. $30 - (4 \times 7)$

8. $30 - (4 + 7)$

9. $100 - (9 \times 8)$

10. $30 + (4 \times 7)$

11. If a dealer buys 50 melons at 18¢ each and sells them at 20¢ each, what is his gain?

12. If a man buys 9 dozen oranges at 15¢ a dozen and sells them at 21¢ a dozen, what is his gain?

13. If I buy 21 acres of land at $85 per acre and sell the same at $93 per acre, how much do I gain?

14. If a boy buys 23 quarts of chestnuts at 5¢ a quart and sells them at 10¢ a quart, how much does he gain?

EXERCISE 33

REVIEW

1. Subtract 27,816 from 100,000.

2. Eva has saved $113. How much more will she need to save before she has $200?

3. Multiply 8165 by 247. By 307. By 798.

4. How many times can $213 be subtracted from $1000?

5. How many times will Mr. X need to put $215 in the bank before he has $2000 there?

6. The product is 1,528,550 and the multiplier is 475. What is the multiplicand?

7. The dividend is 284,888 and the quotient is 596. What is the divisor?

8. The product is 8000 and the multiplicand is 400. What is the multiplier?

9. The divisor is 29, the quotient is 37, and the remainder is 12. Find the dividend.

10. Mr. Carter bought a house for $10,000. He paid $1000 down and the rest in 6 equal installments. How large was each installment?

11. In 76,254, what is the value of each figure? How many items of work does each figure do?

12. Express each of the following in the Roman notation: 27; 301; 156.

13. Mr. X built a telephone line from his farmhouse to the railroad station 4 miles distant. How many poles did he use, if the poles were placed 150 feet apart?

Compute each of the following in the shortest way:

14. $224 × 25

15. 824 × 5 in.

16. $\frac{287}{15} \times \frac{613}{15}$

17. $212 \times 15 - 210 \times 15$

18. $117 \times 5 + 113 \times 5 + 70 \times 5$

19. $\frac{324}{4} - \frac{300}{4}$

20. If 8 acres of land cost $216, what will 1 acre cost? 7 acres?

21. Find the difference between $200 and the value of 9 tons of hay worth $15 a ton.

22. A boy went to a store and bought 4 pounds of sugar at $.06 a pound and 2 dozen eggs at $.42 a dozen. What change did he receive for $5?

23. On 12 acres of land, without the use of fertilizer, a farmer raised a crop of 18 bushels of corn per acre worth 70 cents a bushel. On a like field of 12 acres, he used 9 loads of fertilizer costing $2.10 a load and raised a crop of 32 bushels of corn worth 70 cents a bushel. What did he gain on the second crop by use of the fertilizer? Did the fertilizer have any further value besides its benefit to this one crop?

24. John has 65 marbles, and Harry and William each have 9 times as many as John. How many marbles do all have together?

25. A man earns $1200 a year; his expenses per year are $978. In how many years can he save $2000?

26. A farmer, to prevent blight of the vines, sprayed a 40-acre field of potatoes with Bordeaux mixture at a cost of $4 per acre.

The yield on this field exceeded that on an adjoining field by 60 bushels per acre. If the potatoes were worth 65 cents per bushel, how much did he gain by the spraying?

27. If an ordinary gas burner uses 7 cubic feet of gas per hour, and a Welsbach burner uses 4 cubic feet per hour, what will be the difference in the amount of gas consumed by 12 burners in 15 hours? (Point out the long and the short way of making this calculation.)

28. A man bought three houses. He paid for the first $5260, for the second $3585, and for the third as much as for both the others. He sold the three houses for $15,979. How much did he gain or lose?

29. Make up and work an example similar to Ex. 10. To Ex. 11. To Ex. 25.

30. Have a Relay Race with examples in addition, subtraction, multiplication, and division. (See p. 393.)

Let the class drill in the process in which they most need practice.

Use of the Telegraph

39. **The Cost of Sending a Telegram** varies with the number of words in the telegram. Thus, for a short distance the cost is 25¢ for the first ten words and 2¢ for each additional word; or put in a brief form the rate is expressed as 25−2.

The sign @ means *at the rate of*, or *at*. Thus, "at the rate of 25−2" may be shortened to "@ 25−2."

When a telegram is paid for, the words in the address and signature are not regarded as part of the message.

The cost of sending a telegram varies also with the distance. Thus, the rate from New York to San Francisco is about double what it is from New York to Chicago.

The cost of sending a telegram may vary also according to the time of day at which the telegram is sent. Night rates are usually less than day rates. Thus, the day rate in a certain case is 30−2, while the night rate is 25−1. Why should the night rate be less?

A *night letter* is a telegram sent at night at a special rate. The cost for the first 50 words is the same as that for a day message of 10 words; for every 10 words above 50, there is an additional charge equal to $\frac{1}{5}$ of the rate for 50 words. A *day letter* may also be sent under certain conditions.

EXERCISE 34

Find the cost of a telegram containing

1. 17 words @ 50−3
2. 10 words @ 100−7
3. 15 words @ 60−4
4. 23 words @ 40−3
5. 23 words @ 25−2
6. 13 words @ 30−2
7. 19 words @ 25−1
8. 16 words @ 75−5
9. 7 words @ 60−4
10. 17 words @ 100−7

11. Find the cost of telegraphing from your home to New York City, and also to other important cities. Make up and work five examples concerning these rates.

12. Get a telegraph blank and write on it the following telegram. Compute the cost of the telegram at 50 – 3.

9 A. M. BELLEPLAIN, MD., Dec. 18, 1911.
To Mr. ALFRED HOOPER,
17 Washington Ave., St. Louis, Mo.
Vacation begins a day early. Will be home to-morrow night. Meet me eight forty evening at station.
ELSIE HOOPER.

13. Now write the message of Ex. 12 in ten words. How much money will this save?

14. Write five other telegrams and compute their cost at rates selected from those used in Exs. 1 – 10.

15. Find the cost of a night letter at a 50 – 3 rate, if the letter contains 50 words. 60 words. 83 words. 35 words.

16. By sending a telegram to Chicago, a miller in Louisville was able to buy 1000 bushels of wheat 2¢ a bushel cheaper than he could if he had ordered the wheat by letter. If the telegram cost 40¢, how much did he save by the use of the telegraph?

17. If a person's life is saved by calling a surgeon from a distant city by a telegram, can we measure in money the value of the telegraph in such a case?

18. Name other instances in which the value of the telegraph is so great that it cannot be measured in money.

CHAPTER IV

FACTORING AND CANCELLATION

EXERCISE 35

Meanings and Uses

1. What two numbers multiplied together make 6? What are the factors of 6? What numbers will divide 6 without a remainder?

2. What are the factors of 15? Of 21? Of 35?

3. Name the factors of 49. Of 16. 24. 45.

4. Tell what numbers will divide 14 without a remainder. 63. 27. 56.

5. What numbers will divide 21 without a remainder?

6. Name some number which has no exact divisor but itself and 1.

7. Name all such numbers between 1 and 10. Between 10 and 20. Between 20 and 30.

8. Tell which of the following numbers have no exact divisor except the number itself and 1:

17 26 33 31 47 65 23 83 93

9. Compare the work of writing $\frac{12}{18}$, with that of writing $\frac{2}{3}$. How do we change $\frac{12}{18}$ into $\frac{2}{3}$?

10. Compare the work of dividing 1638 by 78, with that of dividing 273 by 13.

11. $18 is what part of $27? Does a knowledge of the factors of numbers help us in making comparisons like this?

12. Why do we now make definitions and rules?

40. The **factors** of a number are the numbers which, when multiplied together, produce the given number.

Thus, 7 and 3 are the factors of 21.

The word *factors* means *makers*. In one sense the factors of a number are the makers of the number.

A **prime number** is a number which has no exact divisor except itself and 1.

Name four prime numbers.

Numbers are said to be *prime to each other* when they have no common factor.

Thus, 10 and 21 are prime to each other. They are not prime numbers however.

A **composite number** is a number which has exact divisors besides itself and 1.

A **prime factor** is a factor which is itself a prime number.

Thus, the factors of 15 are 3 and 5; and since 3 and 5 are prime numbers, they are prime factors. 9 and 4 are factors of 36, but they are not prime factors.

Factoring is the process of separating a number into its prime factors.

An **even number** is a number which is exactly divisible by 2.

Name four even numbers.

An **odd number** is a number which is not exactly divisible by 2.

Name four odd numbers.

41. Uses of Factoring. (1) Factoring saves labor in multiplication and division by enabling us to make the numbers smaller by removing a factor common to both divisor and dividend. (See Ex. 10, p. 68. Make up and work a similar example.)

(2) Factoring saves labor in the solution of problems involving comparison by making the comparisons simpler.

(3) Factoring saves labor in the use of fractions by reducing fractions to simple forms.

EXERCISE 36

AIDS IN FACTORING

1. Write five even numbers. Also write five odd numbers.

2. Make a list of all the even numbers between 40 and 60. The last figure of each of these numbers is divisible by what small number? Is the last figure of any odd number divisible by 2?

3. Compare the work of dividing 153,278 by 2, with that of dividing 8 by 2. Will the division of 8 by 2 show you that 153,278 is exactly divisible by 2?

4. Make a list of all the numbers between 60 and 100 which are exactly divisible by 5. The last figure of each of these numbers is one of what two figures?

5. Tell which of the following numbers are exactly divisible by 5: 225, 312, 310, 590, 503, 1782, 65.

Hereafter in this chapter we shall shorten "exactly divisible" to "divisible."

6. Add the figures of 27. Is this sum divisible by 3? Is 27 divisible by 3?

Add the figures of 81. Is this sum divisible by 3? Is 81 divisible by 3?

Add the figures of 46. Is this sum divisible by 3? Is 46 divisible by 3?

7. Make a list of all numbers between 80 and 100 which are

divisible by 3. Is the sum of the figures of each of these numbers divisible by 3?

8. Test in like manner 5217, 1011, 234,672, and 20,139.

9. Compare the work of dividing 7281 by 3, with that of finding the sum of the digits 7, 2, 8, 1, and dividing that sum by 3.

42. Tests of Divisibility by 2, 3, and 5; Prime Factors. We have the following aids or tests in trying to factor a given number:

A number is divisible by 2, *if the last figure is an even number or* 0;
By 3, *if the sum of the figures is divisible by* 3;
By 5, *if the last figure is* 0 *or* 5.

To determine the prime factors of a number, *determine the smallest exact divisor of the given number, then the smallest exact divisor of the quotient,* and so on.

Ex. What are the prime factors of 2520?

$$\begin{array}{r} 2\overline{)2520} \\ 2\overline{)1260} \\ 2\overline{)630} \\ 3\overline{)315} \\ 3\overline{)105} \\ 5\overline{)35} \\ 7 \end{array}$$

$2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$, *Factors*

Since the last figure of 2520 is 0, the number is divisible by 2. In like manner, the quotient, 1260, is divisible by 2, as is the next quotient, 630. Since 315 is not divisible by 2, we test it to find whether it is divisible by 3, and find that it is, since the sum of its figures, 9, is divisible by 3. Similarly, 105 is divisible by 3. The smallest divisor of the next quotient, 35, is 5. Hence the factors of 2520 are 2, 2, 2, 3, 3, 5, 7.

EXERCISE 37

1. In the following list of numbers tell wnich are prime:
6 9 5 4 3 15 13 18 7 11 24 2 1 16

2. Make a list of all the prime numbers between 0 and 25.

Find the prime factors of:

3. 12 18 20 24 28 32 36
42 48 50 56 60 64 66

4. 70 72 76 80 81 84 88
90 92 96 98 99 100 1224

Tell which of the following are divisible by 2. By 3. By 5.

5. 46 85 72 428 936 1234 1235 1236 4625

6. 7131 4685 7779 34,275 916,674 712,446

7. What even number is also a prime number?

8. Name three factors of 24 which are not prime.

9. Separate each of the following into two factors in all possible ways:

12 16 24 20 18 16 30 36 40 32

10. On squared paper mark out a rectangle 4 spaces long and 3 spaces wide. Also a rectangle 6 spaces long and 2 spaces wide. By use of these rectangles, show that $4 \times 3 = 6 \times 2$.

11. In like manner, show that $8 \times 3 = 12 \times 2$.

12. "2 times 5" is contained in "6 times 5" how many times? Show this by splints. Show it also on squared paper.

13. $\frac{6 \times 5}{2 \times 5} = ?$ $\frac{6}{2} = ?$ Does $\frac{6 \times 5}{2 \times 5} = \frac{6}{2}$?

14. Make up and work an example similar to Ex. 12.

15. State your example in the form given in Ex. 13.

16. 2 nickels are contained in 6 nickels how many times? Show this with coins. Does $\frac{6 \times 5}{2 \times 5} = \frac{6}{2}$?

43. Cancellation is a way of shortening work by dividing a dividend and its divisor by the same number.

Striking out the same factor from both the divisor and the dividend does not alter the quotient.

Ex. Divide 391 × 5 by 17 × 5.

$$\frac{391 \times 5}{17 \times 5} = \frac{391}{17} \qquad \begin{array}{r l} 23 & \textit{Quotient} \\ 17\overline{)391} & \\ \underline{34} & \\ 51 & \\ \underline{51} & \end{array}$$

44. The Golden Rule of Arithmetic. So important is cancellation as a means of saving labor that the following has been called the Golden Rule of Arithmetic:

In examples involving only multiplication and division, first use cancellation as far as possible.

Ex. 1. Find the value of $\frac{6 \times 10}{20}$.

$$\frac{\overset{3}{\cancel{6}} \times \cancel{10}}{\underset{\cancel{2}}{\cancel{20}}} = 3 \textit{ Ans.}$$

We first divide both 10 and 20 by 10. We then divide both 2 and 6 by 2 and get the quotient 3.

Ex. 2. If 6 horses cost \$450, what will 15 horses cost?

We first indicate the work and then cancel.

Cost of 6 horses = \$450

Cost of 1 horse $= \frac{\$450}{6}$

$$\text{Cost of 15 horses} = \frac{\$450 \times 15}{6} = \frac{\overset{\$225}{\cancel{\$450}} \times \overset{5}{\cancel{15}}}{\underset{\cancel{2}}{\cancel{6}}} = \$225 \times 5 = \$1125 \textit{ Ans.}$$

EXERCISE 38

Compute the following by cancellation. Also work Exs. 1, 2, and 3 without cancellation, and in each of these examples compare the amount of work by the two processes.

1. $\frac{15 \times 7}{5 \times 7}$
2. $\frac{\$15 \times 12}{\$5 \times 12}$
3. $\frac{9 \times 40}{18}$
4. $\frac{9 \times 250}{6}$
5. $\frac{9 \times 250}{12}$
6. $\frac{8 \times 25}{4 \times 5}$
7. $\frac{9 \times 150}{4 \times 5}$
8. $\frac{18 \times 30}{12 \times 5}$
9. $\frac{60 \times 75}{25 \times 12}$
10. $\frac{1365}{21 \times 5}$
11. $\frac{2850}{25 \times 5}$
12. $\frac{3236}{16 \times 16}$
13. $\frac{28 \times 56 \times 15}{14 \times 5 \times 3}$
14. $\frac{112 \times 40 \times 18}{56 \times 3 \times 4}$
15. $\frac{48 \times 72 \times 20}{48 \times 15 \times 7}$
16. $\frac{54 \times 36 \times 25}{45 \times 7 \times 30}$
17. $\frac{176 \times 15 \times 12}{9 \times 5 \times 44}$
18. $\frac{14 \times 15 \times 56}{8 \times 45 \times 7}$

19. If 21 acres yield 462 bushels of wheat, what will 1 acre yield? What will 32 acres yield?

20. A farmer exchanges 4 tons of hay worth $15 a ton for coal worth $6 a ton. How many tons of coal will he receive?

21. If 8 horses cost $1800, what will 7 horses cost?

22. If a railroad train goes 29,725 miles in 41 days, how far will it go in 85 days?

23. 8 periods of 45 minutes each are equal to how many hours?

24. How many dozen eggs at 25¢ per dozen must be given in exchange for 20 pounds of sugar at 5¢ per pound?

25. Make up and work an example similar to Ex. 19. To Ex. 20. To Ex. 23.

26. What is the Golden Rule of Arithmetic? Give an example of its use.

Divide, using factoring and cancellation:

27. 12 × 15 × 77 by 55 × 14

28. 15 × 18 × 35 by 63 × 25

29. 18 × 22 × 25 by 99 × 10

30. 24 × 270 by 18 × 30 × 4

31. Divide the product of 48, 55, and 63 by the product of 88, 30, and 7.

32. A rectangle 12 spaces long and 10 spaces wide is how many times as large as one 5 spaces long and 4 spaces wide? Find this out by factoring and cancellation. Verify your result on squared paper.

33. Make up an example similar to Ex. 32.

34. In 6 days Harry gathered 104 quarts of blackberries. He sold these at 9¢ a quart. How much did he make per day? (Use factoring and cancellation.)

35. How many of the examples in this Exercise can you work at sight?

36. Draw examples in addition from a box and see how many you can work correctly in 15 minutes.

EXERCISE

ORAL WORK: COMPARISON

Cancellations may often be performed mentally.

1. If 3 pounds of sugar cost 20 cents, what will 6 pounds cost?

6 pounds are twice 3 pounds. Hence, 6 pounds will cost twice 20 cents, or 40 cents, *Ans.*

2. If 4 lead pencils cost 25 ¢, what will 8 lead pencils cost? 12 pencils? 16?

3. If 6 car tickets cost 25¢, what will 12 car tickets cost? 18 tickets? 24? 30? 36?

4. If 2 cans of tomatoes cost 25¢, what will 6 cans cost? 12 cans?

5. If 12 oranges cost 25¢, what will 36 oranges cost? 60 oranges? 4 dozen oranges?

6. We have 35 apple trees in our orchard. I have gathered 11 bushels of apples from 5 trees. How many bushels will all the trees produce at the same rate?

7. If 100 oysters cost 60¢, what will 50 oysters cost? 75 oysters?

8. If 6 oranges cost 15¢, what will 4 oranges cost?

9. If 4 lead pencils cost 25¢, how many pencils can be bought for 50¢? For $1?

10. If 2 cans of tomatoes cost 25¢, how many cans may be bought for 50¢? For 75¢? For $1?

11. If 8 feet of wire cost 10¢, how many feet of wire can be bought for 20¢? For 50¢? 80¢?

12. If 3 spools of thread cost 10¢, how many spools can be bought for 30¢? For 40¢? 50¢?

13. If 4 hats cost $5.20, what will 8 hats cost?

14. If a train runs 610 miles in 14 hours, how far will it run in 56 hours at the same rate?

15. Practice rapid addition of columns of figures.

Let the teacher dictate examples similar to those in Exercise 9, p. 26.

GREATEST COMMON DIVISOR

EXERCISE 40

1. What are the factors of 4? Of 6?

2. What factor have 4 and 6 in common?

3. What common factor have 9 and 12? 24 and 27?

4. What common factor have 4, 6, and 8? 24, 27, and 36?

5. Name three different common factors of 12 and 18. Which of these common factors is the largest?

6. What three numbers will divide both 42 and 360? If we are dividing 360 by 42, which of these three numbers will enable us to save most labor by the use of cancellation?

7. What is the largest number that will divide both 12 and 18? What is the greatest common divisor of 20 and 30? Of 18, 24, and 30?

8. What is the largest common measure of 8 in. and 10 in.? Show this on the foot rule.

9. What is the largest common measure of 6 spaces and 9 spaces? Show this on squared paper.

10. What is the largest common measure of $2\frac{1}{2}$ bushels and 3 pecks?

45. A common factor, or **common divisor**, of two or more numbers is a number which will exactly divide the given numbers.

Give an example of a common divisor of two numbers.

The **greatest common divisor** of two or more numbers is the greatest number that will exactly divide all the given numbers.

"Greatest common divisor" is often shortened to G. C. D.

Name a common divisor of 12, 18, and 24, which is not the greatest common divisor of the numbers.

Now name the greatest common divisor of 12, 18, and 24.

46. Uses of the G. C. D. A knowledge of the G. C. D. of two or more numbers aids us to make the best use of factoring, as in reducing a fraction to its lowest terms. (See Art. 59, p. 94.)

47. Method of Finding the G. C. D. of two or more numbers.

Arrange the given numbers in a row and divide by any number greater than 1 *that is exactly contained in all the given numbers;*

In like manner, divide all the quotients obtained, till there is no number which is exactly contained in all the quotients;

The product of all the divisors will be the G. C. D.

Ex. Find the G. C. D. of 42, 63, 105.

$$\begin{array}{r|rrr} 3 & 42, & 63, & 105 \\ \hline 7 & 14, & 21, & 35 \\ \hline & 2, & 3, & 5 \end{array}$$

$3 \times 7 = 21$ G. C. D.

We first divide all the numbers by 3, obtaining 14, 21, 35 as quotients. Each of these can be divided by 7, giving 2, 3, 5 as quotients. There is no number larger than 1 which will divide 2, 3, 5. Hence, we have 3 × 7, or 21, as the G. C. D.

EXERCISE 41

Find the G. C. D. of

1. 12, 42, 54
2. 15, 21, 30
3. 18, 48, 56
4. 24, 60, 84
5. 10, 90, 110
6. 27, 99, 117
7. 60, 105, 165
8. 36, 84, 108
9. 175, 325, 450

10. Find the G. C. D. of 4 quarts and 6 quarts. (Show the meaning of your result by actual measurement of sand or water.)

11. What is the G. C. D. of 6 splints and 9 splints?

12. Make up and work an example similar to Ex. 10. To Ex. 11.

13. What is the largest measure which could be used in measur-

ing exactly the contents of three tanks containing 30 gal., 28 gal., and 24 gal. respectively?

14. What is the length of the longest equal pieces into which you can cut three ribbons measuring 30 in., 3 ft. 6 in., and 4 ft.?

15. What are the largest equal bills which can be used in paying three debts of $20, $35, $40?

16. Two schools of 48 boys and 64 boys are to be divided into equal companies of the largest possible size. How many boys will there be in each of these companies?

17. What is the largest measure which can be used exactly in emptying tanks containing 12 gal., 18 gal., and 24 gal.? How many times would the measure be used in emptying the three tanks?

18. Name two numbers of which 3 is the G. C. D. Of which 5 is the G. C. D.

19. Name three numbers of which 4 is the G. C. D. Of which 5 is the G. C. D.

20. What is the greatest number which will divide 10, 20, and 35 exactly?

21. Name three numbers of which 4 is the G. C. D. Of which 2 oz. is the G. C. D. 3 sq. in.

22. Make up and work an example similar to Ex. 13. To Ex. 14. To Ex. 15. To Ex. 20.

23. How many of the examples in this Exercise can you work at sight?

24. Draw examples in subtraction from a box and see how many you can work correctly in 15 minutes.

LEAST COMMON MULTIPLE

48. A **multiple** of a number is a number which is exactly divisible by the given number.

Thus, 12 is a multiple of 3.

A **common multiple** of two or more numbers is a number which is exactly divisible by all of the given numbers.

Thus, 12 is a common multiple of 2 and 3.

The **least common multiple** of two or more numbers is the least number which is exactly divisible by all of the given numbers.

Thus, 6 is the least common multiple of 2 and 3.

"Least common multiple" is often shortened to L. C. M.

EXERCISE 42

1. Give all the multiples of 2 from 2 to 20. Of 3 from 3 to 20.

2. Give the first five multiples of 4. Also of 6.

3. Is there any number which occurs in both of the lists obtained in Ex. 2? What multiple have 4 and 6 in common?

4. What number will contain both 4 and 6 exactly as divisors? 8 and 12? 6 and 8? 12 and 18? 3 in. and 6 in.?

5. What number will exactly contain 4, 6, and 8 as divisors? 3 in., 5 in., and 6 in.?

6. What is the least number which is exactly divisible by both 4 and 6? Name some other number which is exactly divisible by both 4 and 6.

7. What is the least number which is exactly divisible by 4, 6, and 8?

8. By 5, 10, and 15? By 6, 9, 12? By 4, 8, 10?

9. On the foot rule show that 2 in. is contained an exact num-

ber of times in 6 in. Also that 3 in. is contained an exact number of times in 6 in. Is 6 in. a common multiple of 2 in. and 3 in.?

10. Make up and work an example similar to Ex. 9.

49. Uses of the L. C. M. By using the L. C. M., we save labor in reducing fractions to the same unit, and hence in adding fractions. (See Art. 61, p. 99.)

Other uses of the L. C. M. are shown in Ex. 19–21, p. 82, and similar examples.

50. Method of Finding the L. C. M. To find the least common multiple of two or more numbers, find all the factors which each number contains and multiply these factors together, taking each factor as many times as it occurs in any one number. To do this,

Arrange the given numbers in a row;

Divide by any prime number that will divide at least two of them, bringing down each undivided number along with the quotient;

Continue the process until there is no number which will exactly divide any two of the quotients;

The L. C. M. will be the product of all the divisors and the final quotients.

Ex. Find the L. C. M. of 14, 20, 21, 35.

$$\begin{array}{r|rrrr} 2) & 14, & 20, & 21, & 35 \\ \hline 7) & 7, & 10, & 21, & 35 \\ \hline 5) & 1, & 10, & 3, & 5 \\ \hline & 1, & 2, & 3, & 1 \end{array}$$

Since 2 will divide both 14 and 20, we divide by it, bringing down 21 and 35 undivided. Since 7 will divide 7, 21, and 35, we divide by it, bringing down 10 undivided. Similarly we divide by 5, bringing down 3.

The divisors and final quotients include every factor of 14, 20, 21, and 35, and they include each factor as many times as it occurs in any one number. Hence, the L. C. M. = 2 × 7 × 5 × 2 × 3, or 420 *Ans.*

The process may be shortened by striking out any number which is exactly contained in any other number in a given set. Thus, the L. C. M. of 7, 10, 14, 20 is the same as the L. C. M. of 14 and 20, because 7 is contained in 14, and 10 in 20.

EXERCISE 43

Find the L. C. M. of

1. 12 and 18
2. 9 and 21
3. 40 and 180
4. 72 and 132
5. 8, 12, 18
6. 9, 15, 35
7. 66, 42, 77
8. 48, 78, 80
9. 4, 10, 16, 25
10. 18, 15, 40, 72
11. 7, 12, 15, 40
12. 6, 7, 8, 9, 10

13. How can we most quickly determine the L. C. M. of three prime numbers, as of 3, 5, and 7? Of three numbers, no two of which have a common factor, as of 10, 21, 11?

14. On squared paper mark off an oblong which is the L. C. M. of 3 small square spaces and 4 small square spaces.

15. Also of 2 square spaces and 5 square spaces.

16. Also of 6 square spaces and 8 square spaces.

17. Make up and work a similar example.

18. Find the L. C. M. of 30 minutes and 50 minutes.

19. Trolley cars leave a central point in a certain city on one route every 30 minutes, and on another route every 40 minutes. If a car leaves on each route at 2 o'clock, how many minutes after that will it be before cars leave again on the two routes at the same time?

SUGGESTION. Find the L. C. M. of 30 minutes and 40 minutes.

20. A car also leaves on a third route at 2 o'clock and every 50 minutes. At what time after 2 o'clock will cars leave on all three routes together?

SUG. Find the L. C. M. of 30, 40, and 50 minutes.

21. Make up and work an example similar to Ex. 19.

22. Find the L. C. M. of the numbers below the line (i. e. the denominators) of the fractions $\frac{5}{6}$, $\frac{3}{8}$, $\frac{1}{12}$.

23. Find the smallest number which will contain both 4 and 6. 3 in. and 4 in.

24. Name two numbers of which 18 is the L. C. M. Name three numbers.

25. Name two numbers of which 12 inches is the L. C. M. Name three numbers.

26. Name two numbers whose product is also their L. C. M. Name three numbers of which the same is true.

27. How many examples in this Exercise can you work at sight?

28. Draw examples in multiplication from a box and see how many you can work correctly in 15 minutes.

EXERCISE 44

REVIEW

1. Find the G. C. D. of 6 in. and 9 in. Also their L. C. M. By aid of the foot rule, show the meaning of each of these results.

2. Find the G. C. D. and the L. C. M. of 12 and 32.

3. Find the G. C. D. and the L. C. M. of 10, 35, 60.

4. A boy by working 7 hours a day for 5 days picked 63 quarts of huckleberries. He sold them for 10 cents a quart. How much did he earn per hour? (Work this problem first without factoring and cancellation, and afterward by the aid of these processes. Compare the work in the two processes.)

5. Trolley cars leave the city hall every 40 minutes on one route and every 45 minutes on another route. If a car leaves on each route at 12 o'clock, how long will it be before cars start again at the same time on the two routes?

6. Find the sum of all the prime numbers between 60 and 100.

7. Multiply one thousand one by one hundred one.

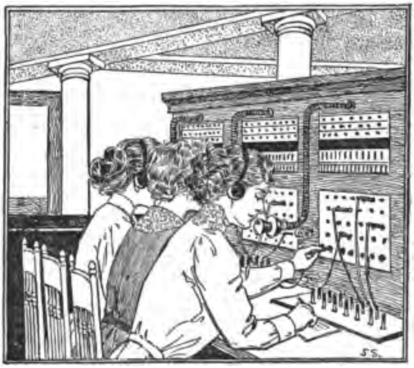
8. A ribbon 1 ft. 6 in. long and another 2 ft. 4 in. long are to be cut into the largest possible pieces all of equal length. Find the length of these pieces. Have you found a G. C. D. or an L. C. M.?

9. Make up and work an example similar to Ex. 3.

10. Also an example similar to Ex. 4.

11. Draw examples in division from a box and see how many you can work correctly in 15 minutes.

Let the pupils make up examples and write them on cards.



USE OF THE TELEPHONE

51. **The Cost of Long-Distance Telephoning** is usually a certain sum for the first three minutes, with an additional charge for every minute the telephone is used after that. Thus, in a certain case the charge is 75 cents for the first three minutes and 25 cents for each minute after that. This charge is written briefly 75 – 25.

EXERCISE 45

1. What will it cost to talk over a telephone for 10 minutes @ 75–25?

2. How much will it cost to talk from 10.53 A. M. to 11.05 A. M. @ 75–25?

Find the cost of telephoning for

3. 12 min. @ 40 − 10
4. 8 min. @ 20 − 5
5. 7 min. @ 100 − 25
6. 9 min. @ 60 − 15
7. 5 min. @ 40 − 10
8. 4 min. @ 150 − 40

9. A trip from home to a certain city would cost a business man $30 for railroad expenses and $4 for a hotel bill. Instead of making the trip, he uses a telephone for 15 minutes at the rate of $5 − $1.50. How much money does he save? Is there any other advantage to him in the use of the telephone, besides the difference in money?

10. What is the cost of the use of a telephone for three months at $18 a month, with extra charges of $2.40, $5.15, and $4.80 in the three months respectively?

11. Which costs more: to hire an errand boy at $6 a week for the 52 weeks in a year, or to pay $60 for the use of a telephone? What can the errand boy do which the telephone cannot do? What can the telephone do which the errand boy cannot do?

12. Find the cost of telephoning from your home, or a place near your home, to some distant city by long-distance telephone. Make up and work three examples concerning this rate.

13. Find the cost of using the long-distance telephone, 12 minutes @ 60 − 15, 6 minutes @ 100 − 25, and 8 minutes @ 150 − 40. Also find the cost of three trips by railroad costing

$37.50, $28.75, and $40.60 respectively. How much is gained by using the telephone in the ways described, instead of taking the three trips?

14. Which gives a more permanent record of the message sent, the telegraph or the telephone?

15. Name some advantage which the telephone has over the telegraph.

16. Practice making change as in a telegraph office.

If the teacher desires, this may be converted into a business game.

CHAPTER V

FRACTIONS

Numeration and Notation

EXERCISE 46

1. How many quarters are there in a dollar? In a half-dollar?

2. What is the short way of writing three fourths?

3. In $\$\frac{7}{10}$, into how many equal parts is \$1 divided? How many of these equal parts are taken?

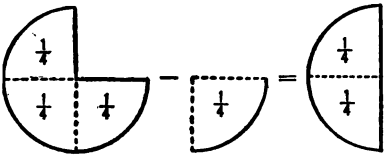
4. Compare the work of writing seven tenths, with that of writing $\frac{7}{10}$. In like manner, compare 7 times $\frac{1}{10}$, with $\frac{7}{10}$.

5. Why do we ever use $\frac{1}{10}$ of a dollar (or a dime)? Why do we not do all our buying and selling with dollars alone?

6. $\frac{3}{4} - \frac{1}{4} = ?$

7. Compare the work of writing $\frac{3}{4} - \frac{1}{4}$, with that of writing $\frac{1}{2}$.

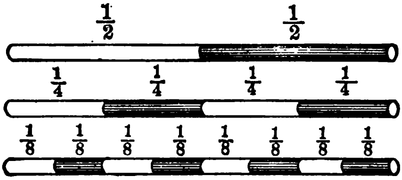
8. On the shaded rods on p. 88 show that $\frac{6}{8} = \frac{3}{4}$. Show the same on the row of dots. On the oblong of dots.

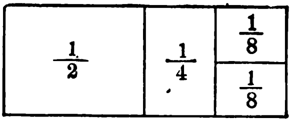


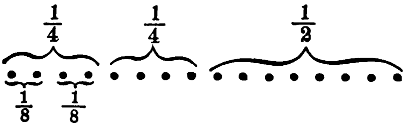
$\frac{3}{4} - \frac{1}{4} \quad = \frac{2}{4}$ or $\frac{1}{2}$

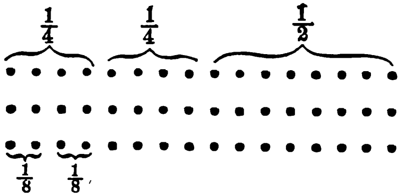
9. Fold a small oblong of paper into fourths. Fold again into eighths. Now on the paper show that $\frac{6}{8} = \frac{3}{4}$.

10. Draw a circle and by lines drawn from the center show that $\frac{6}{8} = \frac{3}{4}$.









11. Show on the foot rule that $\frac{2}{8} = \frac{1}{4}$ $\frac{6}{8} = \frac{3}{4}$ $\frac{4}{8} = \frac{1}{2}$

12. Why do we now make definitions and rules?

52. A **fraction** is one or more of the equal parts into which a unit (or group of units) is divided. As two thirds, three fifths.

A fraction is expressed by one figure written over another figure with a line between. As $\frac{2}{3}$, $\frac{3}{5}$.

The **denominator** (or namer) of a fraction is the number which shows into how many equal parts the given unit (or group) is divided. The denominator is written below the line. Thus, in $\frac{2}{3}$, the denominator is 3.

The **numerator** (or numberer) of a fraction is the number which shows how many of the equal parts of the given unit are taken. The numerator is written above the line. Thus, in $\frac{2}{3}$, the numerator is 2.

In $\frac{2}{3}$, we regard $\frac{1}{3}$ as a new unit called the *fractional unit.* The numerator, 2, tells how many times this fractional unit is taken.

Terms of a fraction is a general name for both numerator and denominator. Thus, 2 and 3 are the terms of $\frac{2}{3}$.

A fraction may also be regarded as division not yet performed.

Thus, $\frac{12}{4}$ means 12 divided by 4, and $\frac{3}{5}$ means 3 divided by 5.

53. Uses of Fractions. (1) Fractions give smaller units to work with.

(2) Fractions also enable us to save labor by the use of cancellation.

54. Kinds of Fractions; Integers.

A **proper fraction** is a fraction whose numerator is less than the denominator. As $\frac{2}{3}$, $\frac{1}{7}$.

An **improper fraction** is a fraction in which the numerator is equal to or greater than the denominator. As $\frac{4}{4}$, $\frac{9}{7}$.

A **mixed number** is a number composed of a whole number and a fraction. As $2\frac{3}{4}$, $16\frac{5}{8}$.

An **integer** is a whole number. As 7, 12, 1.

EXERCISE 47

1. One cent is what fraction of a dime?

2. Three quarts are what fraction of a gallon?

3. Make up an example similar to Ex. 2, concerning a number of inches and 1 yard. Also one concerning ounces and 2 pounds. Concerning quarts and 3 pecks.

4. In $\frac{7}{12}$ ft., into how many parts is 1 foot divided? How many of these parts are taken? Draw a line to represent one foot and on the line mark off the parts taken.

5. Treat in the same way $\frac{5}{8}$ peck; $\frac{5}{8}$ gallon; $\frac{11}{12}$ year; $\frac{7}{16}$ lb.; $\$\frac{3}{4}$.

6. Show on the foot rule that $3 \times \frac{1}{4}$ in. $=$ 3 in. $\div$ 4, or $\frac{3}{4}$ in.

7. Which is greater: $\frac{1}{3}$ of anything, or $\frac{1}{9}$ of it? Show this on the yard stick or on squared paper.

8. Which is larger: $\frac{1}{4}$ of a dollar, or $\frac{1}{20}$ of a dollar?

9. Make up and answer an example similar to Ex. 8.

10. Does a large denominator mean a large or small fractional unit? What does a large numerator mean?

11. In a fraction, what is the office, or use, of the denominator? Of the numerator?

12. In reading a fraction, do you read the numerator or the denominator first? Give an example.

13. In reading a mixed number, where is the word *and* put? Give an example.

Read the following fractions and tell what kind each fraction is:

14. $\frac{3}{4}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{7}$ $\frac{16}{23}$ $\frac{27}{19}$ $1\frac{4}{7}$ $2\frac{1}{2}$ $\frac{13}{15}$ $\frac{19}{11}$ $\frac{78}{28}$ $\frac{15}{91}$

15. $\frac{42}{29}$ $1\frac{17}{18}$ $43\frac{2}{11}$ $71\frac{5}{101}$ $\frac{58}{97}$ $\frac{43}{21}$ $\frac{19}{20}$ $\frac{49}{50}$ $\frac{79}{10}$ $\frac{429}{100}$

16. $3\frac{1}{17}$ $5\frac{3}{100}$ $\frac{9}{1000}$ $\frac{99}{25}$ $2\frac{4}{75}$ $\frac{9}{4}$ $\frac{273}{438}$ $\frac{438}{273}$ $\frac{279}{569}$

17. Write 5 proper fractions. 5 mixed numbers.

18. Write in figures: Two ninths. Seven thirds. Fourteen fiftieths. Eighteen forty-sevenths. Three and six elevenths. Five and twenty sixty-firsts. Ten and seven hundredths. Ninety-seven thousandths. Ninety and seven thousandths.

Which is the numerator in each of these fractions? Which is the denominator?

Write the following, expressing the fractions in figures:

19. Cork is one fourth as heavy as water.

20. The diameter of a circle is seven twenty-seconds as long as the circumference.

21. The area of Texas is more than one twelfth of the area of the United States.

22. Ice is ninety-two hundredths as heavy as water. Iron is seven and one half times as heavy as water.

23. Gunpowder is one tenth sulphur, three twentieths charcoal, and three fourths niter.

REDUCTION OF FRACTIONS

55. A Reduction of a Fraction is the process of changing the form of a fraction without changing its value, as when we write $\frac{5}{3}$ for $1\frac{2}{3}$.

I. *Reduction of a Mixed Number to an Improper Fraction*

EXERCISE 48

1. How many quarters are there in \$2? In $\$\frac{3}{4}$?
2. How many quarters, therefore, are there in $\$2\frac{3}{4}$?
3. How many $\frac{1}{4}$ in. in 2 inches? In $\frac{3}{4}$ in.? In $2\frac{3}{4}$ in? Show this on the foot rule.
4. How many fifths in 2? (Show this on squared paper.) How many fifths in $2\frac{1}{5}$? In $4\frac{3}{5}$?
5. How many sevenths in 3? In $3\frac{4}{7}$?
6. How many ninths in 5? In $5\frac{7}{9}$?

56. Hence the **Method of Reducing a Mixed Number to an Improper Fraction** is to

Multiply the whole number by the denominator of the fraction; Add the numerator of the fraction to this product, and set the result over the denominator.

Ex. Reduce $7\frac{5}{8}$ to an improper fraction.

$7 = \frac{56}{8}$
$\frac{56}{8} + \frac{5}{8} = \frac{61}{8}$
$7\frac{5}{8} = \frac{61}{8}$ *Ans.*

In 1 there are 8 eighths. Hence, in 7 there are 7 times 8 eighths, or 56 eighths. To $\frac{56}{8}$ we add the $\frac{5}{8}$, and obtain $\frac{61}{8}$ as the result.

EXERCISE 49

Reduce to an improper fraction:

1. $5\frac{1}{2}$ yd.
2. $30\frac{1}{4}$ sq. ft.
3. $365\frac{1}{4}$ da.
4. Express $13\frac{2}{7}$ as some number divided by 7.

Reduce to an improper fraction:

5. $2\frac{1}{4}$ $3\frac{1}{7}$ $5\frac{2}{3}$ $4\frac{1}{9}$ $6\frac{2}{5}$ $7\frac{2}{3}$ $16\frac{1}{5}$ $12\frac{6}{7}$ $11\frac{5}{6}$ $13\frac{3}{8}$

6. $6\frac{1}{7}$ $8\frac{2}{5}$ $9\frac{3}{11}$ $4\frac{7}{12}$ $9\frac{7}{9}$ $8\frac{4}{7}$ $12\frac{3}{11}$ $10\frac{1}{10}$ $8\frac{5}{9}$ $7\frac{8}{11}$

7. How many fifths in $7\frac{2}{5}$? $9\frac{3}{5}$? $11\frac{4}{5}$? $12\frac{1}{5}$?

8. How many eighths in $8\frac{3}{8}$? $5\frac{1}{8}$? $13\frac{5}{8}$? $9\frac{7}{8}$? $11\frac{3}{8}$?

Reduce to an improper fraction:

9. $11\frac{7}{13}$ $9\frac{3}{16}$ $13\frac{5}{14}$ $23\frac{4}{5}$ $32\frac{7}{15}$ $46\frac{13}{21}$ $81\frac{4}{11}$

10. $74\frac{5}{12}$ $83\frac{7}{18}$ $98\frac{7}{15}$ $111\frac{5}{11}$ $73\frac{5}{42}$ $8\frac{7}{135}$ $48\frac{9}{101}$

11. How many of the examples in this Exercise can you work at sight?

12. Make up and work an example similar to Ex. 3. To Ex. 4. To Ex. 8.

13. Work again Exercise 21 (p. 45) or similar examples dictated by the teacher.

14. Practice rapid addition.

Let the teacher dictate examples similar to those on p. 23.

II. *Reduction of an Improper Fraction to a Whole or Mixed Number*

EXERCISE 50

1. Walter has 7 quarter-dollars. These are equal to how many whole dollars and how many quarter-dollars as a remainder?

2. Show that $\frac{9}{4}$ of a square equals twice the square plus $\frac{1}{4}$ of the square.

3. Show on your foot rule that $\frac{13}{4}$ in. $=$ $3\frac{1}{4}$ in.

4. How many ones in $\frac{13}{5}$ and how many fifths left over?

5. How many ones in $\frac{22}{7}$ and how many sevenths left over?

6. Which is easier to make a mental picture of: $\$\frac{13}{4}$ or $\$3\frac{1}{4}$? $\frac{25}{8}$ in. or $3\frac{1}{8}$ in.? $\frac{13}{5}$ ft. or $2\frac{3}{5}$ ft.?

57. Hence the **Method of Reducing an Improper Fraction to a Whole or Mixed Number** is to

Divide the numerator by the denominator;

Set the remainder, if there is one, over the denominator.

Ex. Reduce $\frac{23}{7}$ to a mixed number.

$\frac{23}{7} = 23 \div 7 = 3\frac{2}{7}$ *Ans.*

There are 7 sevenths in 1 whole unit. Hence, in $\frac{23}{7}$ there are as many whole units as 7 is contained times in 23, or 3, with a remainder of $\frac{2}{7}$.

58. Uses of Mixed Numbers and Improper Fractions. In general, it is easier to form a mental picture of a quantity in the form of a mixed number, while it is easier to operate with the same quantity in the form of an improper fraction.

Thus, \$$5\frac{2}{5}$ is easier to picture to yourself than \$$\frac{27}{5}$, but it is easier to divide \$100 by \$$\frac{27}{5}$ than by \$$5\frac{2}{5}$.

EXERCISE 51

Reduce to a mixed number:

1. $\frac{19}{4}$ in. \$$\frac{23}{5}$ $\frac{13}{2}$ gal. $\frac{17}{4}$ ft. $\frac{10}{3}$ ft.
2. $\frac{23}{4}$ $\frac{15}{4}$ $\frac{19}{4}$ $\frac{23}{5}$ $\frac{37}{5}$ $\frac{42}{5}$ $\frac{44}{7}$ $\frac{59}{7}$
3. $\frac{16}{3}$ $\frac{28}{5}$ $\frac{49}{9}$ $\frac{73}{8}$ $\frac{58}{11}$ $\frac{79}{8}$ $\frac{110}{9}$ $\frac{88}{7}$ $\frac{49}{11}$
4. $\frac{97}{12}$ $\frac{67}{4}$ $\frac{83}{5}$ $\frac{127}{8}$ $\frac{73}{11}$ $\frac{99}{5}$ $\frac{300}{7}$ $\frac{400}{11}$ $\frac{500}{9}$
5. Express 28 divided by 3 as a fraction, and also as a mixed number.

Reduce to a mixed number:

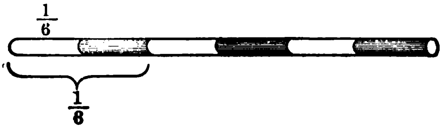
6. $\frac{43}{9}$ $\frac{87}{7}$ $\frac{56}{11}$ $\frac{48}{5}$ $\frac{77}{5}$ $\frac{123}{11}$ $\frac{110}{9}$ $\frac{150}{7}$
7. $\frac{187}{9}$ $\frac{198}{5}$ $\frac{427}{3}$ $\frac{876}{5}$ $\frac{876}{11}$ $\frac{935}{12}$ $\frac{863}{12}$
8. Make up and work an example similar to Ex. 1. To Ex. 5.
9. How many of the examples in this Exercise can you work at sight?

10. Work again Exercise 25 (p. 52) or similar examples dictated by the teacher.

III. *Reduction of a Fraction to its Lowest Terms*

EXERCISE 52

1. Try to make a mental picture of $\frac{12}{18}$ of an apple. Now, of $\frac{2}{3}$ of an apple. Which is easier?



2. Compare the work of writing $\frac{12}{18}$ with that of writing $\frac{2}{3}$.

3. On the shaded rod show that $\frac{2}{6} = \frac{1}{3}$. Also that $\frac{3}{6} = \frac{1}{2}$.

4. Draw a circle and divide it into 6 equal parts by lines from the center. On this circle show that $\frac{2}{6} = \frac{1}{3}$.

5. Fold a small oblong or square of paper into thirds; fold again into sixths. Now show on your paper that $\frac{2}{6} = \frac{1}{3}$.

6. On squared paper mark off a rectangle 12 squares long and 4 squares wide. How many square spaces does the rectangle contain? On this rectangle show that $\frac{9}{12}$ (of 48) = $\frac{3}{4}$ (of 48). Also that $\frac{6}{12}$ (of 48) = $\frac{1}{2}$ (of 48).

7. On squared paper show that $\frac{8}{10} = \frac{4}{5}$. That $\frac{18}{24} = \frac{3}{4}$.

59. Reducing a Fraction to its Lowest Terms is making both numerator and denominator as small as possible without altering the value of the fraction. The method is to

Strike out from numerator and denominator every factor common to them both.

Ex. Reduce $\frac{18}{27}$ to its lowest terms.

$\frac{18}{27} = \frac{2}{3}$

Dividing both numerator and denominator of a fraction by the same number does not change the value of the fraction (see Art. 43, p. 73). Hence we divide 18 and 27 by 9, the largest number that will exactly divide them both, and obtain $\frac{2}{3}$ as the result.

The principle given in the first part of this article may also be stated in the following form:

If both the numerator and the denominator of a fraction are multiplied or divided by the same number, the value of the fraction is not changed.

EXERCISE 53

Reduce to lowest terms:

1. $\frac{14}{21}$ $\frac{36}{24}$ $\frac{28}{36}$ $\frac{15}{45}$ $\frac{18}{45}$ $\frac{21}{28}$ $\frac{40}{48}$ $\frac{25}{60}$ $\frac{12}{72}$ $\frac{18}{54}$

2. $\frac{36}{72}$ $\frac{48}{80}$ $\frac{32}{96}$ $\frac{36}{100}$ $\frac{144}{156}$ $\frac{108}{228}$ $\frac{132}{156}$ $\frac{169}{195}$ $\frac{209}{253}$

3. Harry bought a bushel of chestnuts and has sold 18 quarts. What fraction of a bushel has he sold? What fraction of a bushel is left?

4. Mr. Foster had a farm containing 240 acres; he sold 200 acres. What fraction of the farm did he sell?

5. In a book containing 360 pages, Mary has read 240 pages. What fraction of the book has she read? What part remains for her to read?

6. $\frac{1}{2} = \frac{?}{8} = \frac{?}{16} = \frac{?}{24}$

7. Make and work an example for $\frac{2}{3}$, similar to Ex. 6. For $\frac{3}{4}$.

8. The distance from New York to Philadelphia by railroad is 90 miles; by river and ocean the distance is 240 miles. What fraction is the first distance of the second?

9. If a cubic foot of stone weighs 150 lb. and a cubic foot of iron weighs 480 lb., what fraction is the first weight of the second?

10. In changing $\frac{9}{12}$ to $\frac{3}{4}$, do we increase or diminish the size of the fractional unit? Do we increase or diminish the number of fractional units?

11. Change $\frac{126}{168}$ to an equivalent fraction having the largest possible fractional unit.

12. If we had 12 bananas on the table and have eaten 9 of them, state in smallest terms what fraction of the bananas we have eaten.

13. John has 90 marbles and William has 25. What part of John's number is William's?

14. If it costs 8¢ a pound to mail a book, what is the cost per ounce?

15. If it costs 25¢ to send a telegram of 10 words, what is the cost per word?

16. Make up and work an example similar to Ex. 3. Also to Ex. 11. Ex. 13.

17. How many of the examples in this Exercise can you work at sight?

18. Practice making change as in Exercise 13 (p. 33). Let the pupils take turns in dictating examples.

IV. *Reduction of Fractions to their Least Common Denominator*

EXERCISE 54

1. Which would you rather have: $\frac{1}{3}$ of a melon, or $\frac{4}{9}$ of a melon? $\frac{3}{9}$ or $\frac{4}{9}$?

2. Which is easier to compare: $\frac{1}{3}$ with $\frac{4}{9}$, or $\frac{3}{9}$ with $\frac{4}{9}$?

3. Are the fractional units alike in $\frac{1}{3}$ and $\frac{4}{9}$? In $\frac{3}{9}$ and $\frac{4}{9}$?

4. What, then, is one of the advantages of being able to reduce two or more fractions to the same unit?

5. Which is greater: $\frac{1}{2}$ or $\frac{3}{10}$? $\frac{1}{2}$ or $\frac{4}{9}$?

6. Can you add $\frac{1}{3} + \frac{1}{6} + \frac{5}{12}$? Can you add $\frac{4}{12} + \frac{2}{12} + \frac{5}{12}$?

7. Are the fractional units alike in $\frac{1}{3}+\frac{1}{6}+\frac{5}{12}$? In $\frac{4}{12}+\frac{2}{12}+\frac{5}{12}$?

8. What is another advantage in being able to reduce fractions to like units?

9. Which is simpler: $\frac{4}{12}+\frac{2}{12}+\frac{5}{12}$, or $\frac{12}{36}+\frac{6}{36}+\frac{15}{36}$? In which set of fractions is the fractional unit smaller?

60. Similar Fractions and Least Common Denominator.

Similar fractions are fractions which have the same fractional unit. They have the same denominator, or, as it is called, a *common denominator*.

Thus, $\frac{12}{36}$, $\frac{6}{36}$, and $\frac{15}{36}$ have the same fractional unit $\frac{1}{36}$, or the common denominator 36.

The **least common denominator** of two or more fractions is the least common multiple of their denominators.

"Least common denominator" is often abbreviated to L. C. D.

The method of changing two or more fractions to the least common denominator is to

Find the L. C. M. of all the denominators;

Divide this L. C. M. by the denominator of each fraction;

Multiply the numerator and denominator of each fraction by the quotient obtained for that fraction.

Ex. Reduce $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{7}{12}$ to their least common denominator.

$$\begin{array}{r|rrr} 4) & 4, & 8, & 12 \\ \hline & 1, & 2, & 3 \end{array}$$

$4 \times 2 \times 3 = 24$, L. C. D.

$$\frac{3}{4}=\frac{3\times 6}{4\times 6}=\frac{18}{24}$$

$$\frac{5}{8}=\frac{5\times 3}{8\times 3}=\frac{15}{24}$$

$$\frac{7}{12}=\frac{7\times 2}{12\times 2}=\frac{14}{24}$$

We find first the least common multiple of the denominators (4, 8, 12), which is 24.

4 is contained in 24, 6 times. Hence, to reduce $\frac{3}{4}$ to 24ths, we multiply both the numerator and the denominator by 6.

Similarly 8 is contained in 24, 3 times, and the terms of the second fraction are multiplied by 3. Etc.

EXERCISE 55

Reduce to equivalent fractions having the least common denominator:

1. $\frac{2}{3}, \frac{5}{6}$ **3.** $\frac{3}{4}$ in., $\frac{5}{6}$ in. **5.** $\frac{5}{8}$ ft., $\frac{1}{6}$ ft. **7.** $\frac{1}{4}, \frac{1}{24}$

2. $\frac{1}{2}, \frac{3}{10}$ **4.** $\frac{3}{10}$ ft., $\frac{2}{15}$ ft. **6.** $\frac{2}{3}$ gal., $\frac{1}{12}$ gal. **8.** $\frac{2}{7}, \frac{3}{21}$

9. $\frac{2}{3}, \frac{1}{6}, \frac{3}{4}$ **12.** $\frac{3}{4}, \frac{5}{12}, \frac{1}{6}$ **15.** $\frac{2}{3}, \frac{4}{5}, \frac{7}{10}$

10. $\frac{1}{3}, \frac{5}{6}, \frac{1}{4}$ **13.** $\frac{5}{8}, \frac{7}{12}, \frac{3}{4}$ **16.** $\frac{8}{9}, \frac{5}{6}, \frac{2}{3}$

11. $\frac{2}{3}, \frac{5}{6}, \frac{4}{9}$ **14.** $\frac{9}{16}, \frac{11}{24}, \frac{7}{8}$ **17.** $\frac{1}{2}, \frac{2}{3}, \frac{4}{5}$

Observe that if the denominators in a set of fractions are all prime to each other (see Art. 42, p. 71), the shortest way to reduce the fractions to their L. C. D. is to multiply each numerator by the product of the denominators of all the other fractions, and then write each result over the product of all the denominators.

18. $\frac{2}{5}, \frac{5}{6}, \frac{3}{7}$ **20.** $\frac{3}{7}, \frac{9}{11}, \frac{11}{12}$ **22.** $\frac{4}{7}, \frac{3}{11}, \frac{7}{9}$

19. $\frac{4}{5}, \frac{1}{6}, \frac{7}{11}$ **21** $\frac{1}{8}, \frac{8}{9}, \frac{10}{11}$ **23.** $\frac{3}{13}, \frac{11}{12}, \frac{4}{5}$

24. Which is greater: $\frac{5}{8}$ or $\frac{13}{16}$? $\frac{5}{8}$ or $\frac{4}{7}$?

25. Which of the following is greatest: $\frac{2}{3}$, $\frac{7}{12}$, or $\frac{5}{8}$?

26. Which would you rather own: $\frac{7}{25}$ or $\frac{24}{100}$ of a sailing vessel? $\frac{3}{20}$ or $\frac{4}{25}$? To what like fractional unit do you reduce these fractions?

27. Pick out the groups of similar fractions in the following:
$\frac{7}{24}$ $\frac{5}{18}$ $\frac{6}{7}$ $\frac{1}{4}$ $\frac{2}{7}$ $\frac{7}{18}$ $\frac{5}{24}$ $\frac{3}{4}$ $\frac{5}{8}$ $\frac{11}{24}$ $\frac{1}{24}$ $\frac{1}{18}$ $\frac{1}{7}$

28. Reduce all the following to 48ths: $\frac{2}{3}, \frac{5}{16}, \frac{7}{12}, \frac{11}{24}$.

29. Similarly make a list of fractions which will reduce to 18ths. To 32ds.

30. What are the three steps in reducing a group of fractions to their L. C. D.?

31. How many of the examples in this Exercise can you work at sight?

32. Practice rapid addition and subtraction, using blackboard cards. (See p. 391.)

PROCESSES WITH FRACTIONS

I. *Addition of Fractions*

EXERCISE 56

1. 5 apples + 2 apples = ? apples.

2. 5 eighths + 2 eighths = ? eighths.

3. $\frac{5}{8} + \frac{2}{8} = ?$

4. Compare the work of writing $\frac{5}{8} + \frac{2}{8}$ with that of writing $\frac{7}{8}$.

5. Which is easier to picture or to use mentally: $\frac{5}{8} + \frac{1}{4}$, or $\frac{7}{8}$?

6. Can you add two fractions whose denominators are unlike, as $\frac{1}{2}$ and $\frac{1}{3}$? How do you reduce such fractions to like units? Add $\frac{1}{2}$ and $\frac{1}{3}$.

7. Find $\frac{3}{4}$ of 48¢. (First find $\frac{1}{4}$ of 48¢ and then $\frac{3}{4}$.) Now find $\frac{3}{8}$ of 48¢. Now find $\frac{7}{8}$ of 48¢. Add your results.

Now find $(\frac{3}{4} + \frac{3}{8} + \frac{7}{8})$ of 48¢, or 2 × 48¢.

Compare the work in the two processes.

8. Which is easier to visualize: $\frac{1}{2} + \frac{3}{4}$, or $1\frac{1}{4}$?

9. What, then, are some of the uses of addition of fractions?

61. **The Method of Adding Fractions** is to

Reduce, if necessary, the given fractions to simplest like units (i. e. to their L. C. D.);

Add the numerators and set the result over the L. C. D.;

Simplify the result.

Ex. Add $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{3}{8}$.

$$\begin{aligned}&\tfrac{2}{3}+\tfrac{5}{6}+\tfrac{3}{8}\\&=\tfrac{16}{24}+\tfrac{20}{24}+\tfrac{9}{24}\\&=\tfrac{45}{24}=\tfrac{15}{8}\ \textit{Sum}\end{aligned}$$

The L. C. D. of 3, 6, and 8 is 24. Reducing each fraction to this denominator, and adding the numerators, we obtain $\frac{45}{24}$, or $\frac{15}{8}$, as the sum.

EXERCISE 57

1. Find the value of $\frac{1}{2}$ in. + $\frac{1}{4}$ in. Show the process on the foot rule. Show it also by paper folding.

2. Add and show the process on the foot rule:

(*1*)	(*2*)	(*3*)	(*4*)	(*5*)	(*6*)
$\frac{1}{2}$ in.	$\frac{1}{4}$ in.	$\frac{5}{8}$ in.	$\frac{3}{2}$ in.	$\frac{1}{5}$ in.	$\frac{3}{10}$ in.
$\frac{3}{4}$ in.	$\frac{3}{8}$ in.	$\frac{1}{2}$ in.	$\frac{3}{8}$ in.	$\frac{1}{10}$ in.	$\frac{2}{5}$ in.

Find the value of

3. $\frac{1}{5}+\frac{7}{10}$

4. $\frac{4}{9}+\frac{5}{6}$

5. $\frac{7}{12}+\frac{5}{8}$

6. $\frac{2}{3}+\frac{3}{4}$

7. $\frac{5}{6}+\frac{7}{8}$

8. $\frac{1}{3}+\frac{5}{6}+\frac{3}{4}$

9. $\frac{3}{5}+\frac{7}{10}+\frac{3}{4}$

10. $\frac{4}{9}+\frac{5}{6}+\frac{2}{3}$

11. $\frac{11}{12}+\frac{5}{8}+\frac{1}{6}$

12. $\frac{3}{7}+\frac{11}{14}+\frac{1}{4}$

13. $\frac{8}{9}+\frac{7}{12}+\frac{5}{6}$

14. $\frac{7}{9}+\frac{11}{12}+\frac{3}{8}$

15. $\frac{3}{8}+\frac{5}{16}+\frac{5}{24}$

16. $\frac{3}{10}+\frac{7}{100}$

17. $\frac{3}{10}+\frac{7}{1000}$

18. Harold grew $\frac{3}{4}$ in. in his ninth year, $\frac{7}{8}$ in. in his tenth year, and $\frac{7}{8}$ in. in his eleventh year. How much did he grow in all three years?

19. Edna has enough ruching for one sleeve of a dress. She needs $\frac{1}{4}$ yd. for the other sleeve and $\frac{3}{8}$ yd. for the neck. How much does she need in all?

20. The ice froze $\frac{3}{4}$ in. one night, $\frac{3}{4}$ in. the next night, and $\frac{7}{8}$ in. the next. How much did it freeze on all three nights?

21. Mr. Adams owned $\frac{7}{64}$ of a sailing vessel and bought $\frac{1}{64}$ more. What fraction of the vessel did he then own?

22. In a certain Cape Cod cranberry bog, a father owned $\frac{1}{4}$ and his three sons owned $\frac{3}{32}$, $\frac{1}{16}$, and $\frac{1}{32}$ respectively. What part of the bog was owned by the family?

23. On squared paper mark off a square having 10 spaces on each side. Tell how many small square spaces there are in the whole square. In $\frac{1}{10}$ of it. In $\frac{1}{5}$. In $\frac{2}{5}$. In $\frac{1}{25}$. In $\frac{2}{5}+\frac{1}{10}$. $\frac{1}{4}+\frac{1}{50}$. $\frac{3}{4}+\frac{2}{25}$.

24. On your square in Ex. 23 show that

$$\left(\frac{1}{5}+\frac{1}{10}\right) \text{ of } 100 = \frac{3}{10} \text{ of } 100.$$

Also that

$$\left(\frac{3}{20}+\frac{4}{25}\right) \text{ of } 100 = \frac{31}{100} \text{ of } 100.$$

25. On squared paper mark off a square having 6 spaces on each side, and make up and work examples concerning $\frac{1}{3}+\frac{1}{2}$, $\frac{1}{3}+\frac{5}{12}$, and other fractions.

26. Point out the three steps in adding $\frac{1}{2}$ and $\frac{1}{6}$.

27. Give an example in the addition of fractions in which the first step is not used.

28. Give another example in which the last step is not used.

29. In three successive years Edwin grew $\frac{3}{2}$ in., $\frac{5}{6}$ in., and $\frac{7}{8}$ in.; and Albert grew $\frac{5}{8}$ in., $\frac{3}{2}$ in., and $\frac{3}{4}$ in. Which boy grew more in the three years?

30. Make up and work an example similar to Ex. 18. To Ex. 20. To Ex. 21.

31. Make up and work two problems entirely your own involving the addition of fractions.

EXERCISE 58

ORAL WORK

Give the value of

1. $\frac{1}{2}$ in. + $\frac{1}{10}$ in.

2. $\frac{1}{4}$ in. + $\frac{1}{2}$ in.

3. $\frac{3}{4}$ in. + $\frac{1}{2}$ in.

4. $\frac{1}{3}+\frac{1}{6}$

5. $\frac{1}{3}+\frac{5}{6}$

6. $\frac{3}{4}+\frac{1}{8}$

7. $\frac{1}{3}+\frac{2}{9}$

8. $\frac{5}{6}+\frac{1}{12}$

9. $\frac{1}{8}+\frac{1}{24}$

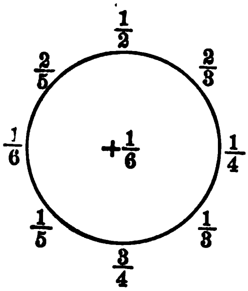
10. $\frac{1}{4}+\frac{1}{2}+\frac{3}{4}$

11. $\frac{2}{3}+\frac{1}{2}+\frac{1}{3}$

12. $\frac{3}{4}+\frac{1}{3}+\frac{1}{4}$

13. How many 12ths in $\frac{1}{2} + \frac{1}{4} + \frac{1}{12}$?

14. I have a board $\frac{1}{2}$ in. thick and another $\frac{3}{4}$ in. thick. How thick are the boards taken together?



15. If Harry eats $\frac{1}{3}$ of a pie and Mary eats $\frac{1}{6}$, how much do they eat together?

16. As a pupil points, add the fraction inside the ring to each fraction on the outside.

Can you think of any short way of adding the fractions when the denominators have no common factor, as in adding $\frac{1}{5} + \frac{1}{6}$?

17. Change the fraction inside the ring to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, etc., and add as before.

18. How many examples in Exercise 57 can you work at sight?

62. The Method of Adding Mixed Numbers is to

Find the sum of the fractions;

Then find the sum of the whole numbers;

Then obtain the sum of the two results in its simplest form.

Ex. Find the value of $11\frac{5}{8} + 20\frac{7}{12} + 6\frac{1}{2}$.

$$\begin{array}{r|l}
 & 24 = \text{L. C. D.} \\
\hline
11\frac{5}{8} & 15 \\
20\frac{7}{12} & 14 \\
6\frac{1}{2} & 12 \\
\hline
37 & \frac{41}{24} \\
\hline
38\frac{17}{24} & \textit{Sum}
\end{array}$$

We write the given mixed numbers in a column, and the L. C. D. of the fractions above another column to the right. This last column contains the numerators of the fractions after they have been reduced to their L. C. D.

The sum of the fractions is $\frac{41}{24}$. The sum of the whole numbers is 37. But $\frac{41}{24} = 1 + \frac{17}{24}$. Hence, the final sum is $37 + 1 + \frac{17}{24}$, or $38\frac{17}{24}$.

EXERCISE 59

1. Find the value of $2\frac{1}{2}$ in. $+ 1\frac{3}{8}$ in. Show the process on the foot rule.

2. Add each of the following and show the meaning of the process on the foot rule:

(1) $\begin{array}{r} 2\frac{1}{4}\text{ in.} \\ \underline{1\frac{3}{8}\text{ in.}} \end{array}$ *(2)* $\begin{array}{r} 3\frac{3}{4}\text{ in.} \\ \underline{2\frac{1}{8}\text{ in.}} \end{array}$ *(3)* $\begin{array}{r} 1\frac{1}{2}\text{ in.} \\ \underline{2\frac{3}{4}\text{ in.}} \end{array}$ *(4)* $\begin{array}{r} 3\frac{3}{4}\text{ in.} \\ \underline{2\frac{3}{8}\text{ in.}} \end{array}$ *(5)* $\begin{array}{r} 2\frac{1}{2}\text{ in.} \\ \underline{3\frac{7}{8}\text{ in.}} \end{array}$

Add the following:

3. $\begin{array}{r} 1\frac{1}{2} \\ \underline{\frac{1}{4}} \end{array}$

4. $\begin{array}{r} 2\frac{1}{2} \\ \underline{3\frac{1}{4}} \end{array}$

5. $\begin{array}{r} 12\frac{1}{2} \\ \underline{23\frac{1}{4}} \end{array}$

6. $\begin{array}{r} 2\frac{2}{3} \\ \underline{1\frac{2}{3}} \end{array}$

7. $\begin{array}{r} 5\frac{2}{3} \\ \underline{7\frac{2}{3}} \end{array}$

8. $\begin{array}{r} 15\frac{2}{3} \\ \underline{27\frac{2}{3}} \end{array}$

9. $\begin{array}{r} 5\frac{4}{5} \\ \underline{1\frac{8}{10}} \end{array}$

10. $\begin{array}{r} 8\frac{4}{5} \\ \underline{2\frac{3}{10}} \end{array}$

11. $\begin{array}{r} 348\frac{4}{5} \\ \underline{127\frac{3}{10}} \end{array}$

12. $\begin{array}{r} 2\frac{1}{6} \\ \underline{5\frac{7}{8}} \end{array}$

13. $\begin{array}{r} 12\frac{1}{6} \\ \underline{5\frac{7}{8}} \end{array}$

14. $\begin{array}{r} 327\frac{1}{6} \\ \underline{214\frac{7}{8}} \end{array}$

Find the value of

15. $3\frac{1}{4} + 4\frac{2}{3}$

16. $5\frac{1}{2} + 6\frac{1}{3}$

17. $7\frac{3}{8} + 1\frac{1}{12}$

18. $3\frac{5}{15} + 8\frac{3}{8}$

19. $4\frac{4}{9} + 10\frac{5}{6}$

20. $2\frac{1}{3} + 3\frac{3}{4} + 4\frac{5}{6}$

21. $11\frac{1}{2} + 4\frac{3}{8} + 2\frac{1}{6}$

22. $4\frac{7}{8} + 6\frac{11}{12} + 2\frac{1}{6}$

23. $4 + \frac{8}{9} + 17\frac{7}{12}$

24. $3\frac{11}{16} + 8\frac{5}{12} + \frac{5}{24}$

25. $4\frac{7}{10} + 6\frac{18}{100}$

26. $3\frac{3}{10} + 7\frac{128}{1000}$

27. A boy nine years old was 4 ft. $7\frac{1}{2}$ in. tall. In two years he grew $3\frac{3}{4}$ in. How tall was he then?

28. If the ice on the pond was $5\frac{3}{4}$ in. thick and then froze $2\frac{2}{3}$ in. more, how thick did it become?

29. On a field a farmer in one season had successive cuttings of alfalfa as follows: $2\frac{2}{3}$ tons, $1\frac{1}{2}$ tons, $1\frac{1}{3}$ tons, and $1\frac{1}{4}$ tons. How many tons was this in all?

30. A board $2\frac{3}{4}$ in. wide and one $7\frac{1}{2}$ in. wide together will be how wide?

31. If three boards are $4\frac{1}{4}$ in., $5\frac{3}{4}$ in., and $6\frac{7}{8}$ in. wide respectively, together they will equal a board of what width?

32. Make up and work an example similar to Ex. 31.

33. Find the length of a fence around a garden $20\frac{2}{3}$ yd. long and $8\frac{1}{3}$ yd. wide.

34. What is the cost of the fence in Ex. 33 at 60¢ a yard?

35. In three successive months the rainfall is $2\frac{3}{10}$ in., $1\frac{3}{5}$ in., and $4\frac{1}{10}$ in. Find the rainfall for the three months.

36. Make up and work an example similar to Ex. 27. To Ex. 28. To Ex. 33.

37. Make up and work two problems entirely your own involving the addition of mixed numbers.

38. Count by $2\frac{1}{2}$'s from $2\frac{1}{2}$ to 20. By $1\frac{1}{2}$'s from $1\frac{1}{2}$ to 21.

39. How many examples in this Exercise can you work at sight?

40. Practice adding small two-figured numbers like those in Exercise 9 (p. 26). Let the pupils take turns in writing rows of these examples on the blackboard for an oral time-test of some pupil named by the writer of the examples.

II. *Subtraction of Fractions*

63. In **Subtracting Fractions** we proceed as in adding fractions, except that instead of adding numerators we subtract them.

Ex. Subtract $\frac{3}{8}$ from $\frac{7}{12}$.

$$\frac{7}{12} - \frac{3}{8} = \frac{14}{24} - \frac{9}{24}$$
$$= \frac{5}{24}\ \textit{Difference}$$

The L. C. M. of 12 and 8 is 24.
$\frac{7}{12} = \frac{14}{24}$, and $\frac{3}{8} = \frac{9}{24}$.
The difference between $\frac{14}{24}$ and $\frac{9}{24}$ is $\frac{5}{24}$.

EXERCISE 60

1. Find the value of $\frac{3}{4}$ in. $-$ $\frac{1}{2}$ in. Show the process on the foot rule.

2. Treat in like manner:

(1) $\frac{3}{2}$ in. $-$ $\frac{1}{4}$ in.	*(4)* $\frac{8}{10}$ in. $-$ $\frac{1}{5}$ in.
(2) $\frac{1}{2}$ in. $-$ $\frac{1}{8}$ in.	*(5)* $\frac{2}{5}$ in. $-$ $\frac{1}{10}$ in.
(3) $\frac{7}{8}$ in. $-$ $\frac{1}{2}$ in.	*(6)* $\frac{4}{5}$ in. $-$ $\frac{3}{10}$ in.

3. Make up and work two examples similar to those in Ex. 2.

Find the value of the following and check each result:

4. $\$\frac{7}{10} - \$\frac{8}{10}$
5. $\frac{7}{12}$ ft. $- \frac{1}{12}$ ft.
6. $\frac{5}{6}$ gal. $- \frac{1}{2}$ gal.
7. $\frac{3}{4}$ in. $- \frac{1}{2}$ in.
8. $\frac{5}{6} - \frac{2}{3}$
9. $\frac{7}{8} - \frac{1}{12}$
10. $\frac{4}{5} - \frac{8}{15}$
11. $\frac{7}{15} - \frac{3}{10}$
12. $\frac{6}{7} - \frac{4}{5}$
13. $\frac{19}{21} - \frac{11}{14}$
14. $\frac{7}{8} - \frac{2}{9}$
15. $\frac{40}{81} - \frac{1}{3}$
16. $\frac{17}{24} - \frac{7}{16}$
17. $\frac{5}{16} - \frac{7}{40}$
18. $\frac{41}{15} - \frac{18}{10}$
19. $\frac{17}{9} - \frac{7}{6}$
20. $\frac{11}{12} - \frac{9}{18}$
21. $\frac{5}{9} - \frac{1}{6}$
22. $2 - \frac{3}{4}$ (or $\frac{2}{1} - \frac{3}{4}$)
23. $3 - \frac{7}{8}$
24. $4 - \frac{2}{3}$

25. Compare the work of writing $\frac{11}{12} - \frac{1}{6}$, with that of writing $\frac{3}{4}$. What, then, is one of the uses of the subtraction of fractions?

26. On squared paper show that $\frac{5}{6}$ (of 48) $- \frac{1}{2}$ (of 48) $= \frac{1}{3}$ (of 48).

27. Also show that $\frac{3}{10}$ (of 100) $- \frac{1}{10}$ (of 100) $= \frac{1}{5}$ (of 100). Also that $\frac{7}{20} - \frac{3}{25} = \frac{23}{100}$.

28. Make up and work an example similar to Ex. 27.

29. Mary grew $\frac{7}{8}$ in. one year and $\frac{5}{6}$ in. another year. How much more did she grow during the first year than during the second?

30. Mr. X owns $\frac{17}{64}$ of a sailing vessel and his brother owns $\frac{3}{8}$. How much greater is Mr. X's share than his brother's?

31. What are the three steps in finding the value of $\frac{1}{2} - \frac{1}{6}$?

32. Give an example of the subtraction of fractions in which the first step is not used.

33. Give another example in which the third step is not used.

34. Make up and work an example similar to Ex. 19. Ex. 29. Ex. 30.

35. Make up and work two problems entirely your own involving the subtraction of fractions.

36. Count by $2\frac{1}{2}$'s from 20 to $2\frac{1}{2}$. By $1\frac{1}{2}$ from 21 to $1\frac{1}{2}$.

37. How many of the examples in this Exercise can you work at sight?

38. Practice adding and subtracting two-figured numbers. See how rapidly you can work at sight the examples on p. 31.

64. Subtraction of Mixed Numbers. To subtract one mixed number from another mixed number,

Subtract the fractions, then subtract the whole numbers, and combine the results.

Ex. 1. Subtract $\$3\frac{1}{4}$ from $\$5\frac{1}{3}$.

$$\begin{array}{rl} 5\frac{1}{3} = & 5\frac{4}{12} \\ 3\frac{1}{4} = & \underline{3\frac{3}{12}} \\ & 2\frac{1}{12} \text{ \textit{Difference}} \end{array}$$

$\frac{4}{12} - \frac{3}{12} = \frac{1}{12}$. $5 - 3 = 2$.
Hence, the entire difference is $2\frac{1}{12}$.

If the fraction in the subtrahend is larger than the fraction in the minuend, it is necessary to *increase the fraction in the minuend by borrowing* 1 *from the whole number in the minuend.*

Ex. 2. Find the value of $17\frac{1}{4} - 12\frac{2}{3}$.

$$\begin{array}{rlll} 17\frac{1}{4} & = 17\frac{3}{12} & = & 16\frac{15}{12} \\ 12\frac{2}{3} & = 12\frac{8}{12} & = & \underline{12\frac{8}{12}} \\ & & & 4\frac{7}{12} \text{ \textit{Difference}} \end{array}$$

Since $\frac{8}{12}$ is larger than $\frac{3}{12}$, we borrow 1, or $\frac{12}{12}$, from 17, and add it to $\frac{3}{12}$, making $\frac{15}{12}$. This leaves 16 as the whole number in the minuend. We then subtract as usual.

EXERCISE 61

1. Find the value of $2\frac{1}{2}$ in. $-$ $1\frac{1}{4}$ in. Show the process on the foot rule.

2. Subtract and show the meaning of the process on the foot rule:

(*1*)	(*2*)	(*3*)	(*4*)	(*5*)
$2\frac{3}{4}$ in.	$3\frac{1}{2}$ in.	$2\frac{1}{2}$ in.	$2\frac{1}{8}$ in.	$3\frac{3}{8}$ in.
$1\frac{1}{2}$ in.	$1\frac{1}{8}$ in.	$1\frac{3}{4}$ in.	$1\frac{7}{8}$ in.	$1\frac{3}{4}$ in.

Subtract the following and check each result:

3. $\begin{array}{r} 1\frac{1}{2} \\ \underline{\frac{1}{4}} \end{array}$

4. $\begin{array}{r} 3\frac{1}{2} \\ \underline{1\frac{1}{4}} \end{array}$

5. $\begin{array}{r} 15\frac{1}{2} \\ \underline{3\frac{1}{4}} \end{array}$

6. $\begin{array}{r} 2\frac{2}{3} \\ \underline{1\frac{1}{6}} \end{array}$

7. $\begin{array}{r} 5\frac{2}{3} \\ \underline{2\frac{1}{6}} \end{array}$

8. $\begin{array}{r} 23\frac{2}{3} \\ \underline{11\frac{1}{6}} \end{array}$

9. $\begin{array}{r} 3\frac{2}{5} \\ \underline{1\frac{7}{10}} \end{array}$

10. $\begin{array}{r} 6\frac{2}{5} \\ \underline{3\frac{7}{10}} \end{array}$

11. $\begin{array}{r} 23\frac{2}{5} \\ \underline{14\frac{7}{10}} \end{array}$

12. $\begin{array}{r} 8\frac{1}{6} \\ \underline{5\frac{7}{8}} \end{array}$

13. $\begin{array}{r} 18\frac{1}{6} \\ \underline{12\frac{7}{8}} \end{array}$

14. $\begin{array}{r} 223\frac{1}{6} \\ \underline{52\frac{7}{8}} \end{array}$

Find the value of

15. $4\frac{3}{5} - 2\frac{1}{10}$
16. $7\frac{5}{6} - 3\frac{5}{9}$
17. $5\frac{7}{8} - 3\frac{5}{12}$
18. $10\frac{1}{9} - 7\frac{2}{3}$
19. $9\frac{4}{5} - 4\frac{9}{10}$
20. $8\frac{3}{5} - 5\frac{2}{3}$
21. $10\frac{5}{7} - 3\frac{5}{6}$
22. $7\frac{5}{12} - 5\frac{19}{24}$
23. $9\frac{11}{18} - 4\frac{11}{12}$

24. From $5\frac{17}{10}$ take $3\frac{19}{100}$.

25. Yesterday the ice on the pond was $5\frac{3}{4}$ in. thick. This morning it is $7\frac{1}{8}$ in. thick. How much has it frozen during the night?

26. John weighed $58\frac{3}{4}$ lb. two years ago. He now weighs $73\frac{1}{4}$ lb. How much has he gained in two years?

27. Two years ago John was 4 ft. $6\frac{3}{4}$ in. tall. Now he is 4 ft. $8\frac{1}{8}$ in. tall. How much has his height increased in two years?

28. Mr. X does not want to cut ice until the ice is $7\frac{1}{2}$ in. thick. The ice is now $5\frac{3}{4}$ in. thick. How much must its thickness increase before it will be ready to cut?

29. If the skirt of a dress is to be $27\frac{1}{2}$ in. long in all, and the ruffle at the bottom is $3\frac{1}{4}$ in. wide, what will be the length of the skirt above the ruffle?

30. By how much does the thickness of a 2-inch board exceed the length of a $1\frac{1}{8}$-inch screw?

31. How wide a board must I place by the side of a board $5\frac{3}{8}$ in. wide, in order that the two boards together may be 8 in. wide?

32. Make up and work an example similar to Ex. 31.

33. If $5\frac{1}{2}$ yd. of cloth are necessary to make a dress and Mary now has $3\frac{7}{8}$ yd., how much must she buy?

34. A recent record for the hundred-yard dash is $9\frac{3}{5}$ seconds and Harold can run the distance in $12\frac{4}{5}$ seconds. How much longer is his time than that of the record?

35. From a barrel which contained $31\frac{1}{2}$ gal. of vinegar, 5 gal. and 3 qt. have been sold. How can I tell how much vinegar remains in the barrel, without the labor of measuring it?

36. From a farm which contained $150\frac{1}{4}$ acres, the owner has sold $26\frac{1}{2}$ acres for building lots. How can he tell how many acres he now has, without measuring what remains?

37. Make up and work three examples similar to such of the examples in this Exercise as you think are most interesting or instructive.

38. How many of the examples in this Exercise can you work at sight?

39. Have a Time Race with Exs. 7–14 on p. 107, or with similar examples dictated by the teacher. (See p. 393.)

EXERCISE 62

REVIEW

Find the value of

1. $\frac{3}{4} + \frac{1}{2} - \frac{5}{8} + \frac{1}{6}$

2. $\frac{5}{12} - \frac{1}{8} + \frac{1}{6} - \frac{2}{3} + \frac{3}{4}$

3. $21\frac{1}{4} + 17\frac{1}{2} - 2\frac{3}{6}$

4. $18\frac{1}{9} - 5\frac{1}{12} + 2\frac{3}{4} - 3\frac{1}{6}$

5. Which is greater: $\frac{2}{9}$ or $\frac{3}{13}$? How much greater is it?

6. From the sum of $\frac{1}{2}$ and $\frac{1}{3}$, subtract their difference.

7. From a barrel containing $31\frac{1}{2}$ gal. of molasses, a dealer sold $5\frac{1}{2}$ gal. on Monday, $3\frac{3}{4}$ gal. on Tuesday, and $2\frac{1}{4}$ gal. on Wednesday.

How can the dealer tell how much remains in the barrel, without measuring it?

8. Mr. Swift in one year burned $8\frac{1}{2}$ tons of coal in one furnace, $6\frac{3}{4}$ tons in another, and $7\frac{1}{4}$ tons in the kitchen range. He had $3\frac{1}{2}$ tons at the beginning of the year. How many tons did he need to buy during the year?

9. Compute $(\frac{3}{8} + \frac{5}{12} + \frac{4}{16}) - (\frac{5}{2} + \frac{6}{18} + \frac{3}{12})$.

10. From 2 take $1\frac{1}{7} - \frac{11}{14}$.

11. Find the difference between $\frac{1}{9}$ and $\frac{1}{5}$. Between $\frac{4}{9}$ and $\frac{2}{5}$.

12. If there is a set of scales in the schoolroom, show that $\frac{3}{16}$ lb. + $\frac{5}{16}$ lb. = $\frac{1}{2}$ lb.

13. Make up and work an example similar to Ex. 7. To Ex. 8. Ex. 9.

14 Work again Exercise 31 (p. 61) or similar examples dictated by the teacher.

III. *Multiplication of Fractions*

EXERCISE 63

1. On squared paper mark off a square with 3 spaces on each side. What fraction of the whole is one small square? Hence, show that 5 times $\frac{1}{9} = \frac{5}{9}$.

2. If you divide a pie into 6 equal parts and take 5 of them, how many sixths do you take? Show this by dividing a circle into 6 equal parts.

3. 5 times $\frac{1}{6}$ = ?

4. Compare the work of writing $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$, with that of writing $\frac{5}{6}$. Compare 5 times $\frac{1}{6}$ with $\frac{5}{6}$.

5. Show on the foot rule that 5 times $\frac{1}{8}$ in. = $\frac{5}{8}$ in. That 5 times $\frac{3}{8}$ in. = $\frac{15}{8}$ in.

6. $\frac{1}{8}$ in. × 6 = ? $\frac{3}{4}$ in. × 6 = ?

7. On squared paper mark off an oblong 10 spaces long and 3 spaces wide. On this show that 7 times $\frac{1}{10}$ (of 30) = $\frac{7}{10}$ (of 30). Also that 3 times $\frac{1}{10}$ (of 30) = $\frac{3}{10}$ (of 30).

65. Multiplication of a Fraction by a Whole Number. From the preceding examples it is evident that, to get the product of a fraction and a whole number, we

Multiply the numerator of the fraction by the whole number, using cancellation as far as possible.

Ex. 1. Multiply $\frac{7}{12}$ by 15.

$$\frac{7}{\underset{4}{\cancel{12}}} \times \overset{5}{\cancel{15}} = \frac{35}{4} = 8\tfrac{3}{4} \textit{ Product}$$

EXERCISE 64

Multiply, using cancellation wherever possible:

1. $\frac{5}{18}$ by 24
2. $\frac{13}{16}$ by 20
3. $\frac{5}{24}$ by 30
4. $\frac{6}{27}$ by 36
5. $\frac{8}{35}$ by 49
6. $\frac{11}{36}$ by 48
7. $\frac{12}{25}$ by 55
8. $\frac{13}{45}$ by 81
9. $\frac{41}{60}$ by 100
10. $\frac{5}{8}$ in. by 20
11. $\frac{8}{16}$ lb. by 24
12. $\frac{3}{8}$ gal. by 16
13. $\frac{5}{27}$ ft. by 18

14. What will it cost to mail a book weighing 12 oz., at $\frac{1}{2}$¢ per oz.? Also another book weighing 1 lb. 4 oz.?

15. What will it cost to send by mail newspapers weighing 8 oz., at $\frac{1}{4}$¢ per oz.?

16. If a boy gains $\frac{7}{8}$ in. per year in height on the average, how much will he gain in 3 years? How much less than 3 inches is this? How much will he gain in 5 years? How much less than 5 inches is this?

17. Set down $\frac{2}{3}$ five times and add. Now multiply $\frac{2}{3}$ by 5. Compare the work in the two processes.

18. What must a newsboy pay for 40 newspapers at $\frac{3}{5}$¢ each?

19. If a train travels 18 miles in 24 minutes, what fraction of a mile does it go in 1 minute? How far will it go in 1 hour?

20. If there is a set of scales in the schoolroom, show that $3 \times \frac{3}{16}$ lb. $= \frac{9}{16}$ lb.

21. What does it cost to launder 50 collars, at $1\frac{1}{2}$¢ per collar?

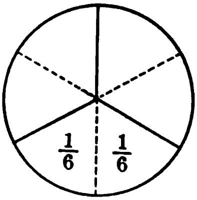
22. Compute $\frac{3}{4} \times 67 + \frac{5}{4} \times 67$ in the shortest way.

23. Make up and work an example similar to Ex. 18. To Ex. 19. To Ex. 22.

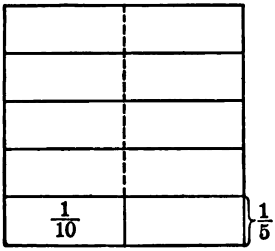
24. How many of the examples in this Exercise can you work at sight?

EXERCISE 65

1. Draw a circle and divide it into thirds. Now divide each third into 2 equal parts. What fraction of the whole circle is each of these smaller parts?



2. $\frac{1}{2}$ of $\frac{1}{3}$ = ?



3. Draw a square and divide it into 5 equal parts by lines drawn as in the diagram. Then divide one of these fifths into 2 equal parts. Hence $\frac{1}{2}$ of $\frac{1}{5}$ = ?

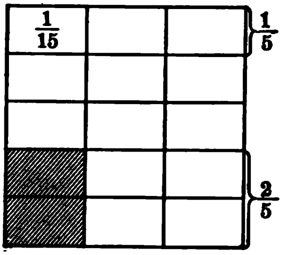
4. On squared paper mark off a rectangle 5 spaces long and 2 spaces wide. On this rectangle show that $\frac{1}{2}$ of $\frac{1}{5} = \frac{1}{10}$.

The word *of* between two fractions is equivalent to the sign of multiplication. Thus, $\frac{5}{6}$ of $\frac{4}{7}$ means the same as $\frac{5}{6} \times \frac{4}{7}$.

5. $\frac{1}{2} \times \frac{1}{5} = ?$ 7. $\frac{1}{2} \times \frac{1}{6} = ?$ 9. $\frac{1}{2} \times \frac{1}{15} = ?$

6. $\frac{1}{2} \times \frac{1}{3} = ?$ 8. $\frac{1}{2} \times \frac{1}{7} = ?$ 10. $\frac{1}{3} \times \frac{1}{4} = ?$

11. Draw a square and divide it into 5 equal parts. Divide each of these into 3 equal parts. Show that $\frac{1}{3}$ of $\frac{2}{5} = \frac{2}{15}$.



12. Multiply $\frac{1}{3}$ by $\frac{2}{5}$. What is $\frac{1}{3}$ of $\frac{1}{5}$? What is $\frac{1}{3}$ of $\frac{2}{5}$? $\frac{2}{3}$ of $\frac{2}{5}$?

13. $\frac{1}{3} \times \frac{4}{5} = ?$ 15. $\frac{1}{5} \times \frac{4}{3} = ?$

14. $\frac{1}{3} \times \frac{2}{7} = ?$ 16. $\frac{1}{7} \times \frac{5}{8} = ?$

17. On squared paper mark off a rectangle 10 spaces long and 6 spaces wide. On it show that $\frac{1}{3}$ of $\frac{1}{5}$ (of 60) $= \frac{1}{15}$ (of 60).

18. Practice making examples in telegraphing, and in making change for one, two, and five-dollar bills used in paying for telegrams. (See Exercise 34, p. 66.) Let either the teacher or a pupil make examples, and let the pupils take turns in acting as cashier.

66. Multiplication of a Fraction by a Fraction.

Ex. Multiply $\frac{2}{3}$ by $\frac{4}{7}$.

$$\frac{2}{3} \times \frac{4}{7} = \frac{2 \times 4}{3 \times 7} = \frac{8}{21} \textit{ Product}$$

$\frac{1}{3} \times \frac{1}{7} = \frac{1}{21}$. But $\frac{4}{7}$ is 4 times as large as $\frac{1}{7}$. Hence, $\frac{1}{3}$ of $\frac{4}{7} = 4$ times $\frac{1}{3} \times \frac{1}{7}$, or $\frac{4}{21}$.

$\frac{2}{3}$ of $\frac{4}{7} = 2$ times $\frac{4}{21}$, or $\frac{8}{21}$.

Hence, to multiply one fraction by another,

Multiply the numerators together for the numerator of the product, and the denominators together for the denominator of the product;

Cancel wherever possible.

EXERCISE 66

Find the value of

1. $\frac{1}{3} \times \frac{3}{8}$ in. Show this on the foot rule.
2. $\frac{2}{3} \times \frac{3}{8}$ in. Show this on the foot rule.
3. Make up and work a similar example for yourself.

Find the value of

4. $\frac{7}{8}$ of $\frac{8}{5}$
5. $\frac{4}{7} \times \frac{3}{11}$
6. $\frac{2}{9} \times \frac{3}{4}$
7. $\frac{5}{6} \times \frac{8}{15}$
8. $\frac{7}{8} \times \frac{24}{35}$
9. $\frac{4}{15} \times \frac{5}{16}$
10. $\frac{14}{15} \times \frac{20}{21}$
11. $\frac{9}{16} \times \frac{8}{27}$
12. $\frac{24}{25}$ of $\frac{35}{32}$
13. $\frac{3}{7} \times \frac{5}{11}$
14. $\frac{33}{36}$ of $\frac{19}{65}$
15. $\frac{42}{55} \times \frac{11}{7}$
16. $\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}$
17. $\frac{4}{7} \times \frac{14}{15} \times \frac{5}{8}$
18. $\frac{9}{14} \times \frac{21}{22} \times \frac{44}{27}$
19. $\frac{8}{15}$ of $\frac{3}{7}$ of $\frac{5}{16}$
20. $\frac{24}{25} \times \frac{50}{51} \times \frac{34}{64}$
21. $\frac{5}{9} \times \frac{11}{32} \times \frac{12}{7}$

22. Mary had $\frac{1}{2}$ of a pie and gave $\frac{1}{3}$ of what she had to her brother. What fraction of the pie did she give him? If she had $\frac{2}{3}$ of a pie and gave him $\frac{1}{4}$ of her piece, what fraction did she give him?

23. A man owning $\frac{2}{3}$ of a business, sold $\frac{3}{4}$ of his share. What fraction of the whole business did he sell?

24. In one year Walter grew $\frac{7}{8}$ of an inch. How much did he grow in $\frac{1}{4}$ of a year? In $\frac{3}{4}$ of a year?

25. Compare the work of writing $\frac{2}{3}$ of $\frac{6}{5}$, with that of writing $\frac{4}{5}$.

26. Compare the work of finding $\frac{2}{5}$ of $\frac{15}{8}$ of 5120, with that of finding $\frac{3}{4}$ of 5120.

27. What are some of the uses of the multiplication of fractions?

28. On squared paper show that $\frac{3}{4}$ of $\frac{5}{6}$ (of 24) $\doteq$ $\frac{5}{8}$ (of 24).

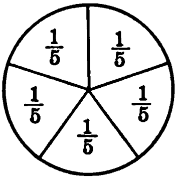
29. Make up and work an example similar to Ex. 22. To Ex. 23. To Ex. 26.

30. How many of the examples in this Exercise can you work at sight?

31. Practice sight multiplication of a fraction by a fraction, using some form of oral drill. (See p. 103.)

IV. *Division of Fractions*

EXERCISE 67



1. Draw a circle to represent a pie and divide the circle into 5 equal parts.

2. Take 4 of these equal parts, or $\frac{4}{5}$ of the pie.

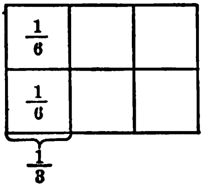
3. $\frac{1}{2}$ of 4 fifths of a pie = ? $\frac{4}{5} \div 2 = ?$

4. Divide $\frac{4}{7}$ by 2. $\frac{6}{7}$ by 3.

5. On squared paper mark off an oblong 10 spaces long and 6 spaces wide. On this oblong show that $\frac{4}{5}$ (of 60) $\div 2 = \frac{2}{5}$ (of 60).

6. Compare the work of writing $\frac{4}{5} \div 2$, with that of writing $\frac{2}{5}$.

7. $\frac{1}{3} \div 2$ equals what fraction? Draw a circle and divide it to show this.



8. On squared paper mark off a rectangle 3 spaces long and 2 spaces wide. On this rectangle show that $\frac{1}{3} \div 2 = \frac{1}{6}$.

9. If $\frac{1}{2}$ of a cake is divided equally between 2 boys, what part does each boy receive? What part, if the cake is divided equally among 3 boys? Among 5 boys?

10. What is the value of $\frac{1}{3} \div 4$? $\frac{1}{5} \div 4$? $\frac{2}{5} \div 4$?

67. Division of a Fraction by a Whole Number. From the preceding examples we see that, to divide a fraction by a whole number, the method is as follows:

If possible, divide the numerator by the whole number;
If not, multiply the denominator by the whole number;
Use cancellation as far as possible.

Ex. Divide $\frac{10}{7}$ by 15.

$$\frac{10}{7} \div 15 = \frac{\overset{2}{\cancel{10}}}{7 \times \underset{3}{\cancel{15}}} = \frac{2}{21} \textit{ Quotient}$$

EXERCISE 68

Divide:

1. $\frac{8}{5}$ in. by 2. By 4. Also $\frac{5}{32}$ in. by 2.
2. $\frac{12}{17}$ by 6
3. $\frac{9}{16}$ by 6
4. $\frac{24}{29}$ by 40
5. $\frac{8}{27}$ by 20
6. $\frac{32}{33}$ by 48
7. $\frac{48}{71}$ by 120
8. $\frac{3}{4}$ by 5
9. $\frac{4}{7}$ by 5
10. $\frac{5}{6}$ by 10
11. $\frac{10}{11}$ by 3
12. $\frac{30}{47}$ by 8
13. $\frac{99}{100}$ by 121
14. $\frac{1}{7}$ by 3
15. $\frac{2}{11}$ by 8
16. $\frac{12}{19}$ by 36
17. $\frac{15}{22}$ by 75
18. $\frac{75}{22}$ by 15
19. $\frac{144}{625}$ by 360

20. If William spaded $\frac{8}{9}$ of the garden in 4 days, what part of it did he spade in 1 day?

21. If Lambert grew $2\frac{2}{5}$ in. in 3 years, how much did he grow in 1 year? In 2 years?

22. If $\frac{6}{7}$ of a pie were divided among 3 children, what part of the pie would each child receive?

23. If there is a set of scales in the schoolroom, show that $\frac{9}{16}$ lb. $\div 3 = \frac{3}{16}$ lb.

24. Divide each of the following by 2:

$$\frac{2}{3} \quad \frac{1}{3} \quad \frac{4}{5} \quad \frac{1}{5} \quad \frac{3}{5} \quad \frac{1}{21} \quad \frac{1}{4} \quad \frac{3}{7} \quad \frac{4}{7} \quad \frac{5}{9} \quad \frac{4}{9} \quad \frac{5}{6} \quad \frac{1}{6} \quad \frac{6}{7}$$

25. Also divide each of the fractions in Ex. 24 by 3. By 4. By 5. By 6.

26. Make up and work an example similar to Ex. 20.

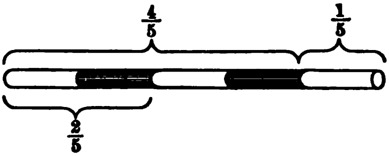
27. Also make an example similar to Ex. 21.

28. Make up and work two problems entirely your own involving the division of a fraction by a whole number.

29. How many of the examples in this Exercise can you work at sight?

30. Practice making examples in telephoning, and in making change when the telephone calls are paid for by bills of different sizes. (See Exercise 45, p. 85.)

EXERCISE 69



1. 4 fifths contains 2 fifths how many times?

$\frac{4}{5}$ in. ÷ $\frac{2}{5}$ in. = ? Show this on a foot rule.

2. $\frac{2}{5}$ in. is contained in $\frac{6}{5}$ in. how many times? Show this on the foot rule.

3. Divide a pie into 9 equal parts. How many times is $\frac{2}{9}$ of the pie contained in $\frac{8}{9}$ of the pie? $\frac{3}{9}$ in $\frac{9}{9}$? $\frac{3}{9}$ in $\frac{2}{3}$ or $\frac{6}{9}$?

4. $\frac{8}{9} \div \frac{2}{9} = ?$ $1 \div \frac{3}{9} = ?$ $\frac{2}{3} \div \frac{3}{9} = ?$

5. $\frac{1}{2}$ in. is contained in 3 in. how many times? $3 \div \frac{1}{2} = ?$ Show this on the foot rule.

6. $\frac{1}{2}$ apple is contained in 3 apples how many times? $3 \div \frac{1}{2} = ?$

7. $\frac{3}{4}$ in. is contained in 6 in. how many times? 6 in. ÷ $\frac{3}{4}$ in. = ? Show this on the foot rule.

SUG. We have 6 in. = $\frac{24}{4}$ in. Hence, $\frac{24}{4}$ in. ÷ $\frac{3}{4}$ in. = ?

8. On squared paper show that $\frac{9}{10}$ (of 100) ÷ $\frac{3}{10}$ (of 100) = 3.

9. Make up and work an example similar to Ex. 8.

68. Division by a Fraction.

Reduce the fractions (or the whole number and the fraction) *to the same denominator* (or unit);

Divide the numerator of the dividend by the numerator of the divisor.

Ex. Divide $\frac{5}{6}$ by $\frac{3}{10}$.

$$\frac{5}{6} \div \frac{3}{10} = \frac{25}{30} \div \frac{9}{30} = \frac{25}{9} = 2\frac{7}{9} \textit{ Quotient}$$

Later we shall learn another and often briefer method of dividing a fraction by a fraction. (See Art. 122, p. 261.)

EXERCISE 70

1. Divide $\frac{9}{8}$ in. by $\frac{3}{8}$ in. Show this on the foot rule.

2. Similarly divide $\frac{3}{4}$ in. by $\frac{1}{4}$ in. $\frac{9}{16}$ in. by $\frac{3}{16}$ in. $\frac{3}{2}$ in. by $\frac{3}{4}$ in.

Divide and check your answer in each example:

3. $\frac{9}{10} \div \frac{3}{10}$
4. $\frac{16}{5} \div \frac{4}{5}$
5. $\frac{9}{32} \div \frac{3}{32}$
6. $\frac{1}{2} \div \frac{1}{5}$
7. $\frac{1}{2} \div \frac{2}{5}$
8. $\frac{3}{4} \div \frac{1}{3}$
9. $\frac{1}{2} \div \frac{1}{6}$
10. $\frac{2}{3} \div \frac{1}{6}$
11. $\frac{3}{4} \div \frac{1}{8}$
12. $\frac{3}{4} \div \frac{3}{8}$
13. $\frac{7}{10} \div \frac{7}{12}$
14. $\frac{5}{16} \div \frac{7}{24}$

15. Divide 2 in. by $\frac{1}{4}$ in. Show this on the foot rule.

16. Divide 3 in. by $\frac{3}{4}$ in. By $\frac{3}{8}$ in. By $\frac{3}{16}$ in.

Find the value of

17. $10 \div \frac{5}{2}$
18. $20 \div \frac{5}{4}$
19. $30 \div \frac{5}{6}$
20. $8 \div \frac{3}{4}$
21. $12 \div \frac{2}{3}$
22. $24 \div \frac{5}{8}$

23. Subtract $\frac{3}{4}$ in. from 12 in. as many times as you can. Now divide 12 in. by $\frac{3}{4}$ in. Compare the work in the two processes.

24. A piece of ribbon $5\frac{1}{2}$ ft. long may be divided into how many strips, each $\frac{1}{2}$ ft. long?

25. How many pieces of ribbon, each $\frac{2}{3}$ yd. long, can be cut from 6 yd.?

26. A plank 2 in. thick is how many times as thick as a board $\frac{3}{4}$ in. thick? As a board $\frac{7}{8}$ in. thick?

27. Eugene grew $\frac{3}{4}$ in. in the past year and Harold grew $\frac{3}{8}$ in. Eugene's growth was how many times as great as Harold's?

28. How many steps are there in the process of dividing $\frac{2}{3}$ by $\frac{1}{6}$? What is the first step? The second step? Can you make an example in division of fractions where the first step is not needed?

29. It costs 2¢ an ounce to mail a letter and $\frac{1}{2}$¢ an ounce to mail a book. Letter postage is how many times as much as postage on a book?

30. On squared paper mark off a square each side of which is 6 spaces. On this square show that $\frac{2}{3}$ (of 36) $\div$ $\frac{1}{6}$ (of 36) $= 4$.

31. On a square each side of which is 10 spaces, show what is meant by $\frac{3}{5} \div \frac{1}{5}$. Also by $\frac{3}{5} \div \frac{1}{10}$. By $\frac{4}{25} \div \frac{3}{20}$.

32. Make up and work an example similar to Ex. 24. Also one similar to Ex. 26.

33. How many of the examples in this Exercise can you work at sight?

34. Play the Tally Game with examples in addition, subtraction, multiplication, and division. See in which process you can make the best record.

V. *Combinations of Operations with Fractions*

EXERCISE 71

1. What is a parenthesis? For what is the parenthesis used in arithmetic? Give an example showing how the use of the parenthesis may save labor.

2. By use of the parenthesis, write in a shorter way: $\frac{1}{6}$ of $720 + \frac{5}{6}$ of 720.

3. Compute $\frac{1}{6} \times 720 + \frac{5}{6} \times 720$. Now compute $(\frac{1}{6} + \frac{5}{6}) \times 720$ by first adding the fractions inside the parenthesis. Compare the amount of work in the two processes. Which process is more likely to be correct?

By aid of the parenthesis, write each of the following in a briefer form:

4. $\frac{3}{2} \times \frac{9}{8} + \frac{1}{2} \times \frac{9}{8}$

5. $\frac{7}{8}$ of \$640 − $\frac{1}{8}$ of \$640

6. $\frac{5}{9} \times \frac{14}{9} - \frac{2}{9}$ of $\frac{14}{9}$

7. $\frac{3}{8}$ of $\frac{4}{9} + \frac{1}{8}$ of $\frac{4}{9}$

8. 3 less the sum of $\frac{1}{2}$ and $\frac{1}{3}$.

9. The sum of $\frac{2}{3}$ and $\frac{1}{2}$ less their difference.

10. The sum of $\frac{2}{3}$ and $\frac{1}{2}$ multiplied by their difference.

11. The sum of $\frac{2}{3}$ and $\frac{1}{2}$ divided by their difference.

12. Compute Ex. 4 by the short way.

13. Compute Exs. 5–11.

Indicate the work in the following by use of the parenthesis and then compute in the short way:

14. I bought 3000 lb. of sugar at $4\frac{1}{2}$¢ per pound and sold the same at $4\frac{3}{4}$¢ per pound. What was my gain?

15. I bought 6000 lb. of wire at $3\frac{1}{6}$¢ and sold the same at $3\frac{1}{2}$¢. What was my gain?

16. Make up and work an example similar to Ex. 4. To Ex. 14. Also two examples similar to such other examples in this Exercise as the teacher may indicate.

17. Work again Exercise 39 (p. 76) or similar examples dictated by the teacher.

VI. *Aliquot Parts of 100*

EXERCISE 72

1. What is the short way of multiplying 212 by 25? By 50? (See Art. 28, p. 44.)

2. Multiply the following by 25 in the short way.

(*1*) 612 (*2*) $540 (*3*) 96 ft. (*4*) 5280 ft.

Also multiply each of the above numbers by 50 in the short way.

3. $\frac{100}{3} = ?$ Hence, $33\frac{1}{3} = 100 \div$ what number?

4. Multiply 627 by $33\frac{1}{3}$. Also multiply 627 by $\frac{100}{3}$. Compare the work in the two processes.

$$\overset{209}{\cancel{627}} \times \frac{100}{\cancel{3}} = 209 \times 100 = 20900$$

$$\begin{array}{rl} 627 & \\ 33\frac{1}{3} & \\ \hline 209 & (= \frac{1}{3} \text{ of } 627) \\ 1881 & \\ 1881 & \\ \hline 20900 & \textit{Product} \end{array}$$

5. Multiply each of the following by $33\frac{1}{3}$ in the short way, using cancellation wherever possible:

(*1*) 921 (*2*) $1032 (*3*) 729 ft. (*4*) 6282 yd.

6. $\frac{200}{3} = ?$ Hence, $66\frac{2}{3} = 200 \div$ what number?

7. Multiply 627 by $66\frac{2}{3}$. Also multiply 627 by $\frac{200}{3}$. Compare the work in the two processes.

8. Multiply each of the following by $66\frac{2}{3}$ in the short way:

(*1*) 918 (*2*) $1035 (*3*) 935 ft. (*4*) 3282 yd.

9. $\frac{100}{8} = ?$ Hence, $12\frac{1}{2} = 100 \div$ what number?

10. Multiply 48 by $12\frac{1}{2}$. Now multiply 48 by $\frac{100}{8}$. Compare the work in the two processes.

11. Multiply each of the following by $12\frac{1}{2}$ in the short way:

(*1*) 424 (*2*) $864 (*3*) 1016 ft. (*4*) 720 ft.

12. $\frac{300}{8} = ?$ Hence, $37\frac{1}{2} = 300 \div$ what number?

13. Multiply 48 by 37½. Now multiply 48 by $\frac{300}{8}$. (Use cancellation.)

14. Multiply each of the following by 37½ in the short way:

(*1*) 360 (*2*) $728 (*3*) 1032 ft. (*4*) 736 yd.

15. $\frac{500}{8} = ?$ Hence, 62½ = 500 ÷ what number?

16. Multiply 48 by 62½. Now multiply 48 by $\frac{500}{8}$. (Use cancellation.) Compare the work in the two processes.

17. Multiply each of the following by 62½ in the short way:

(*1*) 368 (*2*) $736 (*3*) 1832 ft. (*4*) 3736 yd.

69. An Aliquot Part of a number is any number which will exactly divide the given number.

Thus, 12 is an aliquot part of 36. Also 33⅓ is an aliquot part of 100, since 33⅓ is contained in 100 exactly 3 times.

When one number is a simple fractional part of another number, it is sometimes called an aliquot part of the given number.

Thus, 66⅔ is not exactly contained in 100, but since it is ⅔ of 100, it is termed an aliquot part of 100.

EXERCISE 73

Multiply each of the following in the shortest way. Use cancellation wherever possible.

1. 224 by 12½ By 37½ By 62½ 87½ 25

2. $24 by 33⅓ By 66⅔ By 12½ 37½ 87½

3. 648 by 37½ By 33⅓ By 62½ 66⅔ 12½

4. 1080 by 12½ By 33⅓ By 37½ 66⅔ 25

5. 1250 by 12½ By 66⅔ By 25 37½ 33⅓

6. 2448 by 33⅓ By 37½ By 50 25 66⅔

7. 6000 by 12½ By 33⅓ By 87½ 37½ 50

Find the value of

8. 77 lb. of beef at $16\frac{2}{3}$¢. At $12\frac{1}{2}$¢.

9. 236 bu. of rye at $66\frac{2}{3}$¢. At $62\frac{1}{2}$¢.

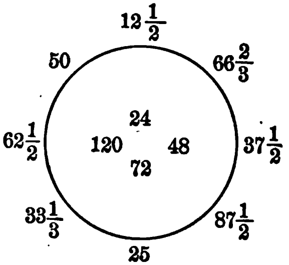
10. 375 bu. of corn at $62\frac{1}{2}$¢. At 75¢.

11. 566 yd. of carpet at $66\frac{2}{3}$¢. At $87\frac{1}{2}$¢.

12. 438 yd. of cloth at $37\frac{1}{2}$¢. At $33\frac{1}{3}$¢.

13. 2 doz. baseballs at $66\frac{2}{3}$¢ for one baseball. At $87\frac{1}{2}$¢.

14. A dealer bought 2464 bu. of wheat at 75¢ per bushel and sold it at $87\frac{1}{2}$¢ per bushel. Compute his gain in the shortest way.



15. As the teacher points, multiply at sight a number inside the ring by a number outside. (The numbers inside may be changed to 96, 240, 144, etc.)

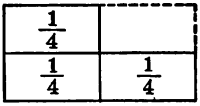
16. Make up and work an example similar to Ex. 6.

17. Also make an example similar to Ex. 7. To Ex. 8.

18. How many of the examples in this Exercise can you work at sight?

Analysis with Fractions

70. Given the Value of a Fraction, to Find the Value of the Whole.



Ex. If $\frac{3}{4}$ of a rectangular plot of ground is worth $1200, what is the whole plot worth?

Value of 3 fourths of a plot = $1200
" " 1 fourth of a plot = $\frac{1}{3}$ of $1200 = $400
" " 4 fourths or whole plot = 4 × $400 = $1600 *Ans.*

EXERCISE 74

1. If $\frac{3}{4}$ of a pie is worth 30 cents, what is the whole pie worth? (Draw a diagram to illustrate.)

2. If $\frac{2}{3}$ of a rectangular plot of ground is worth \$600, what is the whole plot worth? (Draw diagram.)

3. If $\frac{5}{6}$ of a pie is worth 25 cents, what is the whole pie worth? (Draw diagram.)

4. If $\frac{4}{5}$ of a rectangular plot of ground is worth \$840, what is the whole plot worth? (Draw diagram.)

5. If $\frac{3}{4}$ of a yard of silk costs 60 cents, what will a yard cost? (Draw diagram.)

6. If I pay \$30 for the use of borrowed money for 3 months, what ought I to pay for its use for a year? (Draw diagram.)

7. If $\frac{5}{8}$ of a vessel is worth \$3500, what is the whole vessel worth?

8. If $\frac{7}{12}$ of an oil well sells for \$8400, what is the entire well worth?

9. After being killed and dressed, a steer weighed 728 lb. This was $\frac{7}{10}$ of its live weight. What was its live weight?

10. A steamer going at $\frac{2}{3}$ her regular speed went 12 miles an hour. What was her regular speed?

11. On an acre of land a farmer raised 25 bushels of corn. This was $\frac{5}{7}$ of what he raised the next year when he used selected seed corn. What was his crop the second year?

12. 18 is $\frac{3}{4}$ of what number? $\frac{6}{7}$ of what number? $\frac{1}{6}$ of what number?

13. Mrs. Seaton sold $\frac{2}{7}$ of her chickens and had 45 chickens left. How many chickens did she have in all?

$\frac{7}{7} - \frac{2}{7} = \frac{5}{7}$

$\frac{5}{7}$ of her chickens $= 45$
$\frac{1}{7}$ " " " $= \frac{1}{5}$ of $45 = 9$
$\frac{7}{7}$ " " " $= 7 \times 9 = 63$ *Ans.*

If from all her chickens, or $\frac{7}{7}$, she sold $\frac{2}{7}$, she had $\frac{7}{7} - \frac{2}{7}$ or $\frac{5}{7}$, left.

If $\frac{5}{7}$ of them = 45 chickens, $\frac{1}{7} = \frac{1}{5}$ of 45, or 9.

Then $\frac{7}{7}$ = 7 times 9, or 63 chickens.

14. After selling $\frac{3}{5}$ of his farm, a farmer had 120 acres left. How many acres did he have at first? (How many fifths did he have left?)

15. A baseball nine lost $\frac{3}{7}$ of its games, and won the rest, or 36 games. How many games did the ball nine play in all?

16. Harriet received $\frac{1}{7}$ of the eggs which she found. She gave her mother 42 eggs. How many did she keep for herself?

17. After selling $\frac{7}{16}$ of my farm, I had 180 acres left. How many acres had I in my farm at first?

18. Two sevenths of my apples spoiled, and I sold the rest, which was 925 bushels. How many bushels were there all together? How many were spoiled?

19. Make up and work an example similar to Ex. 1. To Ex. 4. To Ex. 13.

20. Make up and work two examples entirely your own concerning analysis with fractions.

21. How many of the examples in this Exercise can you work at sight?

22. Practice counting by 2's, 3's, etc. to 9's, beginning with any number and making 144 the limit.

23. Practice rapid addition by decades.

71. Given the Value of a Fraction, to Find the Value of Another Fraction.

Ex. If $\frac{3}{5}$ of an acre of land is worth \$120, what is $\frac{4}{5}$ of an acre worth?

Value of $\frac{3}{5}$ acre = \$120
" " $\frac{1}{5}$ " = $\frac{1}{3}$ of \$120 = \$40
" " $\frac{4}{5}$ " = 4 × \$40 = \$160 *Ans.*

EXERCISE 75

1. If $\frac{3}{5}$ of a pie costs 18 cents, what does $\frac{4}{5}$ of the pie cost? (Draw diagram.)

2. If $\frac{5}{12}$ of a lot of ground costs \$350, what will $\frac{7}{12}$ of the lot cost? (Diagram.)

3. If $\frac{3}{4}$ of a melon costs 24 cents, what will $\frac{1}{2}$ of a melon cost?

4. Mr. A owns $\frac{3}{32}$ of a coal mine and Mr. B $\frac{5}{32}$. If Mr. A can sell his share for \$6000, what is Mr. B's worth?

5. If $\frac{3}{5}$ of a lot of ground sells for \$1500, what is the rest of the lot worth? (Diagram.)

6. A ball nine in one year lost 35 games, which was $\frac{5}{12}$ of the number played. How many games did the ball nine win?

7. After 72 days, or $\frac{2}{3}$, of the summer vacation are gone, how many days of vacation are left?

8. Make up and work an example similar to Ex. 1. To Ex. 4. To Ex. 6.

9. How many of the examples in this Exercise can you work at sight?

10. How many examples in Exercise 49 (p. 91) can you work at sight?

EXERCISE 76

Review

Find the sum of

1. $1\frac{3}{5} + 3\frac{1}{2} + \frac{5}{6} + 8\frac{11}{15}$
2. $\frac{11}{12} + \frac{4}{11} + 7\frac{19}{20} + 5\frac{1}{6}$
3. $\frac{14}{15} + \frac{11}{12} + \frac{19}{20} + 1\frac{13}{30}$
4. $3\frac{1}{8} + 7\frac{11}{24} + 6\frac{1}{9} + \frac{5}{12}$

Find the value of:

5. $\frac{2}{3} - \frac{10}{11}$
6. $1\frac{1}{6} - 2\frac{3}{7}$
7. $3\frac{4}{5} - 5\frac{1}{4}$
8. $\frac{9}{10} - \frac{2}{3}$
9. $1\frac{1}{7} - \frac{5}{6}$
10. $3\frac{7}{8} - \frac{11}{12}$
11. $3\frac{11}{12} - 1\frac{7}{15}$
12. $4\frac{9}{16} - 2\frac{5}{8}$
13. $7\frac{1}{10} - 5\frac{4}{15}$

Collect:

14. $5 + 3\frac{7}{8} - 2\frac{1}{3} + 5\frac{1}{6} - 6\frac{3}{4} + 7\frac{1}{2} - 11\frac{11}{12}$
15. $2\frac{1}{2} + 4 - \frac{7}{8} + 3\frac{3}{4} - 1\frac{1}{16} + \frac{5}{16}$
16. $8\frac{1}{2} + 9\frac{5}{21} + 4\frac{3}{8} - 7\frac{3}{4} - 6\frac{7}{24} - 3\frac{2}{3} + 1\frac{7}{16}$

Multiply:

17. $3\frac{1}{2}$ by $\frac{4}{7}$
18. $\frac{3}{4}$ by $2\frac{2}{3}$
19. $\frac{4}{11}$ by $9\frac{7}{8}$
20. 35 by $3\frac{3}{4}$
21. $17\frac{3}{5}$ by $16\frac{2}{3}$
22. 316 by $2\frac{4}{5}$
23. $\frac{2}{5}$ of $16\frac{1}{2}$
24. $\frac{7}{15}$ of $2\frac{11}{14}$
25. $\frac{5}{12}$ of $8\frac{7}{15}$

Compute in the shortest way:

26. 48 × $12\frac{1}{2}$
27. $\frac{1}{3}$ of 120 − $\frac{1}{5}$ of 120
28. $\frac{3}{4}$ of 120 − $\frac{3}{5}$ of 120
29. 600 × $33\frac{1}{3}$

30. Which is easier to visualize: $\frac{31}{4}$ or $7\frac{3}{4}$?

31. If a girl knits $\frac{1}{3}$ of a necktie a day, how long will it take her to knit 12 neckties?

32. A bottle which holds $\frac{3}{4}$ pint is filled from another containing $2\frac{1}{2}$ pints. How many pints are left in the latter?

33. Will the girl who works 4 examples out of 7, or the one who works 5 out of 9, get the higher grade?

Sug. Find out which fraction is larger, $\frac{4}{7}$ or $\frac{5}{9}$.

34. A grocer bought 288 oranges and sold $\frac{3}{8}$ of them. How many oranges did he have left?

35. One boy can run 100 yards in $13\frac{1}{2}$ seconds and another boy can run the same distance in $12\frac{2}{5}$ seconds. Find the difference between their times.

36. In the quarter-mile relay race the four runners from our school were timed in seconds as follows: $54\frac{1}{5}$, $53\frac{2}{5}$, $54\frac{3}{5}$, $51\frac{2}{5}$. What was their average time?

37. If I buy apples at $\frac{2}{5}$¢ each and sell them at $\frac{2}{3}$¢ each, how much will I gain on 10 dozen apples?

38. If a newsboy buys papers for $1\frac{2}{5}$¢ each and sells them for 2¢ each, how many papers will he have to sell in order to make 50¢?

39. An unskilled farmer one year received $7280 for the products of his farm. Later by rotating his crops he increased his receipts for a year by $\frac{2}{7}$. How much did he then receive?

40. By what number must $12\frac{1}{2}$ be multiplied to equal 100? To equal 250? 750?

41. A grocer bought a hogshead of sugar weighing 744 pounds at $4\frac{1}{8}$¢ per pound, and sold it at $4\frac{1}{2}$¢ per pound. What was his profit?

42. On a field of 20 acres planted without the use of fertilizer, a farmer obtained a crop of 12 bushels of wheat per acre worth $1.10 per bushel. On a field of the same size he used 300 pounds of fertilizer costing $24 a ton, and obtained a crop of 23 bushels of wheat per acre also worth $1.10. Show in the shortest way what he gained by the use of the fertilizer.

What would he have gained if he had used fertilizer on both fields?

43. In a certain year the rainfall in Arizona was 8 inches, in Colorado 12 inches, in New York 36 inches, and in Alabama 54 inches. The rainfall in Arizona was what fraction of that in Colorado? Of that in New York? In Alabama?

44. I buy 84 yards of goods at \$.12$\frac{1}{2}$ a yard. How much do I gain or lose by selling the same for \$11.50?

45. What does 30–4 mean in telegraphing? In telephoning?

46. Practice adding columns of figures.

Let the teacher dictate examples similar to those on p. 25.

EXERCISE 77

Oral Review of Fractions

1. Add $\frac{1}{2}$ to each of the following fractions. (The teacher should name the fractions in this list, or write them on the blackboard and point to each.)

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{1}{6}$ $\frac{5}{6}$ $\frac{1}{8}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$ $\frac{1}{10}$ $\frac{8}{10}$ $\frac{9}{10}$

2. Also to the above fractions add in turn: $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{3}{4}$

3. From the fractions in Ex. 1, subtract $\frac{1}{3}$ wherever possible. Subtract $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{2}{3}$

4. Multiply the fractions in Ex. 1 by 2. By 3. By 4 $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$

5. Divide the fractions in Ex. 1 by 2. By 3 4 $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{6}$ $\frac{2}{3}$ $\frac{1}{4}$

6. Practice the addition and subtraction of small fractions by a ring drill. Let the pupils take turns in writing rings.

7. Practice the multiplication of small fractions by a ring drill.

8. Practice the division of small fractions in the same way.

9. Give the value of each of the following:

(1) $12\frac{1}{4} - 3\frac{1}{4}$
(2) $5\frac{1}{2} - 2\frac{1}{3}$
(3) $4\frac{3}{4} + 2\frac{1}{2}$
(4) $7\frac{2}{3} - 1\frac{1}{3}$
(5) $18\frac{1}{2} - 3\frac{1}{3}$
(6) $17 - 2\frac{1}{2}$

10. A piece of wire 6 in. long can be cut into how many pieces each $\frac{1}{2}$ in. long? $\frac{1}{4}$ in.? $\frac{1}{3}$ in.? $\frac{3}{4}$ in.?

11. If $\frac{3}{4}$ of a building lot costs \$150, what will $\frac{1}{4}$ of the lot cost? The whole lot?

12. If $\frac{3}{8}$ of a pie costs 15 cents, what will the whole pie cost?

13. Give the value of each of the following:

(*1*) $\frac{1}{2}+\frac{1}{8}+\frac{1}{8}$	(*3*) $\frac{3}{4}-\frac{1}{2}+\frac{1}{8}$	(*5*) $\frac{2}{3}-\frac{1}{6}+\frac{1}{3}$
(*2*) $\frac{1}{2}+\frac{1}{4}-\frac{1}{8}$	(*4*) $\frac{1}{3}+\frac{1}{6}+\frac{1}{6}$	(*6*) $2\frac{1}{2}+\frac{1}{4}+\frac{1}{4}$

14 If $\frac{2}{5}$ of a yard of ribbon costs \$.27, what will $\frac{4}{5}$ of a yard cost?

SUG. $\frac{4}{5}$ is how many times as large as $\frac{2}{5}$?

15. If $\frac{1}{8}$ of a farm is worth \$3200, what is $\frac{1}{2}$ of the farm worth?

16. If $\frac{1}{2}$ of a barrel of oil weighs 108 lb., what will $\frac{1}{6}$ of a barrel weigh?

17. If a boat travels 26 miles in $2\frac{1}{2}$ hours, how far does it travel in 10 hours at the same rate?

18. If I go to a store and buy 3 lb. of sugar at 6¢ a pound and 2 lb. of coffee at 35¢ a pound, what change shall I receive for \$1?

19. How many examples in Exercise 51 (p. 93) can you now work at sight? In Exercise 59 (p. 102)?

EXERCISE 78

POST OFFICES

1. Commit to memory the following rates of postage:

(*1*) $\frac{1}{16}$¢ per oz. (that is, 1¢ per lb.) for newspapers and magazines mailed by the publishers.

(*2*) $\frac{1}{4}$¢ per oz. for newspapers and magazines mailed by anyone but the publishers.

(*3*) $\frac{1}{2}$¢ per oz. for books.

(*4*) 1¢ per oz. for general merchandise (as clothing, candy, athletic goods, etc.).

(*5*) 2¢ per oz. for letters.

For every fraction of a cent of postage due, a whole cent must be paid.

Thus, the postage due on a book weighing 9 oz. is 5 cents, since we cannot pay $4\frac{1}{2}$ cents.

2. Find the cost of the postage on the following articles, assuming that the wrappings are included in the weights given:

(*1*) a book weighing 12 oz.

(*2*) a book weighing 13 oz.

(*3*) a sweater weighing 17 oz.

(*4*) a pair of slippers weighing $13\frac{1}{2}$ oz.

(*5*) a letter weighing $\frac{3}{4}$ oz.

(*6*) 1 doz. handkerchiefs weighing 7 oz.

(*7*) a magazine (mailed privately) weighing $8\frac{1}{2}$ oz.

(*8*) $\frac{1}{4}$ lb. candy.

(*9*) a book weighing $13\frac{1}{2}$ oz.

(*10*) $1\frac{1}{2}$ lb. spices.

3. Emily takes to the post office for mailing a package of newspapers (9 oz.), a letter ($\frac{1}{4}$ oz.), a magazine ($7\frac{1}{2}$ oz.), and a pair of gloves (3 oz.). What must she pay? What change will she get for a dollar bill?

4. Make up and work an example similar to Ex. 3.

5. If there is a set of scales in the school, find how much postage you would pay if you had your Arithmetic mailed to you. Weigh other articles and compute the postage.

6. Guess the amount of postage required for certain articles, and then correct your guess by weighing them.

7. Find out how many sheets of the letter paper which you commonly use can be put into an envelope and sent by mail for 2¢.

8. Find out the value of the different stamps sold at the post office. If in mailing a book that weighs 37 oz. you use the fewest stamps possible, what different stamps will you use?

9. Make up and work two examples similar to the latter part of Ex. 8.

10. If each copy of a newspaper weighs $3\frac{1}{2}$ oz., what will it cost the publishers to mail 20,000 copies? What would it cost you to mail 1 copy? To mail 10 copies in one package?

11. If I receive by mail a book with 2 two-cent stamps and a one-cent stamp on it, what is the greatest possible weight of the book?

12. If I receive a bundle of newspapers with the same stamps as in Ex. 11, how much may the newspapers weigh? If I receive a box of candy with 4 two-cent stamps on it, what does the candy weigh?

13. How many times more expensive is it for you to send general merchandise than books by mail? Letters than books? Letters than magazines? Letters than newspapers? Books than newspapers? Magazines than newspapers?

14. Is it cheaper for me to send a manuscript weighing 3 lb. 4 oz. by mail, or by express at a cost of 40¢? How much cheaper? (The rate for manuscripts is the same as for letters.)

15. There were 903 post offices in the United States in the year 1800, and 76,688 in 1900. What was the increase in 100 years? In 1900 there were how many times as many post offices as in 1800?

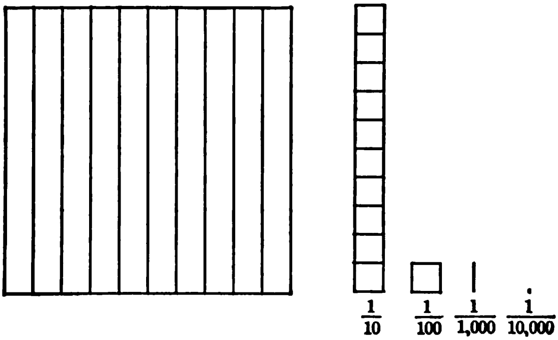
16. Do you know the meaning of the picture on p. 131? Can you name some of the advantages of the Rural Free Delivery to people living in the country?

17. Practice multiplication by the aliquot parts of 100 as in Exercise 73 (p. 111). The teacher or pupils may dictate examples.

CHAPTER VI

DECIMAL FRACTIONS

NUMERATION AND NOTATION TO THOUSANDTHS



EXERCISE 79

1. If a square is divided into 10 equal parts, what is each part called? How many tenths in a whole?

2. If one tenth of the square is divided into 10 equal parts, what is each part called? How many hundredths in one tenth?

3. If one hundredth of the square is divided into 10 equal parts, what is each part called? How many thousandths in one hundredth?

4. What is $\frac{1}{10}$ of $\frac{1}{10}$? What is $\frac{1}{10}$ of $\frac{1}{100}$?

5. How many tenths in 1? How many hundredths in $\frac{1}{10}$?

6. A dime is what part of a dollar? A cent is what part of a dime? A cent is what part of a dollar?

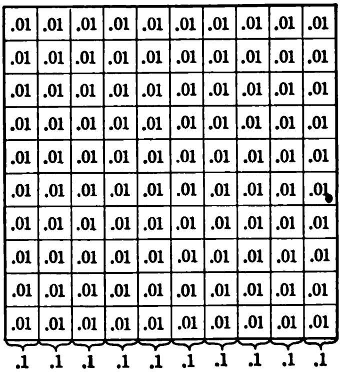
Write 7 dimes and 3 cents by use of the decimal point.

7. In 6777, the 7 in the units' place has what part of the value of the 7 in the tens' place? The 7 in the units' place has what part of the value of the 7 in the hundreds' place?

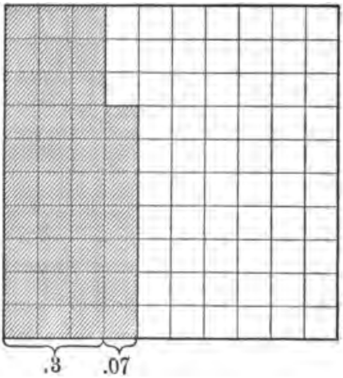
8. These examples suggest a short way of writing tenths, hundredths, etc. For $\frac{7}{10}$ we write .7. For $\frac{7}{100}$ the 7 must be written one place further from the dot, that is, in the form .07, since $\frac{7}{100}$ has $\frac{1}{10}$ the value of $\frac{7}{10}$. Similarly, $\frac{7}{1000}$ is written .007.

9. Compare the work of writing $\frac{7}{10}$, with that of writing .7. Write $\frac{73}{100}$ and .73 How many figures are there in $\frac{73}{100}$? In .73?

10. On squared paper mark off a square having 10 spaces on each side. Divide the square into 100 equal parts as in the first diagram below.



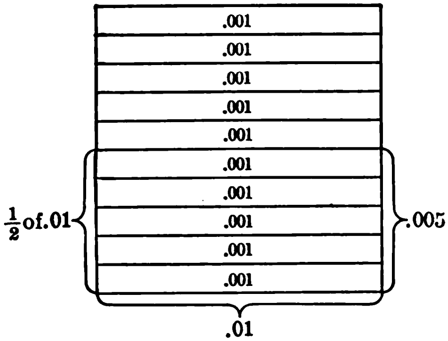
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.1	.1	.1	.1	.1	.1	.1	.1	.1	.1



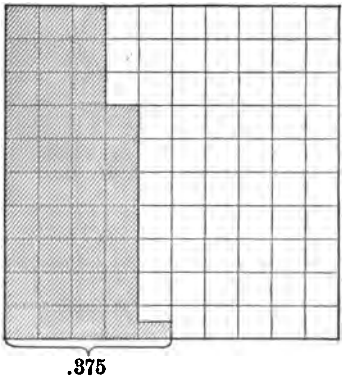
.37 in all is shaded.

11. Make a square of the same size, leaving the spaces unmarked. Shade the part of this square which stands for $\frac{3}{10}$, or .3, of the whole. Next to this, shade the part which stands for $\frac{7}{100}$, or .07, of the whole. You have now shaded the part which stands for .37 of the whole. Keep this square to use again.

12. Draw a small square and let it represent .01. Divide this square into 10 equal parts as in the first diagram below. Mark the thousandths on this square. Show that $\frac{1}{2}$ of .01 = .005.



The above square represents $\frac{1}{100}$ part of the square to the right drawn to a large scale, in order to show the meaning of thousandths.



Shaded part is .375 of the whole.

13. On the square you used in Ex. 11, next to the part already shaded mark off also the part which stands for $\frac{5}{1000}$, or .005. Now shade this and show the entire part which stands for .375 of the whole.

14. On squared paper by shading show the meaning of .25. Of .55. Of. 555. Of .685.

15. On squared paper in like manner show the meaning of

.9 .27 .676 .75 .183 .07 .03 .025

16. In .762, what does the 7 stand for? The 6? The 2?

17. In .38, what does the 3 stand for? The 8? If we put the figure 4 between the decimal point and 3, we get .438. What change has been made in the value of 3? Of 8? Hence, what three items of work does the 4 do?

18. In 527, what does the 5 denote? The 2? The 7? If we put a decimal point to the left of 5, what change is made in the meaning of 5? Of 2? Of 7? What three items of work does the decimal point do?

19. What items of work does the zero do in .067?

20. Compare the work of writing $\frac{7}{10} + \frac{3}{100} + \frac{9}{1000}$, with that of writing .7 + .03 + .009. Also with that of writing .739.

21. Why do we now make definitions and rules?

72. A **decimal fraction**, or a **decimal**, is a fraction whose numerator only is written, and whose denominator is 10, or 100, or 1000, or some similar multiple of 10.

A **decimal point** is a dot placed to the left of the numerator of a decimal fraction to take the place of the denominator.

It is sometimes necessary to insert one or more zeros between the decimal point and the numerator. Thus, $\frac{7}{100}$ is written in the form .07.

73. **Uses of Decimal Fractions.** (1) In decimal fractions we are saved the labor of writing all denominators.

(2) It is easier to reduce decimal fractions to like units and perform operations with these fractions than with common fractions.

These advantages in the use of decimal fractions lead to other advantages which will become evident as we proceed.

74. **Numeration and Notation of Decimals.** The first place to the right of the decimal point is called the tenths' place; the second place to the right is called the hundredths' place, etc. Hence we have the following

Numeration and Notation Table

Ten-thousands	Thousands	Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths
7	3	9	2	8	.	4	6	7

75. Reading Decimals. The most convenient way of reading decimals is to

Read the decimal as if it were a whole number, and give it the name of the last decimal place.

Thus, to read .37, since 7 is in the hundredths' place, we read "thirty-seven hundredths."

This is shorter than to read "three tenths seven hundredths."

In reading a number consisting of a whole number and a decimal, use *and* where the decimal point occurs.

Thus, 2.406 is read "two *and* four hundred six thousandths."

The number in the above table is read "seventy-three thousand nine hundred twenty-eight and four hundred sixty-seven thousandths."

EXERCISE 80

Read:

1. .56 .285 7.85 8.209 9.1 7.08 .007 1.001

2. 3.03 .303 30.03 200.06 200.006 .206 .048

3. .004 38.375 95.023 7.006 .076

4. 78.506 325.049 719.4 920.763 71.123

5. One cubic foot of water weighs 62.5 pounds. (What is a cubic foot?)

6. Ice is .92 as heavy as water.

7. One meter equals 39.37 inches.

A *meter* is the unit of length in the metric system of measurement.

Read:

8. The area of New Jersey is .118 that of New York.
9. One square rod contains 30.25 square yards.
10. One year contains 365.242 days.
11. On squared paper show the meaning of Ex. 8.
12. Work again Exercise 21 (p. 45) or similar examples.

76. Writing Decimals. To express a decimal fraction in figures,

Write the numerator in figures;

Place the decimal point so that the tenths of the given decimal fraction are in the tenths' place, hundredths in the hundredths' place, and so on.

Ex. Express in figures sixty-two thousandths.

We first write 62. Since the 2 must come in the thousandths' or third place, it is necessary to prefix a zero before writing a decimal point to the left. Hence we obtain .062.

Another way of writing a decimal is to set down the decimal point and decide how many zeros are needed before writing any figures.

EXERCISE 81

Write the following as decimal fractions:

1. Seven tenths.
2. Sixty-seven hundredths.
3. Eighty-nine thousandths.
4. Nine hundredths.
5. Three thousandths.
6. Nineteen thousandths.
7. Three hundred and seventy-two thousandths.
8. Three hundred seventy-two thousandths.

Write each of the following in figures:

9. One and forty-five hundredths.
10. Six hundred ninety-four thousandths.
11. Six hundred and ninety-four thousandths.
12. Ten and two thousandths.

Rewrite the following, expressing all numbers by the use of figures, and all fractions as decimal fractions:

13. Cork is twenty-four hundredths as heavy as water.

14. Gold is nineteen and three tenths times as heavy as water.

15. The height of Mt. Washington is four hundred four thousandths of the height of Pike's Peak.

16. A cubic foot of water weighs sixty-two and forty-three hundredths pounds.

17. Practice rapid work on the blackboard with examples in the addition and subtraction of United States money.

The teacher should dictate examples similar to those on p. 22, Exs. 4, 5, 9, 10, and 15, and p. 29, Exs. 10, 11, and 19.

EXERCISE 82

1. What does the 6 in .63 mean? The 6 in .063?

2. The value of the 6 in .63 is how many times as great as the value of the 6 in .063? In like manner, compare the 3's in these two numbers.

3. Put a zero between the decimal point and the 6 in .63. What change has been made in the value of the 6? In the value of .63?

4. Hence, what short way is there of dividing a decimal by 10?

5. Remove the zero in .027. What change has been made in the value of the 2? Of the 7?

6. What is the short way of multiplying a decimal by 10?

7. Remove the two zeros in .008. What change has been made in the value of the 8?

8. What is the short way of multiplying a decimal by 100?

77. First Principles of Decimals. As a result of the method used in writing decimals, we find that

1. *Moving the decimal point one place to the right multiplies a decimal fraction by* 10.

Thus, $.063 \times 10 = .63$.

2. *Moving the decimal point one place to the left divides the decimal by* 10.

Thus, $.56 \div 10 = .056$.

3. *Any number of zeros may be annexed to the right of a decimal without changing its value.*

For $\frac{3}{10} = \frac{30}{100} = \frac{300}{1000}$. (See Art. 59, p. 94.)
Hence, $.3 = .30 = .300$.

Hence, to reduce decimal fractions to similar fractions,

Annex enough zeros to each decimal fraction to give it the same number of places as the decimal fraction having the largest number of places.

Thus, to make .27, .7, and .316 similar we write them .270, .700, and .316.

EXERCISE 83

1. Multiply each of the following by 10:

.07 .18 .003 .092 6.13 .7

2. Divide each of the following by 10:

.9 .23 .07 .72 7.3 7.36

3. Reduce the following to similar fractions: .32 .8 .265.

4. Also .7, .27. Also .8, .088. Also 1.3, .07, .081.

5. Reduce $\frac{3}{20}$ and $\frac{7}{25}$ to similar fractions. Now reduce .3 and .61 to similar fractions. Compare the work in the two processes.

6. By using common fractions as in Art. 77, show that

$$.9 = .90 = .900.$$

7. Which is greater: 3 hundredths, or 3 thousandths?

8. Which would you rather have: a share of 3 tenths in a gold mine, or a share of 3 thousandths? Why?

9. How many figures are used in writing 17 hundredths as a decimal fraction? In writing 7 thousandths? 47 thousandths? 4 tenths?

Let the teacher give similar oral questions as a drill.

10. How many examples in this Exercise can you work at sight?

11. Practice writing decimal fractions dictated by the teacher.

PROCESSES WITH DECIMALS

I. *Addition of Decimals*

78. In **Adding Decimals,** we

Write the numbers so that their decimal points are in the same column;

Begin with the right-hand column and add as in whole numbers;

Place a decimal point between the units and the tenths of the result.

Ex. If three fields contain 17.32, 2.363, and 5.7 acres respectively, how many acres are there in the three fields together?

```
17.32
 2.363
 5.7
------
25.383 Sum
```

EXERCISE 84

1. On squared paper (see p. 134) show the meaning of .45. Also of .22. Now add .45 and .22 on squared paper.

2. On squared paper find the sum of each of the following:

.3 + .5	.35 + .25	.3 + .216
.4 + .25	.42 + .16	.32 + .235

Add, checking the work in each example:

3. 12.36 ft., 6.4 ft., 3.12 ft.

4. 2.28 gal., 13.5 gal., 5.29 gal.

5. 3.725 T., 12.56 T., 3.175 T., 4.65 T.

6. 27.05, 3.87, 15.18.

7. 100.001, 38.817, 192.4, 77.082.

8. 99.116, 127.83, 25.009, 130.705.

9. 11.234, 1.1234, 112.34, 1123.4.

10. Find the sum of $4\frac{7}{10} + 5\frac{17}{100} + 6\frac{189}{1000}$ as common fractions. Now write these mixed numbers in the decimal form and add. Compare the work in the two processes.

11. Make up and work an example similar to Ex. 10.

12. Simplify as much as possible: seven tenths + eighty-three hundredths + seventy-six thousandths.

13. Add $\frac{4}{25}$ and $\frac{7}{20}$ as common fractions. Now add .16 and .35. Compare the work in the two processes.

14. In three successive years Walter gained in height 1.42 in., 1.73 in., and 1.64 in. How much did he gain in the three years?

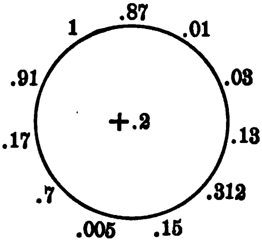
15. If Walter's height (Ex. 14) at the beginning of the three years was 4 ft. 7 in., what was it at the end?

16. The ice on the pond was 3.72 in. thick and it increased in thickness 1.52 in. and .7 in. in two successive days. How thick was the ice then?

17. The sides of a certain field are 224.5 ft., 238.75 ft., 192.6 ft., and 172.8 ft. long. Find the perimeter of the field.

18. If the rainfall during March was 3.72 in., during April 2.16 in., and during May 1.3 in., what was the rainfall during all three of these months?

19. At sight, as the teacher points, add the number inside the ring to each number on the outside. (The numbers may be changed at the discretion of the teacher.)



20. Make up and work an example similar to Ex. 14. To Ex. 15. Ex. 18.

21. Make up and work two problems entirely your own involving the addition of decimal fractions.

22. Practice the rapid written addition of columns of decimal fractions dictated by the teacher.

II. *Subtraction of Decimals*

79. In Subtracting Decimals, we

Write the subtrahend under the minuend, with the decimal points in the same column;

Begin at the right and subtract as in whole numbers;

Place the decimal point in the proper place in the result.

Ex. Subtract 5.802 from 10.09.

$$\begin{array}{r} 10.090 \\ \underline{5.802} \\ 4.288 \end{array}\ \textit{Difference}$$

EXERCISE 85

1. On squared paper show the meaning of .52. Also of .3. Now subtract .3 from .52 on squared paper.

2. On squared paper subtract each of the following:

.2 from .7
.5 from .73
.25 from .55
.16 from .48
.2 from .372
.25 from .592

Find the value of each of the following and check each result:

3. 17.75 ft. – 5.25 ft.

4. 125 ft. – 6.2 ft.

5. 8.27 in. – 2.3 in.

6. 9.652 in. – 2.37 in.

7. 16.73 gal. – 2.73 gal.

8. 3.08 – 1.75

9. 9.35 – 2.7

10. 10.14 – .07

11. .089 – .032

12. .765 – .048

13. 2.8 – .028

14. 4.7 – .09

15. 19 – 13.756

16. 6.327 – .005

17. 17.32 – .217

18. Reduce to a simpler form 16.723 – 5.108.

19. Subtract $\frac{4}{25}$ from $\frac{7}{20}$. Now subtract .16 from .35. Compare the amount of work in the two processes.

20. Last evening the ice on the pond was 5.25 in. thick. This morning it is 6.72 in. thick. How much has it frozen during the night?

21. A year ago Mary was 4 ft. 6.5 in. tall. Now she is 4 ft. 8.2 in. tall. How much has she gained in the year?

22. A year ago Harry could run the 100-yard dash in 13.5 seconds. Now he can run it in 12.8 seconds. How much has his record improved in one year?

23. A gallon of milk weighs 8.622 lb. and a gallon of water 8.355 lb. How much more does the milk weigh than the water?

24. The English shilling is worth \$.243, the French franc \$.193, and the German mark \$.238. What is the difference between each pair of these? Also between each of them and the quarter-dollar?

25. Make up and work two examples similar to such of the examples in this Exercise as you think are most interesting or instructive.

26. Make up a ring and have a sight drill in the subtraction of decimal fractions. (See Ex. 19, p. 143.)

27. Practice the rapid written subtraction of decimal fractions dictated by the teacher.

III. *Multiplication of Decimals*

80. The Multiplication of a Decimal Fraction by a Whole Number.

Ex. Multiply .017 by 3.

$$.017 \times 3 = \frac{17}{1000} \times 3 = \frac{51}{1000} = .051$$

Notice that the number of decimal places in the product equals the number of decimal places in the multiplicand.

Hence, the above multiplication might have been performed as follows:

$$\begin{array}{rl} .017 & \textit{Multiplicand} \\ \underline{3} & \textit{Multiplier} \\ .051 & \textit{Product} \end{array}$$

EXERCISE 86

1. On squared paper mark off a square each side of which is 10 spaces. Show what 1 tenth of this square means. Show the meaning of 2 times 1 tenth. Of 2 × .1. Of 3 × .1.

2. On the same square show what .2 means. Also 3 × .2. Also 4 × .2.

3. On the square show what $\frac{3}{100}$ means. Also what .03 means. Also 2 × .03. 3 × .03. 7 × .03.

4. On the square show what .15 means. Show what 3 × .15 means. Also 4 × .15. 5 × .15.

Find the value of

5. 4 × .07 ft.

6. 4 × .17 in.

7. 4 × .27 cu. in.

8. $1.213 × 7

9. 3.203 ft. × 5

10. 12.65 yd. × 8

11. 17.03 × 8

12. 212.03 × 15

13. 3.706 × 104

14. What is the easiest way to multiply a decimal by 10? By 100?

15. Multiply each of the following numbers by 10:

3.16 .17 .216 17.635 .007

Also multiply each of them by 100.

16. If a boy's average gain in height per year is 1.72 in., how much will he grow in 5 years?

17. If a boy's average gain in weight per year is 7.35 lb., how much will he gain in weight in 3 years?

18. How much does it cost to launder 125 collars, at $.025 per collar?

19. Write .023 twelve times and add. Now multiply .023 by 12. Compare the work in the two processes.

20. Can you tell about how many times as easy it is to multiply 15.062 by 48 as to set down 15.062 forty-eight times and add?

21. If 1 turn of a screw advances the point of the screw .14 in., how far will 5 turns of the screw advance the point? 3 turns? 4 turns?

22. 1 franc is worth $.193. What is the value of 212 francs?

23. Make up and work an example similar to Ex. 16. To Ex. 18. Ex. 19.

24. Practice the written multiplication of decimals by whole numbers, the examples being dictated by the teacher.

25. How many of the examples in this Exercise can you work at sight?

81. The Multiplication of a Decimal by a Decimal.

Ex. 1. Multiply 2.17 by .5.

$$2.17 \times .5 = \frac{217}{100} \times \frac{5}{10} = \frac{1085}{1000} = 1.085 \textit{ Product}$$

Notice that the number of decimal places in the product is equal to the number of decimal places in the multiplier and the multiplicand taken together.

Hence, the above multiplication might have been performed thus:

$$\begin{array}{r} 2.17 \\ .5 \\ \hline 1.085 \end{array}\ \textit{Product}$$

Hence, to multiply a decimal fraction by either a whole number or another decimal fraction,

Multiply as in whole numbers;

Mark off as many decimal places in the product as there are decimal places in both multiplier and multiplicand together, prefixing zeros to the product if necessary.

Ex. 2. Multiply .18 by .5.

$$\begin{array}{r} .18 \\ .5 \\ \hline .090 \end{array}$$

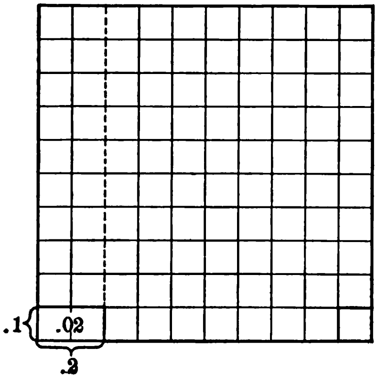
The multiplicand and multiplier together contain three decimal places. To make the product contain three decimal places in the product, we insert a zero between the decimal point and 9.

EXERCISE 87

1. On squared paper mark off a square each side of which is 10 spaces. On this square mark off .2 by a dotted line. Now indicate .1 of .2.

Show that .1 of .2 = .02.

Also that .1 × .2 = .02.



2. On the same or a similar square show what each of the following equals:

.1 × .3   .1 × .4   .1 × .8
.2 × .3   .2 × .2   .3 × .5
.4 × .5   .4 × .6   .7 × .8

Find the value of

3. .32 ft. × .4
4. 16.5 ft. × .5
5. 1.2 × .27 in.
6. 1.36 yd. × .04
7. 2.3 × .15
8. 18.52 × .02
9. 32.7 × .03
10. 6.15 × .017
11. 3.8 × .002
12. 3.82 × .003
13. 1.76 × .172
14. 3.08 × 1.203

15. What is the easiest way to multiply a number by .1? By .01?

16. Multiply each of the following numbers by .1:

3.76 .07 .17 3.06 27. 376.04.

Also multiply each of them by .01.

17. If a boy grows 1.72 in. in a year, how much will he grow in 9 months, or .75 of a year?

18. 1 franc is worth $.193. What are 21.5 francs worth?

19. If 1 lb. of cheese costs $.115, what will 5.5 lb. cost?

20. 1 rod = 16.5 ft. How many feet are there in 3.27 rods?

21. 1 sq. rd. = 30.25 sq. yd. How many square yards are there in 3.76 sq. rd.?

22. Make up and work three examples similar to such of the examples in this Exercise as you think are most interesting or instructive.

23. Write the following arrangement of numbers on the blackboard. Multiply each number in the upper row by each number in the lower row, as the teacher points.

.02	.7	.13	.5	.03	1.2	1.03	.6	.08
3	.1	.2	4	.3	.9	.5	.8	.04

24. What examples in this Exercise can you work at sight?

25. Practice the written multiplication of decimals by decimals, the examples being dictated by the teacher.

IV. *Division of Decimals*

82. The Division of a Decimal Fraction by a Whole Number.

Ex. 1. Divide .126 by 7.

$$.126 \div 7 = \frac{126}{1000} \div 7 = \frac{\overset{18}{\cancel{126}}}{1000 \times \cancel{7}} = \frac{18}{1000} = .018.$$

This work may be put into the following short form:

$$\begin{array}{r} 7\overline{).126} \\ .018 \end{array} \textit{ Quotient}$$

Hence, in dividing a decimal fraction by a whole number,

Divide as in whole numbers;

Mark off as many decimal places in the quotient as there are decimal places in the dividend.

In dividing a decimal fraction by a whole number, place the decimal point for the quotient above or below the decimal point in the dividend (*above* in long division and *below* in short division).

Ex. 2. Divide 17 by 6 and carry the quotient to four decimal places.

$$\begin{array}{r} 6\overline{)17.0000} \\ 2.8333+ \end{array}$$

We may write a decimal point and a number of zeros after the 17, since this does not change the value of 17. We then divide as in Ex. 1.

The answer is to be carried only to four decimal places. Hence, after obtaining 2.8333, we are indifferent to the remainder, but annex a + sign to show that the division is not exact.

EXERCISE 88

1. On squared paper mark off a square each side of which is 10 spaces. On it show that .4 ÷ 2 = .2.

2. On the square show also the meaning of $2\overline{)6 \text{ tenths}}$. Also of $2\overline{).6}$ $2\overline{).8}$ $3\overline{).6}$ $4\overline{).8}$ $2\overline{).04}$ $2\overline{).06}$ $3\overline{).09}$

3. Also show the meaning and value of 2).4 + 2).06. Hence, of 2).46 Also of 2).64 3).69 4).84

4. Make up and work three examples similar to those in Ex. 2 and Ex. 3.

Divide and check each result:

5. 5)12.5 ft.

6. 4)$8.48

7. 7)84.14 yd.

8. 9)157.14 in.

9. 5).08

10. 6)7.266

11. 3)9.45

12. 8).128

13. 9).288

Find the value of

14. 1.32 ÷ 12

15. 14.82 ÷ 13

16. 94.08 ÷ 48

17. 7 ÷ 35

18. 7.776 ÷ 36

19. 1.076 ÷ 20

20. 146.88 ÷ 108

21. 105.3 ÷ 15

22. 92.4 ÷ 24

23. What is the easiest way to divide a decimal fraction by 10? By 100?

24. Divide each of the following by 10:

2.1 31.6 17 .17 4.23 .06

Also divide each of them by 100.

25. Divide 4 by 8. Also by 80. Also by 800.

26. If 12 baseballs cost $18.60, what is the cost of 1 baseball? In order to gain 25¢ on each baseball, at what price per ball must a dealer sell them?

27. If the living expenses of a family for one month (30 days) are $68.40, what are the expenses for one day? If the family wish to reduce their expenses $10 per month, how much must their expenses be reduced per day?

28. If a boy's height in 5 years increases from 4.26 ft. to 5.37 ft., what is the average gain per year?

29. If 1 cu. ft. of water weighs 62.5 lb., what will $\frac{1}{2}$ cu. ft. weigh? $\frac{1}{4}$ cu. ft.? $\frac{1}{5}$ cu. ft.?

30. If 1 turn advances a screw .24 in., how far will $\frac{1}{2}$ turn advance the screw? $\frac{1}{4}$ turn? $\frac{1}{8}$? $\frac{1}{10}$?

31. Divide $\frac{4}{25}$ by 8. Now divide .16 by 8. Which of the two processes is the easier?

32. Divide 14 by 6 to three decimal places.

33. Divide each of the following (to three decimal places, if the division is uneven):

$5\overline{)12}$ $7\overline{)12}$ $8\overline{)10}$ $27 \div 13$ $216 \div 315$

34. The Washington Monument (555 ft.) is how many times as high as the Bunker Hill Monument (221 ft.)?

35. If a cubic foot of iron of a certain quality weighs 462 lb., how many cubic feet will weigh a ton?

36. Make up and work an example similar to Ex. 26. To Ex. 27. To Ex. 35.

37. Place the following arrangement of decimal fractions on the blackboard. As the teacher points, divide each fraction by 2. By 3.

.1	.04	.8	.6
.018	.006	.3	.24
.2	.12	.18	.36

38. How many of the examples in this Exercise can you work at sight?

39. Practice the written division of decimals by whole numbers, the examples being dictated by the teacher.

83. The Division of a Decimal by a Decimal.

Ex. Divide .182 by .13.

$$\frac{.182}{.13} = \frac{.182 \times 100}{.13 \times 100} = \frac{18.2}{13.}$$

Hence, instead of dividing .182 by .13, we may divide 18.2 by 13 and proceed thus:

$$\begin{array}{r} 1.4 \ \textit{Quotient} \\ {}_{\bullet}13)\overline{{}_{\bullet}18.2} \\ \underline{13\,} \\ 52 \\ \underline{52} \end{array}$$

Hence, to divide a decimal by a decimal,

Move the decimal point in both divisor and dividend as many places to the right as there are decimal places in the divisor;

Divide as in whole numbers;

Mark off as many decimal places in the quotient as there are decimal places left in the dividend.

EXERCISE 89

1. On squared paper mark off a square each side of which contains 10 spaces. On this show that .2 is contained in .4 twice. Hence, .4 ÷ .2 = ?

2. On this square show also the meaning and value of

2 tenths)6 tenths   .2).6   .3).6   .2).8   .3).9   .03).06   .02).08

3. Also show the meaning and value of

.02).1   .05).1   .02).4   .05).2   .03).6   .06).3

4. Also of   .02).42   .03).63   .05).35   .8).48   .06).54   .07).49

Divide and check each result:

5. .5 ft.)12.5 ft.
6. .4 in.)8.48 in.
7. .7)84.14 yd.
8. .9)117.18 in.
9. .08 by .05
10. 7.266 by .6
11. 9.45 by .003
12. .128 by .08
13. 1.32 by 1.2
14. 14.82 by .13
15. 94.08 by .048
16. 7 by .35
17. 7.776 by 3.6
18. 14.688 by 1.08
19. 105.3 by .02

20. Divide 1.225 by .05. By .25. By .025.

21. What is the easiest way to divide a number by .1? By .01?

22. Divide each of the following by .1:

.3 2 2.1 .36 .372 5.12 .006 .07

Also divide each of these numbers by .01.

23. Divide 4 by .2. Also by .002. Also divide 4 by 8; by 80; by 800.

24. How many feet of wire, at $.005 a foot, can be bought for $2?

25. Into how many pieces, each .3 in. long, can a yard of wire be cut?

26. If railroad fare is 2.4¢ a mile, $10 will pay the fare for how many miles?

27. Divide $\frac{6}{25}$ by $\frac{3}{10}$. Now divide .24 by .3. Compare the work in the two processes.

28. A certain barrel holds 8.5 gallons. How many bottles can be filled from this, if each bottle holds .125 gallon?

29. If 1 cu. ft. of water weighs 62.5 lb., how many cubic feet will there be in 1000 lb. of water?

30. When a certain screw is turned through a complete revolution, the point of the screw goes forward .132 of an inch. How many times must you turn the screw to drive it into a piece of wood to a depth of 1.716 inches?

31. If the pitch of a screw is .166 inch, how many turns must you give the screw to move it forward a distance of 1.328 inches?

The *pitch* of a screw is the distance the screw moves forward when turned through one complete revolution.

32. Compute in the shortest way $1.379 \div 25 - 1.261 \div 25$.

33. Make up and work an example similar to Ex. 26. To Ex. 27. To Ex. 30.

34. Make up and work two problems entirely your own involving the division of one decimal fraction by another.

35. How many of the examples in this Exercise can you solve as sight examples?

36. Write the following arrangement of decimal fractions on the blackboard. As the teacher points, divide each number by .2. By .3.

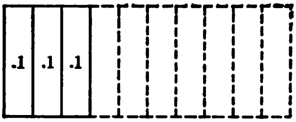
.6	3.6	.14	.12	3.	.06
1.2	.48	.9	.15	.09	.18

37. Practice the written division of decimals by decimals, the examples being dictated by the teacher.

Analysis of Decimals

EXERCISE 90

1. If .3 of a plot of ground is worth $60, what is the whole plot worth?



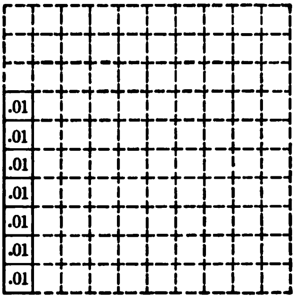
Value of 3 tenths (or .3) of plot = $60
" " 1 tenth (or .1) " " = $\frac{1}{3}$ of $60 = $20
" " 10 tenths (or whole) of plot = 10 × $20 = $200 *Ans.*

2. If .7 of a plot of ground is worth $140, what is the whole plot worth? (Diagram this.) What is .3 of the plot worth?

3. Mrs. Swift sold 40 chickens, which was .2 of all that she had. How many did she have?

4. If I pay $300 for the use of a sum of money for a certain time and this is .2 of the money borrowed, how much have I borrowed?

5. If .07 of a piece of land is worth \$280, what is .04 of the whole piece worth? Also .37? Also the whole piece?



Value of .07 = \$280
“ “ .01 = $\frac{1}{7}$ of \$280 = \$40
“ “ .04 = 4 × \$40 = \$160 *Ans.*
“ “ .37 = 37 × \$40 = \$1480 *Ans.*
“ “ whole = 100 × \$40 = \$4000 *Ans.*

6. If .03 of a piece of land sells for \$600, what should .07 sell for? .32? The whole piece of land? (Diagram this.)

7. A man pays \$40 tax. This is .02 of the value of his property. What is the value of his property?

8. Make up and work an example similar to Ex. 3.

9. Similar to Ex. 5. To Ex. 6.

10. Practice making change as in Exercise 13 (p. 33). You may practice paying for telegraph or telephone messages, or for purchases made at a store.

Common and Decimal Fractions

84. To Change a Decimal to an Equivalent Common Fraction in its lowest terms,

Write the decimal fraction as a common fraction, and reduce the common fraction to its lowest terms.

Ex. Express .75 as a common fraction in lowest terms.

$$.75 = \frac{75}{100} = \frac{3}{4} \textit{ Ans.}$$

EXERCISE 91

1. On squared paper, using the standard square with 10 spaces on each side, show that .25 (of 100 square spaces) = $\frac{1}{4}$ (of 100 spaces). Also that .4 = $\frac{2}{5}$ .75 = $\frac{3}{4}$ .35 = $\frac{7}{20}$ .16 = $\frac{4}{25}$

Change each of the following to an equivalent common fraction in its lowest terms:

2. .25 .4 .04 .16 .72 .84 .36

3. .045 .005 .125 .625 .875 .525 .085

4. 2.15 3.75 10.05 8.125 3.375

5. Express .25 of a year as months. Also .75 yr. .5 yr.

6. Express .25 lb. as ounces. Also .75 lb. .2 lb. .6 lb.

7. Make up and work two examples similar to those in Ex. 2.

8. Also to those in Ex. 5.

9. Also to those in Ex. 6.

10. How many of the examples in this Exercise can you work at sight?

11. Work again Exercise 38 (p. 74) or similar examples dictated by the teacher.

85. To Change a Common Fraction to a Decimal,

Divide the numerator by the denominator.

Ex. 1. Change $\frac{7}{8}$ to the form of a decimal fraction.

$$8\overline{)7.000}$$
$$.875 \text{ Ans.}$$

7 units = 7000 thousandths. $\frac{1}{8}$ of 7000 thousandths is 875 thousandths, or .875.

Ex. 2. Express $\frac{5}{18}$ as a decimal fraction to three decimal places.

$$
\begin{array}{rl}
 & .277\tfrac{7}{9} \text{ or } .277 + \textit{Ans.} \\
18) & \overline{5.000} \\
 & \underline{3\,6} \\
 & 1\,40 \\
 & \underline{1\,26} \\
 & 140 \\
 & \underline{126} \\
 & 14
\end{array}
$$

EXERCISE 92

1. On squared paper show that $\frac{1}{4} = .25$. Also that $\frac{3}{4} = .75$. $\frac{3}{20} = .15$ $\frac{3}{8} = .375$

Change each of the following to the form of a decimal fraction. (If the division is not exact, carry the work to three decimal places only.)

2. $\frac{4}{5}$ $\frac{5}{8}$ $\frac{3}{20}$ $\frac{7}{25}$ $\frac{5}{16}$ $\frac{4}{27}$ $\frac{4}{21}$ $\frac{19}{20}$ $\frac{63}{64}$ $\frac{11}{32}$

3. $\frac{3}{200}$ $\frac{7}{300}$ $\frac{1}{125}$ $\frac{6}{125}$ $\frac{3}{250}$

4. $1\frac{1}{4}$ $7\frac{1}{8}$ $8\frac{7}{8}$ $9\frac{7}{10}$ $5\frac{3}{8}$ $29\frac{3}{16}$ $8\frac{27}{40}$

5. Express 9 months as the decimal part of a year. Also 6 mo. 8 mo. 3 mo.

6. Express 15 days as the decimal part of a month. Also 6 da. 12 da. 2 da. 18 da. 3 da.

7. Express 9 inches as the decimal part of a foot. Also 6 in. 3 in. 2 in. 5 in. 8 in.

8. Make up and work an example similar to Ex. 5, concerning the ounce and pound. Also one concerning the pint and gallon. Also one concerning the quart and bushel.

9. Make up and work two examples similar to those in Ex. 2.

10. How many of the examples in this Exercise can you work at sight?

11. Practice multiplying by aliquot parts of 100 by means of ring drills. (See Exercise 73, p. 121.)

86. Relative Advantages in the Use of Common and Decimal Fractions. By this time you have seen that decimal fractions are generally easier to write and to work with than common fractions.

Thus, .213 contains only one half the figures and signs that $\frac{213}{1000}$ does.

But you will also observe that in special cases common fractions are briefer to write and easier to use than the corresponding decimal fractions.

Thus, $\frac{1}{8}$ is briefer than .125.

Hence, it is customary to substitute decimal fractions for common fractions for general use, but to employ common fractions in special cases.

87. Hundredths in the Per Cent Form.

Ex. 1. Which is greater: $\frac{3}{20}$ or $\frac{4}{25}$?

$\frac{3}{20} = \frac{15}{100} = .15$ Since 16 is greater than 15, $\frac{4}{25}$ (which = .16)
$\frac{4}{25} = \frac{16}{100} = .16$ must be greater than $\frac{3}{20}$ (which = .15).

Ex. 2. One farmer had 3 cows out of 20 condemned as diseased, and another farmer had 4 out of 25 condemned. Whose loss was the greater relatively?

By Ex. 1, $\frac{3}{20} = .15$; hence, the first farmer lost at the rate of 15 out of a hundred. $\frac{4}{25} = .16$; hence, the second farmer lost at the rate of 16 out of a hundred. Since 16 is greater than 15, the loss of the second farmer was greater relatively.

The number per hundred (or number of hundredths) is used, in this way, as a standard of comparison.

This method of comparison has been found so useful that "by the hundred" (or "number of hundredths") has been given a special name, "per cent." It is denoted by a special sign, %.

Thus, 3% is read "three per cent," and is another form for .03.

EXERCISE 93

1. Which is the greater number of hundredths: $\frac{9}{20}$ or $\frac{11}{25}$? Which is the greater per cent?

2. Which is the greater per cent: $\frac{3}{20}$ or $\frac{7}{50}$? $\frac{18}{25}$ or $\frac{17}{20}$?

3. One woman sold 11 chickens out of 25, and another sold 23 out of 50. Which sold the greater number of hundredths, or per hundred? Which sold the greater per cent of her chickens?

4. Which tax rate is more per hundred: $6 on $300 worth of property, or $15 on $500 worth? What is the per cent in each case?

5. On squared paper, using the 10 × 10 square, show the meaning of 7% (i. e. of .07). Also of 5%. Of 30%.

6. 7% = ? hundredths. Compute .07 of $300. Now compute 7% of $300.

7. If I pay $2 tax on every $100 that I own, how much do I pay on $500? On $800? $300? $450?

8. Show that $2 × 4.5 gives the same result as .02 × $450.

9. If I pay $5 for the use of $100 for a year, what must I pay for the use of $300? Of $700? $750? $775? $840?

10. Let one pupil act as clerk in a post office (see p. 129), and let each pupil present some object to be mailed, as a book, letter, newspaper, etc., stating the weight of the object. Let the pupil acting as clerk state the sum due and make change for some coin

or bill, until a mistake is made. The pupil who has offered the package on which the mistake has been made, then begins to act as clerk.

ALIQUOT PARTS OF A DOLLAR

88. Fractional Equivalents for aliquot parts of a dollar.

By the method of Art. 84 (p. 155), we have seen that $\frac{1}{8} = .12\frac{1}{2}$. Hence, $\$\frac{1}{8} = \$.12\frac{1}{2}$. The values of other aliquot parts of a dollar may be obtained in like manner. Hence, we have

$\$.06\frac{1}{4} = \$\frac{1}{16}$
$\$.12\frac{1}{2} = \$\frac{1}{8}$
$\$.16\frac{2}{3} = \$\frac{1}{6}$
$\$.25 = \$\frac{1}{4}$
$\$.33\frac{1}{3} = \$\frac{1}{3}$
$\$.37\frac{1}{2} = \$\frac{3}{8}$
$\$.50 = \$\frac{1}{2}$
$\$.62\frac{1}{2} = \$\frac{5}{8}$
$\$.66\frac{2}{3} = \$\frac{2}{3}$
$\$.75 = \$\frac{3}{4}$
$\$.87\frac{1}{2} = \$\frac{7}{8}$

Ex. How much will 24 bushels cost at $33\frac{1}{3}$¢ per bushel?

LONG PROCESS	SHORT PROCESS
$$\begin{array}{r} 24 \\ (\$)\ \underline{.33\frac{1}{3}} \\ 8 \\ 72 \\ 72 \\ \$8.00 \end{array}$$ *Ans.*	$24 \times \$.33\frac{1}{3} = 24 \times \$\frac{1}{3} = \$8$ *Ans.*

EXERCISE 94

1. On squared paper show that $.12\frac{1}{2} = \frac{1}{8}$. That $.37\frac{1}{2} = \frac{3}{8}$. That $.66\frac{2}{3} = \frac{2}{3}$.

Compute in the shortest way (to the nearest cent when the result is not exact):

2. 48 times $\$.12\frac{1}{2}$. Times $\$.06\frac{1}{4}$. $\$.62\frac{1}{2}$. $\$.33\frac{1}{3}$. $\$.25$.

3. 128 times $\$.66\frac{2}{3}$. Times $\$.62\frac{1}{2}$. $\$.37\frac{1}{2}$. $\$.33\frac{1}{3}$. $\$.16\frac{2}{3}$.

4. 43 times $\$.50$. Times $\$.33\frac{1}{3}$. $\$.66\frac{2}{3}$. $\$.25$. $\$.37\frac{1}{2}$.

5. 24.8 times \$.25. Times \$.87$\frac{1}{2}$. \$.66$\frac{2}{3}$. \$.50. \$.06$\frac{1}{4}$.

6. Make up and work two similar examples for yourself.

Find the cost of

7. 48 lb. of butter at 37$\frac{1}{2}$¢ a pound. At 50¢. At 33$\frac{1}{3}$¢.

8. 23 gal. of molasses at 62$\frac{1}{2}$¢ a gallon. At 50¢. At 87$\frac{1}{2}$¢.

9. 60 bu. of potatoes at 75¢ a bushel. At 66$\frac{2}{3}$¢. At 87$\frac{1}{2}$¢.

10. Multiply 224 by \$.37$\frac{1}{2}$ in the long way. Now multiply in the short way. About how much shorter is the second way? In which process are you less likely to make a mistake, the long or the short one?

11. Find the cost of 24 collars if the price of 1 collar is \$$\frac{1}{8}$.

12. If a merchant buys 245 bu. of potatoes at 50¢ a bushel and sells them at 62$\frac{1}{2}$¢ a bushel, find his profit in the shortest way.

13. How many of the examples in this Exercise can you work at sight?

14. Write the following arrangement of numbers on the blackboard. As the teacher points, give the product of each number in the upper row by each number in the lower row.

24	36	96	48	12	60	72	54
.50	.06$\frac{1}{4}$	.12$\frac{1}{2}$	.25	.75	.66$\frac{2}{3}$	.37$\frac{1}{2}$	.33$\frac{1}{3}$

15. Let the teacher dictate the aliquot parts of a dollar, and let the pupils write the equivalent common fractions. Reverse this order, so that the aliquot parts will be written by the pupils.

89. Price and Cost. In buying and selling, it is convenient to use the terms *price* and *cost* in an exact sense.

The **price** is what must be paid for one unit of the material for sale, as of one pound or one quart.

The **cost** is what must be paid for the entire quantity. Thus, we speak of the "price" of one pound of sugar, but the "cost" of 17 pounds of sugar.

In the bill printed below, point out the price and the cost in each item.

Bills

90. Bills. In buying and selling articles, it is an advantage to have the items of the sale written or printed as a bill.

A **bill** is a written or printed statement of the items of a sale or of services rendered.

The *essential parts* of a complete bill are the *place* of sale, the *time* or date at which the bill is made out, the name of the *buyer*, the name of the *seller*, the *price, quantity,* and *cost* of each item, and the *total cost* of all the items.

Ex. James Smith bought of Ames, Mitchell & Co. 17 lb. of coffee at 38¢ per pound; 75 lb. of sugar at 4½¢ per pound; 20 lb. of oatmeal at 4¢ per pound; and 4 lb. of tea at 80¢ per pound. What was the cost of his purchases?

The purchases may be arranged as a bill, and summed up as follows:

Philadelphia, May 8, 1912.

Mr. James Smith,
Bought of Ames, Mitchell & Co.

17 lb. Coffee	38¢	$6	46
75 lb. Sugar	4½¢	3	375
20 lb. Oatmeal	4¢		80
4 lb. Tea	80¢	3	20
		13	84

Non-essential parts are often placed on a bill as a matter of convenience. Thus, the person or persons who make out the bill may describe on the bill the business they are engaged in, give their telephone number, state the terms of the sale as cash, C. O. D., or 30 days, etc.

Let the pupil obtain and bring to class a bill-head containing some of the non-essential parts just named.

A **balance** due on a bill is the difference between the total amount of the bill and the amount that has been paid on the bill.

91. Uses of Bills. Bills are useful in several ways. For example, the orderly arrangement of the items of a bill makes it easy to detect any error or omission in the items.

A bill is also a record which is permanent and easy to file. It can be readily referred to in case of future forgetfulness or attempt at fraud.

Having purchases "charged" and paying bills at stated times, say once a month, is also a convenience, and saves the labor of paying money and making change many times.

The practice of keeping an account at a store, however, often has the disadvantage of leading a person or a family to make unnecessary purchases.

EXERCISE 95

Copy the following bills and complete them:

1.

Atlanta, Ga., June 1, 1912.

Mrs. John Halifax,
Bought of Frederick, Howland & Co.

	30 lb. Coffee	37½¢		
	15 gal. Oil	12½¢		
	25 lb. Flour	5½¢		
	8 doz. Eggs	25¢		

2.

Trenton, N. J., July 5, 1912.

Miss Alice Carroll,
Bought of Senex Sterling's Sons.

	2 yd. Ribbon	\$.66$\frac{2}{3}$		
	6 Handkerchiefs	.33$\frac{1}{3}$		
	34 yd. Muslin	.06$\frac{1}{4}$		
	20 yd. Gingham	1.20		

Make out bills covering the following items and find the amount of each bill:

3. On Jan. 3, 1913, Mr. Walter Thomas bought of Brown Brothers of Pittsburg, 3 saws @ 75¢, 5$\frac{1}{2}$ doz. screws at 12¢, 7 lb. of nails at 4¢, 2 hatchets @ 62$\frac{1}{2}$¢.

4. On May 8, 1913, Harriet Stoddard bought of Sterling and Co. of Chicago, 10 yd. of serge at 87$\frac{1}{2}$¢, 6 spools of thread at 5¢, 3 doz. buttons at 22¢, 12 yd. of braid at 17¢, 8 yd. of muslin at 11¢.

5. On Oct. 17, 1912, Harper Wallace bought of Charles Morris and Son, Dealers in Fruits and Vegetables, in Denver, the following: 5 barrels of cranberries @ \$8.50; 12 baskets of apples at 40¢; 10 bu. of sweet potatoes at 72¢; 16 bu. of white potatoes at 60¢.

Suppose yourself the proprietor of a grocery business and make out some blank bills which you can use. Fill out these bills with the following sets of items. Let your school companions, or other persons whom you know, be the purchasers.

6. 6 lb. sugar @ 7¢
3 lb. rice @ 8¢
4 lb. prunes @ 12$\frac{1}{2}$¢

7. 3 cans corn @ 11¢
25 lb. flour @ 4$\frac{1}{2}$¢
5 lb. lard @ 11¢

8. 2 lb. butter @ 36¢
$2\frac{1}{2}$ doz. eggs @ 36¢
$\frac{1}{4}$ lb. chipped beef @ 28¢
3 qt. dried beans @ 12¢

9. $\frac{1}{4}$ lb. pepper @ 24¢
2 gal. vinegar @ 25¢
2 gal. molasses @ 60¢
3 lb. crackers @ 20¢

10. Make up the items of two bills similar to Exs. 6–9.

11. In Exs. 1–5, point out the essential parts of each bill, viz.: place, time, persons, and price, quantity, and cost of each item.

12. What is lacking in the following bill? Complete the bill in all essential parts.

Mr. Alfred Cramer,
Bought of Harmen and Son.

	3 Ploughs	$2.15		
	2 Rakes	.75		
	2 Hoes	.40		

13. Let the teacher dictate three other bills in which some essential part is lacking, and let the pupils complete each of these bills.

14. If you should buy several articles at a store and pay for them at the time of purchase, what would be the advantage of having a bill made out covering your purchases and taking it away with you?

92. Bills in the Dr. Form. A person who buys goods and does not pay for them at the time of purchase is said to be a *debtor*. The person who sells the goods is said to be the *creditor*. The short way of writing debtor is *Dr.*; of writing creditor, *Cr.*

Instead of the form given in Art. 90 (p. 162), bills are often written in the form shown on p. 166. Point out the differences between this form and that given in Art. 90.

Philadelphia, May 8, 1912.

Mr. James Smith,
To Ames, Mitchell and Co. Dr.

17 lb. Coffee	38¢	$6	46
75 lb. Sugar	4½¢	3	375
20 lb. Oatmeal	4¢		80
4 lb. Tea	80¢	3	20
		13	84

EXERCISE 96

Review

1. Find the sum of $.5 + \frac{1}{8} + .12\frac{1}{2}$.

2. Find the value of $78 \times \$.12 - 62 \times .375 + 100 \times \$1.25$.

3. Add 5 to .06. To 13.06. To .0785.

4. Find the sum of $.07 + 6\% + 5 + 3\%$.

5. On squared paper show the meaning of $.745 - .125$.

6. Compute $656 \times .75 + 656 \times .15$ in the shortest way.

7. Express $\frac{3}{8}$ as an equivalent decimal fraction. Which of the two forms is easier to write?

8. Express each of the following as a decimal fraction:

$$\frac{5}{8} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{5}{20} \quad \frac{4}{25}$$

9. Reduce by means of cancellation:

$$\frac{1.6 \times 3.6 \times 5}{1.2 \times 4}$$

10. Collect as many different kinds of bills as you can, and note how many are of the form given in Art. 90 (the bill-of-sale form), and how many are of the Dr. form. Also point out what non-essential features are on each bill.

11. Find the cost of $8\frac{1}{2}$ yards of silk at $.68 per yard.

12. How many bushels of wheat at $1.12 can be bought for $81.76?

13. If .325 of mortar is lime, how many pounds of lime are there in one ton of mortar? If the rest of the mortar is sand, how many thousandths of the mortar are sand? How many pounds of sand are there in a ton of mortar?

14. Find the profit on 300 handkerchiefs bought at 3 for 25¢ and sold at 2 for 25¢.

15. What will be the cost of 3750 shingles at $14 a thousand?

SUG. Find the number of thousands, viz.: 3.75, and multiply $14 by this number.

16. Find the cost of 3500 cu. ft. of gas, at the rate of $1.20 for 1000 cubic feet.

17. Find the cost of 74,560 bricks at $9.50 per thousand.

18. Find the cost of 7750 feet of lumber at $42.50 per thousand.

19. If in a given year 4800 people die in a city containing 300,000 inhabitants, what is the death rate per thousand that year?

20. What is the cost of 16 yd. of cloth at $1.37\frac{1}{2}$ per yard? At $1.12\frac{1}{2}$ per yard? At $1.62\frac{1}{2}$ per yard?

EXERCISE 97

ORAL REVIEW

1. Add .2 to each of the following: .3, .5, .48, 3.2, .36, .8.
2. Add .3 to each of the numbers in Ex. 1. Add .9, .02, 1.2, .003, .1.
3. Subtract .1 from each number in Ex. 1. Subtract .2, .02, .005.
4. Multiply each number by .2. By .1, .02, .01, .03, .05.
5. Divide each number by .2. By .1, .02, .01.

6. Give the value of each of the following:

(*1*) $4 \div .2$
(*2*) $4 \times .2$
(*3*) $4 \div .02$
(*4*) $4 \times .02$
(*5*) $.4 \div .2$
(*6*) $.4 \times .2$
(*7*) $.04 \times .2$
(*8*) $.04 \div .02$

7. If .4 of a building lot costs $60, what is the cost of the whole lot at the same rate?

8. If .3 of a business is worth $12,000, what is the whole business worth?

9. Give the value of

(*1*) $60 \times .12\frac{1}{2}$ (*2*) $84 \times .25$ (*3*) $24 \times .37\frac{1}{2}$ (*4*) $48 \times .33\frac{1}{3}$

10. Express each of the following as a decimal fraction:

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{10}$ $\frac{3}{4}$ $\frac{3}{10}$ $\frac{1}{5}$ $\frac{7}{10}$ $\frac{4}{5}$ $\frac{3}{5}$

11. Express as a common fraction:

.4 .5 .25 .75 .6 .8 .12 .05

EXERCISE 98

BUSINESS REVIEW: RAILROAD TRIP FROM NEW YORK TO CHICAGO; FREIGHTS

1. Mr. and Mrs. Morris and their children, Guy and Lucy, traveled from New York to Chicago. When they started from

New York, their watches indicated 8.20 A. M. When they reached Chicago the next day, their watches indicated 6.40 A. M. How many hours and minutes were they on the way? How many hours and what decimal of an hour?

2. Would their watches show the same time as the clock in the railroad station in Chicago? Do you know why this is so?

3. If the distance from New York to Chicago is 912 miles, what was their average speed per hour? (Find to two decimal places.)

4. The fare for a grown person was $20, and for each child half fare was paid. The family occupied 2 lower berths costing $5 each and 2 upper berths costing $4 each. What did their fares and berths cost all together?

5. If each bought three meals on the train and the cost of a meal for one person averaged $1.05, what did they pay for meals?

6. Guy and Lucy were each allowed $1.50 to spend for dinner. The bill of fare contained the following items. Show how each child could select a meal so as to include meat (or fish), two vegetables, and dessert.

Roast beef	$.75	Mashed potatoes	$.25
Roast lamb	$.75	Stewed tomatoes	$.25
Broiled beefsteak	$1.00	Stewed corn	$.25
Broiled chicken	$1.25	String beans	$.25
Broiled fish	$.80	Asparagus	$.40
Celery	$.40	Apple pie	$.15
Lettuce	$.30	Custard pie	$.15
Spinach	$.30	Grapes	$.30
Rice pudding	$.15	Peaches	$.20
Ice cream	$.20	Orange	$.10

7. The family also spent 20¢ for newspapers, 75¢ for magazines, 50¢ for postcards, and 40¢ for a telegram. What were their total expenses on the journey?

8. The distance from New York to Pittsburg is 444 miles. Is this more or less than half way from New York to Chicago? How many miles more or less?

9. Guy saw a freight car on which he read "Gross Weight, 38,840 lb. Capacity 60,000 lb." If the car were loaded to its full capacity, how many tons and what decimal of a ton would it weigh? If one locomotive could pull 8000 tons, how many such cars could it pull? If each car were 40 ft. long, how long a train would these cars make?

10. If it costs $21 to carry 30 tons for 50 miles on a railroad, how many cents does it cost to carry 1 ton 1 mile? If it costs $3.50 to carry 2 tons by horse and wagon on a dirt road for 12 miles, how much does it cost to carry 1 ton 1 mile in this way?

The cost of this transportation by wagon is how many times as great as by railroad?

11. What, then, is one use of railroads? Can you state some other uses of railroads?

12. At Pittsburg on the Ohio River there was a fleet of barges loaded with coal to be floated down the Ohio and Mississippi rivers to New Orleans. Guy's father told him that one such fleet carried 60,000 tons and could be floated down 3000 miles for $6000. How much was the cost of floating 1 ton 1 mile down the river?

How much less was this than the cost of sending 1 ton 1 mile by railroad?

Why, then, do we not send all freight by boat instead of by railroad?

13. If the number of miles of railroad in use in the United States in 1850 was 90,251 and in 1900 was 194,202, what was the

increase? The number of miles in 1900 was how many times as great as the number in 1850?

14. Suppose that you had $50 to spend in traveling. Get time-tables from a railroad station or steamboat line and plan a trip between two cities or places of interest, the starting place being near your home. Make up examples concerning the trip.

CHAPTER VII

COMPOUND NUMBERS

EXERCISE 99

1. Compare the work of using the pint measure 64 times with that of using the bushel measure once.

2. Why do we not weigh coal by the ounce? Why does not a boy state his age as 4337 days, for instance? Why do we not state the distance from New York to San Francisco in yards?

3. In the following list, point out the distances which are best measured by the inch. Also those which are best measured by the foot. By the yard. By the mile.

(*1*) The length and breadth of a window pane.
(*2*) The length of a room.
(*3*) The distance between New York and Philadelphia.
(*4*) The height of Washington Monument.
(*5*) The diameter of the earth.

4. In the following list, point out the materials which are usually weighed by the ounce. By the pound. By the ton.

(*1*) Coal
(*2*) Pepper
(*3*) Sugar
(*4*) Hay
(*5*) Coffee
(*6*) Ginger

5. What unit of money is used in paying for a newspaper? A horse? A farm? A pencil?

6. Which is easier to visualize: a jump of 165 inches, or one of 13 ft. 9 in.?

7. Which is easier to multiply by 23: 13 ft. 9 in., or 165 inches? Try it and see.

8. Why do we now make definitions and rules?

93. **A compound number** is a number expressed in terms of two or more units of the same class but of different size; as 9 feet 7 inches.

A **simple number** is a number expressed in terms of a single unit; as 115 inches.

94. Uses of Compound Numbers. (1) In measuring a large amount of a given material, it often saves labor to use a large unit of measure; and in measuring a small amount, to use a small unit.

Thus, in measuring a large amount of oil we use the gallon or barrel; in measuring a small amount we use the pint or ounce.

(2) In measuring an object, it often saves labor to use a large unit in the first part of the measurement and a small unit in completing the measurement.

Thus, in measuring the distance which an athlete jumps, we measure first the number of feet in the jump, and then the number of inches in the remainder of the jump, obtaining, for instance, 19 feet 7 inches as the entire jump, instead of 235 inches.

Similarly the length of a man's life is expressed in terms of several units of time; as 59 years 8 months and 12 days, for instance.

(3) When a given number is expressed as a compound number, it is easier to form a definite mental picture or estimate of it.

Thus, it is much easier to visualize 19 feet 7 inches than 235 inches; also 59 years 8 months 12 days than 21,787 days.

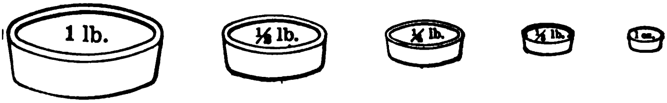
On the other hand, a magnitude expressed as a simple number is much easier to work with.

Thus, to find the area of a room 14 feet 8 inches long and 12 feet 3 inches wide, it is best to reduce the length and breadth of the room either to feet or to inches, before multiplying them.

95. The Different Classes of Compound Numbers in common use are those of weight, length, area, volume (and capacity), value, and time. These may be grouped in three general classes as

1. Weight (and hence value).
2. Length (and hence area, volume, and capacity).
3. Time.

AVOIRDUPOIS WEIGHT


1 lb.
½ lb.
¼ lb.
⅛ lb.
1 oz.

96. The units of **Avoirdupois Weight** are used in weighing all ordinary objects, as groceries, meat, hay, coal.

TABLE

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 hundredweight = 1 ton (T.)
2000 pounds = 1 ton

A pound is also considered as 7000 grains (gr.).

The *long ton* is 2240 lb. It is used in the United States custom houses, and often in weighing large quantities of materials such as coal and grain.

Ex. 1. Reduce 4 lb. 3 oz. to ounces.

4 lb. = 4 × 16 oz. = 64 oz.
64 oz. + 3 oz. = 67 oz. *Ans.* *or*

$$\begin{array}{r} 16\text{ oz.} \\ \underline{4} \\ 64\text{ oz.} \\ \underline{3\text{ oz.}} \\ 67\text{ oz. } \textit{Ans.} \end{array}$$

Ex. 2. Change 67 oz. to pounds and ounces (i. e. to higher units).

67 oz. = $\frac{67}{16}$ lb. = 4 lb. + 3 oz. *Ans.* *or*

$$\begin{array}{r} 4\phantom{\text{ oz.}} \\ 16\text{ oz.}\overline{)67\text{ oz.}} \\ \underline{64}\phantom{\text{ oz.}} \\ 3\text{ oz.} \end{array} \quad 4\text{ lb. } 3\text{ oz. } \textit{Ans.}$$

EXERCISE 100

Reduce:

1. 2 lb. 4 oz. to ounces.
2. 3 T. 125 lb. to pounds.
3. 8 cwt. 60 lb. to pounds.
4. 7 T. 3 cwt. to pounds.
5. $2\frac{1}{4}$ T. to pounds.
6. 2.45 T. to pounds.

Show the meaning of Ex. 1 by weighing sand.

Change to higher units:

7. 100 oz.
8. 1000 oz.
9. 750 lb.
10. 3760 lb.
11. 10,000 lb.
12. 10,000 oz.

13. If there are scales in the schoolroom, weigh $1\frac{3}{4}$ lb. How many ounces in $1\frac{3}{4}$ lb.?

14. Pour into a paper bag what you think is a pound of sand. Now weigh the sand. By how many ounces is your estimate wrong? By what fraction of a pound are you wrong? By what decimal part of a pound are you wrong?

15. Estimate 2 lb. and test your estimate. Also 5 lb.

16. Express as tons and decimals of a ton:

(*1*) 3000 lb. (*2*) 5000 lb. (*3*) 3550 lb. (*4*) 12,500 lb.

17. What will 3000 lb. of hay cost at $14 a ton? What will 2500 lb. cost? 3500 lb.?

18. 5 cwt. of mixed feed will last how many days, if 25 lb. are used per day? If the cost is 80¢ per hundredweight, what is the cost per day?

19. $\frac{3}{4}$ lb. + $\frac{1}{8}$ lb. = $\frac{?}{\text{eighths}}$ lb. = ? ounces.

20. $\frac{1}{4}$ lb. + $\frac{3}{8}$ lb. = ? lb. = ? ounces.

21. 1 ounce = ? grains. $\frac{1}{2}$ lb. = ? grains.

22. If you should use only 1-oz. weights in weighing 5 lb. of sugar, how many weights would you put on the scales? How many, if you used a 3-lb. and a 2-lb. weight?

23. Three carloads of coal contain 37,623 lb., 33,526 lb., and 28,912 lb. respectively. How many tons do they contain together?

24. A wagon and its load of coal weigh 3750 lb. The wagon weighs 1200 lb. What is the coal worth at $6.50 a ton?

25. A cubic foot of water weighs $62\frac{1}{2}$ lb. How many ounces does it weigh?

26. 15 boxes weigh 50 lb. each. What fraction of a ton do they weigh together? What decimal fraction?

27. If a bushel of corn weighs 56 lb., how many bushels are there in a ton?

28. Find the difference between 5 long tons and 5 short tons.

29. If 20 long tons of coal are bought at $4.85 per ton and sold at $6.25 per short ton, what is the gain?

30. Make up and work an example similar to Ex. 18. To Ex. 23. Ex. 27.

31. Choose sides and solve an example in addition, dictated by the teacher, similar to Ex. 11, p. 23. The side wins which first passes in 10 correct papers.

EXERCISE 101

ORAL WORK

1. How many ounces in 1 lb.? In 3 lb.? $\frac{1}{2}$ lb.? $\frac{1}{4}$ lb.? $\frac{3}{4}$ lb.? $\frac{5}{8}$ lb.?

2. How many pounds in 32 oz.? In 80 oz.? 3 cwt.? 1 ton? 4 tons?

3. How many ounces in 1 lb. 3 oz.? In 2 lb. 8 oz.? $3\frac{1}{2}$ lb.?

4. If a pound of butter costs 28 cents, what will 8 oz. cost? 4 oz.? 12 oz.? 2 lb. 4 oz.?

5. If a pound of coffee costs 36 cents, what will $2\frac{1}{2}$ lb. cost? 8 oz.? 3 lb. 12 oz.? $\frac{3}{4}$ lb.?

6. If a pint of water weighs a pound, and a glass of water is half a pint, how much does a glass of water weigh? $\frac{1}{2}$ glass? $\frac{1}{4}$ glass?

7. A certain bottle can be filled four times from a glass of water. How many ounces does the bottle hold?

8. How many tons in 1000 lb.? In 6000 lb.? 60,000 lb.?

9. How many pounds in 6 tons? In 15 tons? $2\frac{1}{2}$ tons?

10. When flour is 4 cents a pound, what will 2 cwt. cost? 8 oz.?

11. What will it cost to mail a book weighing 10 oz.? 18 oz.? 12 oz.?

12. State the cost of mailing a magazine weighing 6 oz. 12 oz.

13. What will it cost to mail a letter weighing 2 oz.? 3 oz.? 4 oz.?

14. How many of the examples in Exercise 100 can you work at sight?

MEASURES OF LENGTH

97. **Linear Units** is a general name for units of length (as one inch, one foot, one yard, etc.) as distinguished from units of area or of volume.

TABLE

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$$\left.\begin{array}{r}5\frac{1}{2}\text{ yards}\\ 16\frac{1}{2}\text{ feet}\end{array}\right\} = 1\text{ rod (rd.)}$$
320 rods = 1 mile (mi.)
Hence, 1 mile = 1760 yd. = 5280 ft.

The foot is frequently divided into tenths and hundredths of a foot instead of into inches. Thus, instead of 2 ft. 6 in., we write 2.5 ft.

Instead of the contractions *ft.* and *in.*, the signs ′ and ″ are frequently used. Thus, instead of 2 ft. 6 in., we write 2′ 6″. The latter method is especially useful on diagrams, where the figures and signs should occupy as little space as possible.

EXERCISE 102

Reduce:

1. 2 ft. 9 in. to inches.
2. 3 yd. 2 ft. to feet. Also to inches.
3. 2 rd. 3 yd. to feet.
4. 3 mi. 100 rd. to rods.
5. 2 mi. 300 yd. to yards.
6. 2′ 7″ to inches.

7. Show the meaning of Exs. 1, 2, and 6 with the foot rule.

Change to higher units:

8. 80 in.
9. 100 in.
10. 14 ft.
11. 600 rd.
12. 2000 rd.
13. 2000 yd.

14. Show the meaning of Exs. 8 and 9 with the foot rule.

15. How many yards long is the quarter-mile run? The half-mile run? Name some place which you think is $\frac{1}{4}$ mile away.

16. How many feet are there in $\frac{2}{3}$ mile? In $\frac{5}{6}$ mile? $\frac{3}{4}$ mile? How many yards in each of these?

17. How many feet in 4.28 miles? In 2.6 miles? How many yards in each of these?

18. If railroad ties on the average are $2\frac{1}{4}$ ft. apart, how many ties will be used in laying a mile of track? What will these cost at $.60 a tie?

19. How many shingles, each 4 in. wide and placed side by side, will be needed to make a row 24 ft. long?

20. Draw on the blackboard a line which you think is a yard long. Now measure this line. By how many inches are you wrong? By what fraction of a yard are you wrong?

21. Show on the side of the schoolroom the distance which you think is a rod. Test your guess by actual measurement.

22. Which is easier to use: the foot rule 33 times, or the rod measure twice? Which measurement is more likely to be accurate? What, then, are some uses of the rod measure?

23. Think of the length of the 100-yard dash (or some other distance 100 yards long). Now name a place which you think is 200 yd. from the schoolhouse. Also one 300 yd. distant.

24. Sound travels 1090 ft. per second. How many yards is this?

While the teacher counts "one — two" slowly, let your mind run over a distance nearly 4 times as long as the 100-yard dash. This will enable you to realize the velocity of sound.

25. If a railroad train travels 60 miles an hour, how many feet is this per second? (Group all operations and use cancellation.) How many yards? How does this velocity compare with the velocity of sound?

Visualize your answer by thinking the proper multiple of 100 yd., while the teacher counts "one — two" slowly.

26. Let the teacher give examples in which a distance on a map or drawing is given, to find the scale.

27. A 12-inch cannon will send a cannon ball 20,000 yd. How many miles is this? (Compute the fraction to 3 decimal places.)

28. Name some place which you think is 1 mile distant. Also one 10 miles distant.

29. Measure the length of the schoolroom by using the foot rule repeatedly. Now measure it by use of the yard measure. Which method is easier? Why? Which is more likely to be accurate?

30. Walter can run the 100-yard dash in 12 seconds. The distance around the bases of the baseball diamond is 150 yd. (allowing for extra distance in turning corners). How long will it take Walter to run the bases when he makes a home run?

31. Make up and work an example similar to Ex. 1. To Ex. 9. Ex. 19.

32. How many examples in Exercises 60 and 61 (pp. 104, 106) can you now work at sight?

EXERCISE 103

ORAL WORK

1. What decimal part of a foot is 6 in.? 9 in.? 3 in.?

2. 9 in. is what fraction of a yard? What decimal part of a yard?

3. How many inches in 2.5 ft.? In 1.25 ft.? 2 ft.? 1 ft.?

4. $\frac{1}{4}$ lb. + $\frac{3}{8}$ lb. = ? lb. = ? oz.

5. Tell how many feet there are in 3 yd. In $6\frac{2}{3}$ yd. 2 rd. 72 in. $\frac{1}{4}$ mi. 10 rd.

6. How many inches in 8 ft.? In 3 yd.? 1 rd.?

7. How many yards in 4 rd.? In 10 rd.? 1 mi.? 24 ft.? 144 in.? 1728 in.? $\frac{1}{2}$ rd.?

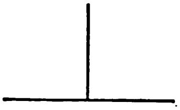
8. Tell how many rods there are in 3 mi. In $\frac{1}{2}$ mi. $\frac{3}{4}$ mi. $\frac{7}{10}$ mi. $1\frac{1}{2}$ mi. $16\frac{1}{2}$ yd. 132 ft.

9. How many inches in 1 ft. 3 in.? In 1 ft. 8 in.? In 2 ft. 5 in.? In 3 ft. 6 in.? In 2 ft. 9 in.?

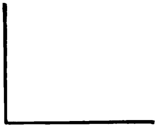
10. How many of the examples in Exercise 102 can you work at sight?

11. Have an oral drill with fraction cards. (See p. 392.)

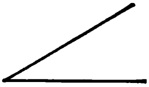
SURFACE OR SQUARE MEASURE



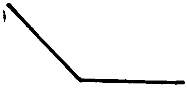
Two Right Angles



Right Angle



Acute Angle



Obtuse Angle

98. An angle is the amount of opening between two lines.

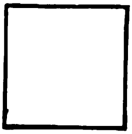
A **right angle** is one of the angles formed by a line meeting another line so as to form equal angles.

An **acute** angle is an angle less than a right angle.

An **obtuse** angle is an angle greater than a right angle.

99. A **surface** is that which has only two dimensions, length and breadth.

A **rectangle** (or oblong) is a flat surface which has four sides and four right angles.



Square

A **square** is a flat surface which has four equal straight lines and four right angles.

The **perimeter** of a rectangle is the distance around the rectangle.

100. A **unit of surface** is a square each side of which is a unit of length; as a square inch or a square foot.

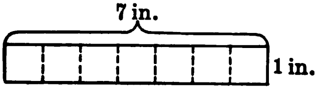
The **area** of a surface is the number of square units which the surface contains.

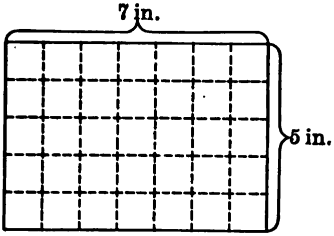
If a rectangle is 7 inches long and 5 inches wide, the area of the rectangle consists of square inches arranged in 5 rows, each row containing 7 square inches.

Hence, area of the given rectangle = (1 sq. in.) × 7 × 5 = 35 sq. in.

Hence, to find the area of a given rectangle, we have the following rule:

Area of rectangle = (1 *sq. unit*) × (*no. of linear units in length*) × (*no. of linear units in width*), like units being used in each case.





It is sometimes convenient to express the area of a rectangle as (number of rows) × (number of square units in a row).

Thus, the area of the above rectangle is expressed as 5 × 7 sq. in.

Point out as many rectangles in the schoolroom as you can. Also as many squares. Name as many rectangles as you can which are not in the schoolroom. Also as many squares.

TABLE

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 sq. ft. = 1 square yard (sq. yd.)
$30\frac{1}{4}$ sq. yd. = 1 square rod (sq. rd.)
160 sq. rd. = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Hence, 1 acre = 43,560 sq. ft. = 4840 sq. yd.

EXERCISE 104

1. On squared paper mark off a rectangle 3 inches long and 4 inches wide. Find the area of this rectangle (1) by counting the number of small squares which the rectangle contains, (2) by the rule on p. 182. Compare the work in the two processes.

2. By lines, divide the surface of the blackboard (or a convenient part of it) into square feet. Count the square feet. Now measure the length and width of the blackboard and get its area by multiplication. Compare the work in the two processes.

3. Make up and work an example similar to Ex. 1.

Reduce:

4. 3 sq. yd. 5 sq. ft. to sq. ft.
5. 3 sq. ft. 56 sq. in. to sq. in.
6. 5 sq. yd. 3 sq. ft. to sq. in.
7. 96 sq. ft. 3 sq. in. to sq. in.
8. 10 sq. ft. $15\frac{1}{2}$ sq. in. to sq. in.
9. 2 sq. yd. 3 sq. ft. to sq. ft.

10. Show the meaning of Exs. 4, 5, and 6 by use of the yard and foot measures.

Change to higher units:

11. 96 sq. ft.
12. 700 sq. in.
13. 1000 sq. in.
14. 500 sq. rd.
15. 2000 sq. rd.
16. 1250 sq. in.

17. Show the meaning of Exs. 12 and 13 by use of the foot measure.

18. A certain pavement is 3 yd. wide and 12 yd. long. What will be the cost of concreting it at $1.10 a square yard? Also of putting a curb in front of it at 24¢ a foot?

19. How many square feet are there in a rug 18 ft. long and 24 ft. wide? How many square yards? What is the cost of this rug at $1.20 a square yard? Find the perimeter of the rug both in feet and in yards.

20. Think of the 100-yard dash. Think of a square each side of which is 100 yd. How many acres does this square contain and what decimal of an acre? (Find to 3 places.)

21. What is the cost of making a macadam road ½ mile long and 9 ft. wide at 12¢ a square yard?

22. Does a square field, each side of which is 70 yd., contain more or less than an acre? What fraction of an acre is the difference?

23. Does a field 100 yd. long and 50 yd. wide contain more or less than an acre?

24. Think of a square each side of which is 1½ miles. How many acres does this square contain? Treat in like manner a square each side of which is 2 miles. 5 miles. 10 miles.

25. Estimate the length and the width of the schoolroom. Using these estimates, find the area of the schoolroom floor. Now measure the length and width of the schoolroom and compute its area. Find the difference between the two results.

26. What is ¼ acre of land worth at $.72 a square foot?

27. How many strips of carpet, each $\frac{3}{4}$ yd. wide, will a room 15 ft. wide require? A room 18 ft. wide? 24 ft. wide? 17 ft.?

In describing a rectangle, "8 inches long and 4 inches wide" may be shortened to 8 inches by 4 inches, or to 8 in. × 4 in., or to 8″ × 4″.

28. How many bricks, each 8″ × 4″, are necessary to lay a pavement 12′ × 6′? (Use cancellation.)

29. A tennis court is 78′ × 27′. Find the area. If a margin of 15 ft. is annexed on each side and end, what will the area be?

30. What is the area of a field 160 yd. long and 30 yd. wide? What is the length of the fence around the field? Answer the same questions for a field 80 yd. long and 60 yd. wide.

How do the areas of these fields compare? Compare also the lengths of their fences. The fence of which field costs less?

On squared paper draw diagrams representing these fields.

31. Construct a right angle on squared paper or by folding a piece of paper.

32. Construct an angle which is equal to half a right angle, either on squared paper or by folding a piece of paper.

33. What decimal part of an acre is the area of a baseball diamond? (See p. 390.)

34. A rectangle is 15 ft. long and its area is 120 sq. ft. What is its width?

SUG. How many rows, each containing 15 sq. ft., are needed to make 120 sq. ft.?

35. Make up and work an example concerning two gardens, similar to Ex. 30.

36. Make up and work an example similar to Ex. 19. To Ex. 21. Ex. 29.

EXERCISE 105

ORAL WORK

1. How many square inches are there in 3 sq. ft.? In 4 sq. ft.? $\frac{1}{2}$ sq. ft.?

2. How many square feet in 4 sq. yd.? In 10 sq. yd.? 12 sq. yd.?

3. $\frac{1}{3}$ ft. + $\frac{1}{4}$ ft. + $\frac{1}{6}$ ft. = ? ft. = ? inches.

4. Does a field 99 yd. long and 40 yd. wide contain more or less than an acre?

5. Is there more or less than an acre in a field 501 ft. × 100 ft.? One 397 ft. × 100 ft.?

6. What kind of angle do the hands of a clock make at 3 o'clock?

7. What angle do they make at 4 o'clock? At 2 o'clock? 5 o'clock?

8. How many examples in Exercise 66 (p. 113) can you work at sight?

101. Formula for the Area of a Rectangle. The rule for computing the area of a rectangle (p. 182) can be put in a simpler form by the use of symbols.

Let A denote the number of square units in the area of a rectangle,

Let l denote the number of linear units in the length of the rectangle,

Let w denote the number of linear units in the width of the rectangle, like units being used in each case. Then,

$$A = l \times w.$$

A *formula* is a rule expressed in symbols.

Ex. 1. Find the area of a rectangle 12 ft. long and 2 yd. wide.

Expressing all units as feet, we have $l = 12$ ft., $w = 6$ ft.

In working an example by use of the formula, after the numbers have been reduced to the same unit, mention of the unit may be omitted until the final result is stated. Hence, we have

$A = 12 \times 6 = 72$, or Area = 72 sq. ft. *Ans.*

Ex. 2. The area of a rectangle is 120 sq. ft. and the length is 5 yd. Find the width.

$$\begin{aligned} A &= 120 \text{ sq. ft.} \qquad l = 15 \text{ ft.} \\ 120 &= 15 \times w \qquad \text{(See Ex. 34, p. 185.)} \\ 120 \div 15 &= 8 \\ \text{Hence, width} &= 8 \text{ ft. } \textit{Ans.} \end{aligned}$$

EXERCISE 106

By use of the formula, find the area of each of the following rectangles:

	Length	Width		Length	Width
1.	126 yd.	12 yd.	4.	3 ft.	9 in.
2.	12 yd.	9 ft.	5.	7 yd.	3 yd. 1 ft. 6 in.
3.	24 ft.	5 ft. 6 in.	6.	9 yd.	5 yd. 2 ft. 1 in.

7. The area of a floor is to be 600 sq. ft. and the width 20 ft. Find the length.

8. The area of a floor is 450 sq. ft. and the length is 30 ft. Find the width.

9. On squared paper draw a diagram for Ex. 2, letting $\frac{1}{4}$ in. stand for 1 yd.

10. Also make diagrams for Exs. 4, 5, and 6.

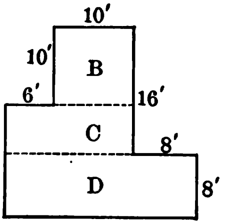
11. The rule given in Art. 100 (p. 182) is about how many times as long as the formula $A = l \times w$? What is the difference between a rule and a formula?

12. If A and w are given, does $l = \dfrac{A}{w}$?

13. Find a similar formula for w when A and l are given.

14. Choose sides and solve two examples in multiplication, dictated by the teacher, similar to those in Exercise 17, p. 39. The side wins which first passes in 10 correct papers.

102. Combinations of Rectangles. Often the area of an irregular surface can be obtained by dividing the given surface into rectangles and finding the area of each rectangle separately.



Ex. Find the area of the floor of a cellar which has the shape and dimensions of the annexed diagram.

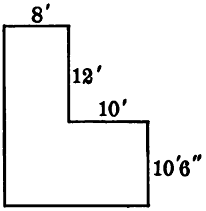
By drawing dotted lines as on the diagram, we divide the given surface into 3 rectangles.

Area of B = 10 × 10 sq. ft. = 100 sq. ft.
" " C = 16 × 6 " " = 96 " "
" " D = 24 × 8 " " = 192 " "
Area of entire figure = 388 sq. ft. *Ans.*

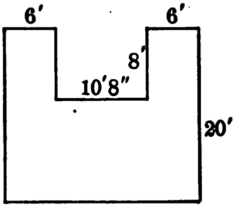
EXERCISE 107

Find the area of each of the following figures by dividing it into rectangles:

1.



2.



3. On your squared paper draw two irregular figures which may be divided into rectangles, and find the area of each figure.

4. A house 20′ × 30′ is on a lot 60′ × 200′. Find the area of the yard about the house. The front of the house is 20′ from the front of the lot, and one side of the house is 10′ from a side of the lot. Draw a diagram, showing the house on the lot, which could be sent to a man who thinks of purchasing the property.

5. In a well lighted room, the window space is at least $\frac{1}{6}$ of the floor space. Measure the windows of your schoolroom and find the area of all of them taken together. Does this equal or exceed $\frac{1}{6}$ of the area of the floor?

6. A certain room is 20 ft. long, 16 ft. wide, and 8 ft. high. Find the area of two opposite sides of the room taken together (windows and doors included). Also the area of the other two sides. Now find the total wall space. Also find the area of the ceiling. What is the area of the walls and ceiling taken together?

7. One book contains 240 pages, each 8″ × 12″. Another book has 360 pages, each 6″ × 9″. How much does the total page surface of one book exceed that of the other?

8. See how many of the examples in Exercise 75 (p. 125) you can work at sight.

CUBIC OR SOLID MEASURE

103. A solid is that which has length, breadth, and thickness.

A **rectangular solid** is a solid bounded by six rectangles.

A **cube** is a solid bounded by six equal squares.

It is often convenient to use the expression "box-shaped solid" for "rectangular solid."

Point out objects which are either cubes or rectangular solids.

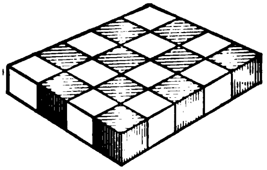
The **volume** (or contents) of a solid is the number of times the solid contains some cube taken as a unit of measure.

TABLE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cu. ft. = 1 cubic yard (cu. yd.)

The teacher should have on the desk a set of 150 inch blocks, half of them dark and half lighter in color. These blocks may be arranged as in the diagrams, or with the dark and light blocks in alternate layers.

104. Method of Finding Volume. A short way to find the number of cubic units in a given box-shaped solid is to multiply the number of cubic units in one layer by the number of layers. (We may think of a solid as made up of layers of cubes.)

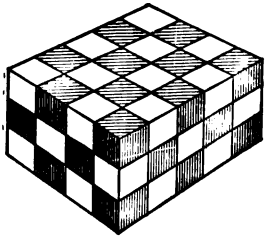


A short way to find the number of cubes in a layer is to multiply the number of cubes in a row by the number of rows.

In the box-shaped solid given in the diagram, a layer contains 5 × 4 small cubes. There are 3 layers. Hence,

Volume = 1 small cube × 5 × 4 × 3 = 60 cubes.

Think how much easier it is to find the length of each of the three edges and take their product than it is to count the 60 small cubes.



Hence, to find the volume of a box-shaped solid, we have the following rule:

Volume of a box-shaped solid = (1 *cu. unit*) × (*no. linear units in its length*) × (*same in its width*) × (*same in its depth*), all units being of the same kind.

Ex. Find the number of cubic yards in a cellar 8 yd. long, 12 ft. wide, and 6 ft. deep.

Reducing all the dimensions to yards, we have

Volume = 1 cu. yd. × 8 × 4 × 2 = 64 cu. yd. *Ans.*

105. Formula for the Volume of a Box-shaped Solid. If V stands for the number of cubic units in the volume of a box-shaped solid, and l, w, and d, stand respectively for the number of linear units in the length, width, and depth of the same solid,

$$V = l \times w \times d.$$

EXERCISE 108

1. How many cubic inches in 5 cu. ft.? In 8 cu. ft.? 16 cu. ft.? 25 cu. ft.? $\frac{1}{3}$ cu. yd.?

2. How many cubic feet in 35,000 cu. in.? In 78,000 cu. in.? 17 cu. yd.?

3. Reduce 18 cu. ft. 152 cu. in. to cubic inches.

4. How many cubic inches in 1 cu. yd.? In 4 cu. yd.?

5. Reduce 7 cu. yd. 15 cu. ft. 300 cu. in. to cubic inches.

6. Change 50 cu. yd. 17 cu. ft. to cubic feet.

7. Put blocks together so as to make a solid 5 inches long, 4 inches wide, and 2 inches deep. How many layers of blocks in the solid? How many blocks in each layer? How many cubic inches in the whole solid?

Which is the shorter way to find the number of cubic inches in this solid: to count them, or to multiply 5 × 4 × 2?

In giving the dimensions of a rectangular solid, "5 inches long, 4 inches wide, and 3 inches deep" may be shortened to $5'' \times 4'' \times 3''$.

8. Write in the shortest way:

   (1) 6 inches long, 4 inches wide, 3 inches deep.
   (2) 8 feet long, 4 feet wide, 9 inches deep.

9. Put blocks together to make box-shaped solids 5″ × 4″ × 4″; 5″ × 5″ × 4″; 5″ × 5″ × 3″; 5″ × 5″ × 5″; and 6″ × 5″ × 5″.

Find the number of small cubes in each solid (i. e. the volume) in the shortest way.

10. Put blocks together and make a layer 12″ × 12″. How many of these layers would be required to make a cubic foot?

11. Put blocks together to make a solid 3″ × 3″ × 3″. If each inch represents 1 foot, what does the whole solid represent?

12. Can you picture a cubic yard? Find some object which you think is about a cubic yard in size. Measure it to see if you are right.

Find the volume of each of the following box-shaped solids:

	l	w	d
13.	3 ft.	2 ft.	1 ft. 8 in.
14.	8 yd.	12 ft.	6 ft.
15.	8 ft.	4 ft.	24 in.
16.	4.5 ft.	3.4 ft.	2.2 ft.
17.	10 in.	6 in.	1 ft. 3 in.

18. A rectangular solid is 8 ft. long and 5 ft. wide and contains 160 cu. ft. How deep is it?

SUG. How many layers, each containing 5×8 cu. ft., or 40 cu. ft., are needed to make 160 cu. ft.?

19. A tank is to be 12 ft. long and 8 ft. wide, and is to contain 384 cu. ft. How deep must the tank be? If V, l, and w are given, does $d = \frac{V}{l \times w}$?

20. A tank is to be 6 ft. deep and 16 ft. long, and is to contain 960 cu. ft. How wide must it be? Make a formula for w when V, l, and d are given.

21. At 45¢ a cubic yard, what will it cost to dig a cellar 24 ft. long, 6 yd. wide, and 6 ft. deep? Also a cellar $20' \times 12' \times 5'$?

22. How many tons of coal can be put into a bin $12' \times 6' \times 5'$, if 40 cu. ft. = 1 ton? What is the advantage in computing the number of tons in this way instead of taking the coal out of the bin and weighing it?

23. How many tons of hay can be put into a mow $40' \times 30' \times 20'$, if 400 cu. ft. of hay = 1 ton?

24. One bin is $25' \times 6' \times 4'$ and another is $20' \times 8' \times 5'$. What is the easiest way of finding out which bin will hold more?

25. Estimate the dimensions of the chalk box. Now measure it and see by what fraction of its volume your estimate is wrong.

26. Estimate the dimensions of the schoolroom. Using your estimates, compute the volume of the schoolroom. Measure the room and find its correct volume. Find the difference between the two results.

27. A space of at least 600 cu. ft. should be allowed for each pupil in a schoolroom. What is the largest number of pupils which should be placed in a room $42' \times 35' \times 12'$?

28. Find the volume of your schoolroom. Is the schoolroom of sufficient size for all the pupils in it? (See Ex. 27.)

29. How many cubic yards are there in a wagon body $9' \times 4' \times 3'$?

30. If 1 cu. ft. of water weighs 1000 oz., how many pounds does it weigh?

31. If 1 cu. ft. of water weighs 62.5 lb., what will be the weight of water in a tank 20′ × 8′ × 4′?

32. Make up and work an example similar to Ex. 19. To Ex. 21. Ex. 27.

33. How many examples in Exercise 94 (p. 160) can you now work at sight?

EXERCISE 109

ORAL WORK

1. How many cubic feet in 2 cu. yd.? In $2\frac{1}{3}$ cu. yd.? $4\frac{2}{3}$ cu. yd.? $1\frac{1}{2}$ cu. yd.?

2. How many cubic feet in 2 cu. yd. 2 cu. ft.? In 3 cu. yd. 1 cu. ft.?

3. How many cubic inches in $\frac{1}{2}$ cu. ft.? In $\frac{1}{3}$ cu. ft.? $\frac{1}{4}$ cu. ft.? $\frac{2}{3}$ cu. ft.?

4. If 1 cu. yd. of marble costs $4.50, what will $\frac{1}{3}$ cu. yd. cost? 18 cu. ft.?

5. How many cubic inches in a brick 8″ × 4″ × 2″?

6. Practice the rapid addition of columns of figures.

DRY MEASURE

106. The units of **Dry Measure** are used in measuring fruit, grain, vegetables, and other loose dry objects.

TABLE

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

A bushel contains 2150.42 cu. in. or a little less than $1\frac{1}{4}$ cu. ft.

EXERCISE 110

Reduce:

1. 25 pk. + 5 qt. to quarts.
2. 1 bu. + 3 qt. to quarts.
3. 2 bu. + 4 pk. to pecks.
4. 3 bu. + 2 pk. to quarts.
5. 2 pk. + 3 qt. to pints.
6. 2 pk. + 3 qt. + 1 pt. to pints.

Change to higher units:

7. 56 qt.
8. 59 qt.
9. 160 pt.
10. 35 pk.
11. 75 qt. + 1 pt.
12. 200 qt.

13. How many quarts in a bushel? How many pints? Did you ever measure a bushel of anything? In doing so, which measure would you prefer to use and why?

14. If a dealer sells 100 quarts of corn to a customer, what measures will it be most convenient for him to use? How often will he have to use each of them?

15. Mr. Arthur bought 6 bu. of oats. If he feeds his horse 1 pk. a day, how many days will the oats last? How many days, if he feeds 6 qt. a day?

16. How many cubic inches in 3 bu.? In 2½ bu.? 5¾ bu.?

17. What will 1 pk. 4 qt. of cherries cost at 12½¢ a quart? Also at 3 qt. for 25¢?

18. If 3 qt. 1 pt. of strawberries cost 42¢, what will 1 qt. 1 pt. cost?

19. If 4 ears of corn make 1 qt. of shelled corn, how many ears will a farmer have to select if he needs 12½ bu. of seed corn?

20. A bushel of corn weighs 56 lb. How many bushels are there in a ton of corn? In 2½ tons?

21. A bushel of potatoes weighs 60 lb. What is the weight of the potatoes in a wagon body 12′ × 4′ × 2′? (Use cancellation.)

22. A storekeeper had 10 bu. of potatoes and has sold to different customers $\frac{1}{4}$ pk., $\frac{1}{2}$ bu., $\frac{3}{4}$ pk., $\frac{1}{2}$ pk., 1 bu., and 1 pk. How can he tell how many he has left without measuring?

23. Walter went chestnutting and got on different days 2 qt., 1 qt. 1 pt., and 3 qt. 1 pt. How many chestnuts did he have in all? He then sold 2 qt. 1 pt., gave away 1 qt., and ate 1 qt. 1 pt. How many chestnuts did he have left?

24. Harry bought 1 bu. of peanuts for $1.20. He roasted them and sold to different customers 1 pt., $\frac{1}{2}$ pt., 1 qt. 1 pt., $1\frac{1}{2}$ pt., and 2 qt. 1 pt. How many peanuts did he have left?

If he received 10¢ a quart for the peanuts that he sold, how much did he receive? How much were the remaining peanuts worth at 10¢ a quart?

25. Make up and work three examples similar to such of the examples in this Exercise as you think are most instructive.

26. Choose sides and solve two examples in long division, dictated by the teacher, similar to Ex. 26, p. 53. The side wins which first passes in 10 correct papers.

EXERCISE 111

ORAL WORK

1. Tell how many quarts there are in 2 pk. In 3 pk. In 1 pk. In 1 bu. In 1 pk. 3 qt. In 2 pk. 2 qt.

2. How many pints in 3 qt.? In 2 qt.? 2.5 qt.? 2 qt. 1 pt.? $1\frac{1}{2}$ pk.?

3. $\frac{1}{2}$ pk. + $\frac{3}{8}$ pk. + $\frac{1}{4}$ pk. = ? pk. = ? qt.

4. $\frac{1}{2}$ bu. + $\frac{3}{16}$ bu. + $\frac{1}{8}$ bu. = ? bu. = ? qt.

5. Hold your hands a foot apart. Now think of one cubic foot. Now think of $1\frac{1}{4}$ cubic feet, or 1 bushel.

6. Tell how many bushels a box will hold, if it contains 5 cu. ft. 10 cu. ft. 15 cu. ft. 25 cu. ft. 100 cu. ft.

7. How many bushels will a bin hold whose dimensions are $4' \times 5' \times 2'$? $10' \times 5' \times 4'$? $6' \times 5' \times 4'$?

8. If a pint of wheat weighs a pound, how many pounds will a bushel of wheat weigh?

9. Practice the oral addition and subtraction of two-figured numbers. (See p. 26 and p. 31.)

LIQUID MEASURE

107. The units of **Liquid Measure** are used in measuring liquids, as oil, water, vinegar, etc.

TABLE

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

A gallon = 231 cubic inches.

A common glass or tumbler holds about $\frac{1}{2}$ pint of liquid.

A barrel (bbl.) usually contains $31\frac{1}{2}$ gallons.

The gill is little used. A pint is usually divided into 16 fluid ounces instead of into gills. Hence a common tumbler contains 8 fluid ounces.

Some liquids, as milk, are frequently measured by the pound. This is an advantage because a large number of pounds can be measured by a single reading of the scales, often without removing the liquid from the cans or the tank in which it is contained.

EXERCISE 112

Reduce:

1. 3 qt. 1 pt. to pints.
2. 14 gal. 2 qt. to quarts.
3. 2 gal. 3 qt. to pints.
4. 5 gal. 2 qt. 1 pt. to pints.

5. Show the meaning of Exs. 1 and 3 by use of the measures named.

Change to higher units:

6. 24 pt.
7. 26 qt.
8. 75 qt. 1 pt.
9. 100 pt.
10. 200 pt.
11. 1000 pt.

12. Show the meaning of Exs. 6 and 7 by use of the actual measures.

13. A milkman has to deliver 200 pints of milk at a certain school. What measures will it be most convenient for him to use? How many times will he use each?

14. Why do we not measure all liquids with the pint measure alone? Of what use is the gallon measure?

Why do we not measure all liquids with the gallon measure? Of what use is the pint measure?

15. How many children can each receive a glass of milk from a pitcher which holds 2 qt. 1 pt. of milk?

16. If a gallon of ice cream costs $1.20, what should 3 pints cost? 2 qt. 1 pt.?

17. If 1 pint is used in each pie, how many gallons and quarts of milk are needed to make 100 custard pies? To make 150 pies? 200?

18. A housekeeper has 2 qt. 1 pt. of milk in one pan, and 3 qt. 1 pt. in another. How can she tell how much she has in all, without putting the milk together and measuring it? Find out for her how much she has.

19. From a barrel of vinegar which holds 31½ gal., a storekeeper sold to customers 1½ gal., 2 qt., 1⅓ gal., and 3 qt. How can he tell how much he has left, without measuring it? Make the computation.

20. If a lamp uses 1 qt. of oil per week, how much will the lamp use in a year? What will this oil cost at 16¢ a gallon?

21. If a family use $\frac{3}{4}$ pt. of kerosene each day, how long will 5 gallons last them?

22. If a box-shaped tank is 2′ 4″ long, 1′ 10″ wide, and 15″ deep, how many gallons will it hold?

23. 32 liquid quarts contain how many cubic inches? 32 dry quarts exceed 32 liquid quarts by how many cubic inches?

24. Make up and work an example similar to Ex. 16. To Ex. 17. Ex. 19.

25. Make up and work two examples entirely your own concerning dry measure.

EXERCISE 113

ORAL WORK

1. How many pints are there in 3 qt.? In a gallon? In 3 gal.? 5 gal.?

2. How many pints in 40 fluid ounces? In 50 qt.? 2.5 qt.?

3. How many ounces in 1 qt.? In $\frac{1}{2}$ pt.? $\frac{3}{4}$ pt.?

4. If a quart of milk costs 6¢, what will 5 gal. 2 qt. cost?

5. If a pint of water weighs a pound, what does a quart weigh? A gallon? 3 qt.?

6. If potatoes are mainly water, about what ought a bushel of potatoes to weigh? What does a bushel of potatoes actually weigh? (See Ex. 21, p. 196.)

7. How many ounces of water in a glassful? How many 2-oz. bottles can be filled from a glassful of fluid?

8. How many of Exs. 1–26 of Exercise 100 (p. 175) can you work at sight?

9. Practice the addition and subtraction of small common fractions.

MEASURES OF TIME

108. The Measures of Time are as follows:

TABLE

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
365 days = 1 common year (yr.)
366 days = 1 leap year
100 years = 1 century

There are 7 days in one week. The year is divided into 12 months, as follows:

		Days			Days
1.	January (Jan.)	31	7.	July	31
2.	February (Feb.)	28 or 29	8.	August (Aug.)	31
3.	March (Mar.)	31	9.	September (Sept.)	30
4.	April (Apr.)	30	10.	October (Oct.)	31
5.	May	31	11.	November (Nov.)	30
6.	June	30	12.	December (Dec.)	31

The following lines, if memorized, will help you to recall the number of days in each month:

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Save February alone,
To which we twenty-eight assign,
Till leap year gives it twenty-nine.

The correct length of the year is $365\frac{1}{4}$ days. It is convenient, however, to make the number of days in each year a whole number. Hence, an ordinary year is considered as containing 365 days.

One year in every four contains 366 days instead of 365, and is called *leap year*. The extra day is added to February, giving it 29 instead of 28 days. Every year divisible by 4 is a leap year; as 1892, 1896. An exception, however, is made in case of years divisible by 100; of such years only those divisible by 400 are leap years. Thus, 1900 was not a leap year, but 2000 will be.

EXERCISE 114

1. Name some of the advantages of having watches and clocks instead of leaving each person to guess the time of day.

2. Name some of the advantages of having all timepieces run together, i. e., indicate the same hour at the same time in different places.

Reduce:

3. 43 min. 27 sec. to seconds.
4. 9 hr. 15 min. to minutes.
5. 2 hr. 15 min. to seconds.
6. 2 hr. 15 sec. to seconds.

Change to higher units:

7. 300 sec.
8. 1200 sec.
9. 1000 min.
10. 3000 sec.

11. Which of the following are leap years: 1802, 1820, 1916, 1922, 2000, 2100, 3000?

12. How many days are there in 17 years, 5 of which are leap years?

13. How many days from Apr. 20 to July 4? (Count July 4 as a day but not Apr. 20.) From Sept. 3 to Dec. 25? From Jan. 3 to Feb. 22?

14. Mary is 13 years 7 months old and her brother is 8 years 5 months old. How much older is Mary than her brother?

15. If a boy gets 12¢ an hour for pulling weeds in a garden, how much will he receive for working 5 hr. 45 min.? 6 hr. 20 min.?

16. How much would this boy (Ex. 15) receive for working from 9.15 A. M. to 11.30 A. M.? From 8.30 A. M. to 1.15 P. M.? (What do A. M. and P. M. mean?)

17. Find out which of the units used in measuring time are natural, that is, are determined by the interval between two events in nature, like two sunrises or two winters. Which are artificial, that is, devised by man, as by the use of a watch or clock?

18. If a clock loses 1 minute 18 seconds from June 5 to Aug. 2, how many seconds does it lose per day?

19. If a clock is losing $2\frac{1}{2}$ seconds per day, how many minutes and how many seconds over will it lose from Sept. 1 to Nov. 1?

20. The following is a portion of a time-table between New York and Albany. How long does it take a train to run from New York to Albany? How long does it take the train to run one mile on the average? (For distance, see p. 389.)

Lv	NEW YORK..........	8$^{\text{A}}_{\text{M}}$45
"	N. Y., 125th St..........	8 57
"	Poughkeepsie..........	10 37
Ar	Albany..........	12$^{\text{P}}_{\text{M}}$00
Ar	Troy..........	......
Lv	Albany..........	12$^{\text{P}}_{\text{M}}$25
"	Troy..........	......
"	Round Lake..........	g1 10
Ar	Ballston..........	1 23
Ar	SARATOGA SPRINGS....	1$^{\text{P}}_{\text{M}}$35

21. Get a time-table for some railroad near your home and from it make and solve 3 examples concerning time.

22. 2.25 yr. = how many years and months?

23. Make up and work an example similar to Ex. 14. To Ex. 15. To Ex. 18.

EXERCISE 115

Oral Work

1. If you count slowly "one — two," etc., the interval between each two words is about one second. Show by counting about how long 10 seconds is. 15 seconds.

2. How many minutes is it since this recitation began? How many hours does school last in the forenoon? In the afternoon?

3. How many hours and minutes is it since breakfast? Until supper?

4. How many minutes in 2 hr.? In $2\frac{1}{2}$ hr.? 5 hr.? 3.25 hr.?

5. $\frac{1}{4}$ hr. + $\frac{1}{2}$ hr. + $\frac{1}{10}$ hr. = how many minutes?

6. How many hours from noon to midnight? From midnight to noon?

7. How many times does the hour hand of a clock go around in a day of 24 hours?

8. Which months contain 31 days? 30 days? 29? 28?

9. How many weeks in a year ? How many days over?

10. If your birthday comes on Wednesday this year, on what day of the week will it come next year?

EXERCISE 116

Review of Compound Numbers

Change to higher units:

1. 126 in.
2. 126 pt. (dry)
3. 250 oz.
4. 450 sec.
5. 450 sq. in.
6. 126 pt. (liquid)
7. 35 sq. ft.
8. 3000 cu. in.
9. 115 in.

Reduce to lower units:

10. 2 gal. 2 qt.
11. 5 yd. 2 ft.
12. 3 bu. 1 pk.
13. 7 wk. 3 da.
14. 5 sq. yd. 3 sq. ft.
15. 3 pk. 2 qt. 1 pt.
16. 1 gal. 1 qt. 1 pt.
17. 5 cu. yd. 1 cu. ft.
18. 2 yd. 1 ft. 3 in.

19. What will 6 bu. 3 pk. 7 qt. of popcorn cost at 3¢ a quart?

20. Find the difference in cubic inches between 1 bushel and 8 gallons.

21. Walter bought 2 bu. 1 pk. of chestnuts at $1.80 a bushel. He roasted them and then sold them at 10¢ a quart. What did he make?

22. If $6\frac{1}{2}$ qt. of shelled corn will plant an acre, how many acres may be planted from 4 bu. of corn?

23. How many days from May 3 to Oct. 20?

24. What will $\frac{3}{4}$ mile of wire fence cost at 8¢ a foot?

25. A man dug a ditch 24 rd. 3 yd. 1 ft. long at the rate of 12¢ a foot. What did he receive?

26. My grocer charges 20 cents for half a peck of potatoes. At this rate, what will a bushel of potatoes cost me?

27. If chestnuts are worth 8¢ a quart, how many bushels can be bought for $10?

28. Practice the multiplication and division of small common fractions by fractions.

EXERCISE 117

GENERAL REVIEW OF PART ONE

1. Fill in the blanks in the following:

3672 = thousands hundreds tens units.

2. In 5623864, how many items of work does the 4 do? Remove the 4; what change has taken place in the value of every other figure?

3. In 5623864, the left-hand figure 6 is how many times as great in value as the right-hand 6?

4. How many periods in 5623864? Mark and name these periods. How many places in this number? Name these places.

5. How many items of work does the zero do in each of the following numbers: 2.076, 3075, 17.208, 3750?

6. New York State contains forty-nine thousand one hundred seventy square miles and Texas contains two hundred sixty-five thousand seven hundred eighty square miles. Can you determine the difference between these two areas by reading or writing the above number words? Now use figures instead of words and determine the difference.

7. Write each of the following in the Roman notation: 1492, 1620, 1776, 1861, 1898. (What important event happened in each of these years?)

8. What principles of the Roman notation are used in writing each of the numbers in Ex. 7?

Find the balance in the following accounts:

9. Dr.	Cr.	10. Rec'd	Paid Out	11. Dr.	Cr.
$101.72	$76.15	$201.62	$99.72	$2000.00	$3712.40
27.62	397.10	17.92	3.65	5542.37	1000.
3721.00	76.42	184.50	314.16	6948.31	1060.92
462.72	819.37	530.43	172.80	374.92	954.83
81.93	62.91	427.11	21.54	7483.00	7211.43
5.96	5.08	1.02	231.76	5029.84	3764.92
703.72	271.37	474.87	42.92	123.40	414.42

12. The receipts of a merchant for the days of a week were as follows:

	M.	T.	W.	Th.	F.	S.
Coal	$762.25	$213.17	$108.17	$325.15	$672.15	$172.15
Lumber . .	37.22	152.37	36.74	172.94	1271.31	12.18
Lime . . .	64.82	122.37	108.25	72.84	176.51	13.09
Fertilizer . .	172.26	91.42	392.76	142.61	51.54	123.72
Seeds . . .	5.42	16.72	39.92	50.00	123.40	17.16

Find the receipts for coal for the entire week; also for lumber, lime, etc. Then find the entire receipts for the week. Also find the entire receipts each day, and for the entire week. Are the final totals obtained by the two methods equal?

13. Mary received the following marks in her studies during one week:

Subjects	M.	T.	W.	T.	F.	Totals
Reading	93	92	85	91	92	
Spelling	65	57	71	81	83	
Arithmetic . . .	81	74	82	76	78	
Geography . . .	90	75	81	74	91	
Penmanship . . .	90	90	85	80	90	
Totals						

On what day did she receive the highest total of marks? In what subject did she receive the highest total of marks during the week?

14. In Ex. 13, find Mary's total marks by both column and row addition. Then find, to two decimal places, her average for the week in each subject. Find the average of these averages (or her average for all studies in the week).

Also find Mary's average for each day, and hence her average for the entire week. How do the two final averages compare?

15. Add the examples on p. 22. Time yourself and make a record of your improvement in speed since you first worked these examples.

Reduce to simplest form:

16. $\frac{5}{6} + \frac{7}{8}$

17. $\frac{5}{12} + \frac{7}{24}$

18. $\frac{3}{20} - \frac{4}{25}$

19. $\frac{8}{9} + \frac{7}{12} - \frac{5}{6}$

20. $7\frac{1}{2} + 3\frac{5}{8} + 2\frac{7}{12}$

21. $4\frac{5}{12} + 7 - 3\frac{5}{6}$

22. $3\frac{1}{10} - 5\frac{7}{15} + 7\frac{4}{20}$

23. $8 + 16\frac{1}{2} + 17\frac{5}{12}$

24. $7\frac{5}{6} + 8\frac{3}{4} + 216\frac{3}{8}$

25. 271.32 + 315.06 + 2.163 − 512.4

26. 3.17 in. + 4.22 in. − 5.3 in. − .072 in.

27. .007 + .03 − .001 + .5 − .205

28. Add $2\frac{3}{100}$ and $3\frac{17}{100}$. Now write these in the decimal form and then add. Compare the amount of work in the two processes.

29. Mr. Stewart on Monday morning had $2763.27 in the bank. On successive days he put into the bank $472.25; took out $216.15; put in $387.24; took out $743; put in $1000; took out $1272.50. How much did he then have in the bank?

30. If a dealer had 57 T. of coal and on successive days sold 41,700 lb., 6200 lb., 4 T. 500 lb., and $2\frac{1}{2}$ T., how can he tell how much he has left, without weighing it? Find this out for him.

31. If the record high jump is 6 ft. $5\frac{5}{8}$ in. and Harry now jumps 5 ft. $2\frac{3}{4}$ in., how much higher must he jump to equal the record?

32.

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)
(*1*)	13	4.8	3.14	427.80	146.785	1000.42
(*2*)	11	14.7	12.37	319.12	127.872	1659.87
(*3*)	9	3.5	2.16	142.76	314.92	1728.42
(*4*)	5	16.1	1.435	319.8	1272.08	3964.75
(*5*)	19	3.2	10.137	817.14	2792.7	8495.67
(*6*)	17	3.6	2.105	376.5	389.27	7592.82
(*7*)	28	14.9	14.275	1297.82	864.93	8347.39

In the above columns of figures, multiply each number in column (*e*) by the number on the same row in column (*a*). Similarly multiply (*e*) by (*b*); also divide (*c*) by (*a*), etc.

The teacher may limit this drill or extend it farther as seems desirable.

33. In Ex. 32, find the sum of each column and the sum of all the columns. Also find the sum of each row and the sum of all the rows. How do the final sums compare?

34. On squared paper (using a square 10 × 10) show that $\frac{3}{20} = \frac{15}{100}$. Also show that $\frac{4}{25} = \frac{16}{100}$. Hence, which is greater: $\frac{3}{20}$ or $\frac{4}{25}$?

35. Reduce three sixteenths plus five eighths minus seven twelfths to the simplest form.

	(*1*)	(*2*)	(*3*)	(*4*)	(*5*)
36. Multiply	204	872.4	746 in.	216.4 in.	4 ft. 3 in.
by	5	5	8	12	2

Reduce to the lowest unit named in each example:

37. 4 gal. 3 qt.

38. 7 yd. 5 in.

39. 7 lb. 13 oz.

40. 5 da. 8 hr. 32 min.

41. 9 bu. 3 pt.

42. 18 rd. 2 yd.

43. I buy from a milkman 3 pt. of milk daily at \$.08 a quart. What will my milk bill be for November, December, and January?

44. A man earns \$110 a month. His expenses are \$856 per year. How much will he save in a year?

45. A farmer took 6 bu. of berries to market and sold them for 10 cents a quart. What did he receive for them?

46. How many prime numbers are there between 1 and 50? See if any member of the class can write this list in one minute.

47. Separate each of the following numbers into prime factors: 231, 5280, 1760, 144, 1728. Why is a knowledge of the prime factors of each of these numbers of especial importance?

48. Find all the exact divisors of 5280 which are less than 25. Treat 1760 and 1728 in the same way.

49. In simplifying $36 \times (752 - 252)$, what operation do we perform first? Why?

50. In simplifying $\frac{144 \times 9 \times 12}{48}$, what do we do first? Why?

51. Compute each of the following in the shortest way:

(*1*) $1212 \times 33\frac{1}{3}$

(*2*) $728 \times 12\frac{1}{2}$

(*3*) $\frac{72 \times 64 \times 25}{60 \times 60}$

(*4*) $728 \times 5 - 728 \times 3$

52. In Ex. 51, in (*1*) multiply in the long way; then in the short way. About how many more figures are required in the one process than in the other? Which process gives more opportunities for making a mistake? Treat (*3*) in the same way.

53. 22 recitations of 55 minutes each equal how many recitations of 40 minutes each?

54. A merchant bought 3000 lb. of nails at $4\frac{1}{4}$¢ a pound and sold them at $4\frac{1}{2}$¢ a pound. How much did he make by the transaction?

55. If 6 suits of clothes cost $96, what will 3 suits cost? 9 suits? 7 suits?

56. How many yards of muslin at 12¢ a yard will be received in exchange for 32 lb. of butter at 33¢ a pound?

57. How many bushels of potatoes at 60¢ a bushel will be needed to pay for 25 yd. of carpet at $1.44 a yard?

Find x, or the missing number, in each of the following:

	l	w	A
58.	3 ft.	4 ft.	x
59.	7 ft.	3 yd.	x
60.	3 ft.	x	12 sq. ft.
61.	x	6 ft.	24 sq. yd.

62. On squared paper make diagrams for Exs. 58–61.

Find x in

	l	w	d	V
63.	8 in.	5 in.	4 in.	x
64.	2 ft.	3 ft. 3 in.	10 in.	x
65.	8 in.	4 in.	x	128 cu. in.
66.	10 ft.	x	4 ft.	200 cu. ft.

67. If $A = 40$ sq. ft. and $l = 10$ ft., find w, using $A = l \times w$.

68. On a field of 23 acres a farmer obtained a crop of 642 bushels of corn. The next year by using a special variety of seed (the Yellow Dent), he increased the yield of the same field to 946 bushels. What was the increase per acre? What was the increase worth per acre at 80¢ a bushel?

69. If gunpowder is $\frac{1}{10}$ sulphur, $\frac{3}{20}$ charcoal, and $\frac{3}{4}$ niter, how many pounds of each of these materials are there in $1\frac{1}{8}$ tons of gunpowder?

70. The largest nugget of gold ever found weighed 184 pounds and was sold for $46,625. How much was this per ounce?

71. James buys a horse for $75, agreeing to pay $11 down and $8 a month afterward. In how many months will he have paid for the horse?

72. If a girl can write 27 words per minute with a pen, and 76 words per minute with a typewriter, how much time can she save by writing 10,000 words with a typewriter?

73. If Porto Rico has an average length of 90 miles and an average width of 40 miles, how does its area compare with that of New Jersey? (See p. 386.)

74. If a motor car is traveling at the rate of 90 miles per hour, how many feet is this per second? How does this speed compare with a bullet from a rifle which goes 2400 feet per second?

75. Our schoolroom contains three windows, each $3' \times 9'$. The floor is $30' \times 40'$. Does the window space equal $\frac{1}{6}$ of the floor space?

76. If the round-trip fare from New York to Philadelphia is $4, how much is saved by sending a letter from one city to the other instead of a messenger who must make the round trip? How much would be saved by telephoning 3 minutes at a 75–25 rate? By sending a telegram of 15 words at a 25–2 rate?

77. If, on the average, there is one voter to every 5 persons in the population, how many voters are there in 87,629 people? How many voters in New York City in the year 1910? (See p. 387.)

78. A tank 1′ 20″ long, 1′ 2″ wide, and 1′ deep will hold how many gallons?

79. If Mr. Taylor pays 25¢ for 4 cigars and smokes 3 cigars a day, how much do his cigars cost in a year? If he had saved this money, how much money would he have saved in 40 years (besides interest on his money)?

80. Is the value of a fraction changed if we divide both numerator and denominator by the same number? Show this by an example on squared paper.

81. Is the value of a fraction changed, if we add the same number to both numerator and denominator? What does the value of $\frac{1}{6}$ become, if we add 4 to both numerator and denominator? Make up and work a similar example.

82. Is the value of a fraction changed, if we subtract the same number from both numerator and denominator? What does $\frac{5}{10}$ become, if we subtract 4 from both numerator and denominator? Make up and work a similar example.

83. Find the difference between the cost of sending a message of 100 words by telegraph at a 25–1 rate and of telephoning the same at a 75–20 rate, if the telephone were in use four minutes?

84. If the divisor is 24 and the quotient is 25, find the dividend.

85. If the minuend is 1000 and the remainder is 726, find the subtrahend.

86. If the product is 456 and the multiplicand 38, find the multiplier.

87. If the sum of two numbers is 3726 and one of the numbers is 1894, find the other number.

88. The letter x stands for a missing number. Find x in each of the following:

(*1*) $\$3000 + x = \$5550$ (*3*) $2567 = 8254 - x$
(*2*) 34 times $x = 1530$ (*4*) $1650 \div x = 46$

89. It costs $40 a month to send 20 children by stage to a certain school on the 21 school days in a month. What is the cost for 1 pupil for 1 day? Is this greater or less than the cost of 2 five-cent carfares in the city each day?

90. One member of a ball nine made 237 base hits out of 920 times at the bat. What was his batting average expressed as a decimal to 3 places?

91. The batting record of the 12 members of a ball team were .314, .309, .302, .292, .287, .276, .254, .252, .243, .241, .237, and .192. Find the average batting record of the whole team.

92. Multiply 786 by 27. Now express these numbers in Roman notation and try to multiply. Why is it difficult to multiply numbers expressed in the Roman notation?

93. From a piece of cloth which originally contained 35 yd., the storekeeper has sold to different customers $3\frac{1}{2}$ yd., $6\frac{3}{4}$ yd., and $1\frac{5}{8}$ yd. How can he tell how much he has left, without measuring it? Do this for him.

94. 2 pk. 4 qt. is what fraction of a bushel? Also what decimal fraction?

95. In division, how do you check, or verify, your work when there is no remainder? Show this by an example. How do you verify if there is a remainder? Show this by another example.

96. Two trains start 1250 miles apart and travel toward each other, one at the rate of 30 miles an hour, and the other at 40 miles. How far apart will they be at the end of 15 hours?

97. A farmer needs $1060 to pay a debt. He has $217.35 in one bank and $462.71 in another. How many bushels of wheat will he need to sell at 92¢ a bushel in order to make up the balance needed?

98. A book contains 300 pages, each of which has 22 lines. Each line contains 10 words on the average, and each word averages 5 letters. How many times is each letter in the alphabet used in the book on the average?

99. If 7.5 cu. ft. of natural gas produce as much heat as 1 lb. of coal, how many cubic feet of gas will produce as much heat as 1 ton of coal? Which is the cheaper fuel, if coal costs $6.50 per ton and gas costs 25¢ per 1000 cu. ft.?

100. Is every rectangle a square? Is every square a rectangle? Draw diagrams on your squared paper to illustrate what you say.

101. If your pace is 2 ft. 6 in. long, how many steps do you take in walking a mile? How long does it take you to walk a mile, if you take two steps in one second?

102. $720 is $\frac{5}{8}$ of what number?

103. If $\frac{3}{4}$ of an acre of land is worth $120, what are 40 acres worth?

104. If the quotient is 27, the divisor 72, and the remainder 17, find the dividend.

105. If I borrow $80 for a year and pay 5¢ for the use of each dollar, what shall I pay in all? What should I pay if I kept the money 2 years?

106. What is the Golden Rule of arithmetic? To what examples on pp. 213 and 214 does it apply?

107. Point out the examples in this Exercise in which work can be saved by the use of cancellation. Those in which work can be saved by the use of parenthesis. Also those in which work can be saved by the use of aliquot parts.

EXERCISE 118

ORAL REVIEW OF PART ONE

1. Write the following arrangement of fractions on the blackboard. As the teacher points, add each pair of fractions.

$\frac{1}{2}$	$\frac{1}{5}$	$\frac{5}{6}$	$\frac{1}{7}$	$\frac{1}{6}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{1}{3}$	$\frac{4}{5}$
$\frac{2}{3}$	$\frac{2}{5}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{3}{5}$	$\frac{3}{7}$	$\frac{1}{10}$

2. State the difference between each pair of fractions in Ex. 1. The product of each pair. The quotient of each pair.

3. Write the following arrangement of decimal fractions on the blackboard. As the teacher points, multiply each pair. Divide each pair. Also add. Also state the difference between each pair.

.2	1.2	.5	.4	3.6	.8	1.8	.6	8	2.4
2	.2	4	.1	.01	.4	.002	5	.5	.001

4. Find the cost of the following in the shortest way:

(*1*) 24 cans of tomatoes at $12\frac{1}{2}$¢. At $16\frac{2}{3}$¢.

(*2*) 32 cans of corn at $12\frac{1}{2}$¢. At $16\frac{2}{3}$¢.

(*3*) 24 baseballs at $33\frac{1}{3}$¢. At $66\frac{2}{3}$¢. 50¢.

(*4*) 16 yards of cloth at $37\frac{1}{2}$¢. At 75¢. $62\frac{1}{2}$¢. $87\frac{1}{2}$¢.

5. Compute in the shortest way:

(*1*) $(12 + 8)\,26$

(*2*) $12 \times 26 + 8 \times 26$

(*3*) $42 \times 26 - 12 \times 26$

(*4*) $80 \times 12\frac{1}{2}$¢

(*5*) $.625 \times 72 - .125 \times 72$

(*6*) $7 \times 12 + 8 \times 12 - 5 \times 12$

(*7*) $\frac{36 \times 5}{12}$

(*8*) $\frac{3.6 \times 5}{12}$

(*9*) $120 \times 16\frac{2}{3}$¢

6. Find the value of $\frac{2}{3} + \frac{1}{6} + \frac{1}{3} + \frac{5}{6}$.

State the results as the successive numerators are added; thus, 4, 5, 7, $\frac{12}{6}$, or 2, *Ans.*

7. In like manner, find the value of

(*1*) $\frac{1}{4} + \frac{1}{2} + \frac{3}{4}$

(*2*) $\frac{3}{8} + \frac{1}{4} + \frac{5}{8} + \frac{3}{4}$

(*3*) $\frac{1}{5} + \frac{3}{10} + \frac{2}{5} + \frac{3}{5}$

(*4*) $\frac{5}{8} + \frac{1}{2} + \frac{3}{4} - \frac{1}{8} + \frac{1}{4}$

(*5*) $\frac{1}{3} + \frac{1}{6} + \frac{1}{2} - \frac{2}{3} + \frac{5}{6}$

(*6*) $\frac{1}{5} + \frac{3}{10} - \frac{1}{2} + \frac{3}{5} + \frac{7}{10}$

8. Give the meaning of each of the following abbreviations. Also recite the table in which each occurs:

pt. T. bu. sq. in. gal. yd. lb. cu. yd.
in. yr. min. cwt. pk. qt. sec. sq. ft.

9. A full set of teeth for an adult person contains 8 incisors, 4 eye teeth, 8 bicuspids, and 12 molars. What fraction of the entire number are the incisors? The molars? The eye teeth? The bicuspids?

10. Find the value of

	(*1*)	(*2*)	(*3*)	(*4*)
(*a*)	$\frac{1}{2} + \frac{1}{4}$	$\frac{5}{2} - 1$	$\frac{2}{3} \times 6$	$2 \div \frac{1}{4}$
(*b*)	$\frac{1}{2} + \frac{3}{4}$	$\frac{11}{2} - \frac{3}{2}$	$\frac{1}{2} \times \frac{3}{5}$	$\frac{3}{2} \div 6$
(*c*)	$\frac{1}{2} + \frac{2}{3}$	$\frac{3}{2} - \frac{1}{4}$	$14 \times 2\frac{1}{2}$	$\frac{1}{4} \div 2$
(*d*)	$4\frac{1}{2} + \frac{2}{5}$	$2\frac{4}{5} - 1\frac{3}{5}$	$12 \times 3\frac{3}{4}$	$.12 \div 3$
(*e*)	$3\frac{1}{3} + 2\frac{1}{2}$	$2\frac{1}{2} - 1\frac{1}{4}$	$.18 \times .6$	$.12 \div .3$

11. If $\frac{1}{4}$ lb. of tea costs 20¢, what will 1 lb. cost? 2 lb.?

12. If $\frac{3}{4}$ lb. of coffee costs 24¢, what will 1 lb. cost? 2 lb.?

13. \$20 is $\frac{1}{4}$ of what sum of money? \$12 is $\frac{2}{3}$ of what sum? \$.50 is $\frac{3}{5}$ of what?

14. If 2 acres yield 400 bushels of potatoes, what will 3 acres yield? 10 acres?

15. State what it costs to mail each of the following:

(*1*) A 2-oz. letter.
(*2*) 1 lb. of candy.
(*3*) Collars weighing 8 oz.
(*4*) Newspapers weighing 6 oz.
(*5*) A book weighing 11 oz.
(*6*) Magazines weighing 7 oz.

16. Divide each of the following fractions by 2:

$\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{2}{7}$ $\frac{3}{7}$ $\frac{4}{7}$ $\frac{5}{7}$ $\frac{6}{7}$ $\frac{2}{9}$ $\frac{4}{9}$ $\frac{3}{10}$
$\frac{5}{9}$ $\frac{8}{9}$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{8}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$

17. Divide each of the fractions in Ex. 16 by 3. By 4. Also multiply each of them by 2. By 3. By 4.

18. Write the following columns of numbers on the blackboard. As the teacher points, make change for \$2. For \$5.

52¢	72¢
\$1.27	\$1.23
98¢	37¢
\$1.67	\$1.16

The teacher may change the numbers at discretion.

19. Find the missing number (or x) in the following:

	l	w	A
(*1*)	2 ft.	3 ft.	x
(*2*)	6 ft.	x	24 sq. ft.
(*3*)	x	8 in.	48 sq. in.
(*4*)	1 ft.	x	48 sq. in.

	l	w	d	V
(*5*)	4 ft.	4 ft.	2 ft.	x
(*6*)	5 in.	4 in.	2 in.	x
(*7*)	5 in.	2 in.	x	40 cu. in.
(*8*)	10 ft.	x	5 ft.	100 cu. ft.

20. On pp. 208, 209, which numbers are abstract and which concrete?

21. Add the following at sight:

(1)	(2)	(3)	(4)	(5)	(6)
49¢	27¢	18 ft.	67 lb.	$1.53	$1.37
16¢	15¢	13 ft.	25 lb.	.25	.24

(7)	(8)	(9)	(10)	(11)	(12)
85	57	147	252	516	46
26	36	23	16	37	217

22. Subtract at sight:

(1)	(2)	(3)	(4)	(5)	(6)
37¢	37¢	32¢	72 in.	88 ft.	52 lb.
16¢	25¢	15¢	24 in.	37 ft.	29 lb.

(7)	(8)	(9)	(10)	(11)	(12)
$1.00	$1.41	299	162	223	516
.23	.15	87	35	45	47

23. Simplify in the shortest way:

(1) $7 + 7 + 7 + 7 + 3 + 5$
(2) $4 + 4 + 3 + 4 + 4 + 4$
(3) $6 + 5 + 6 + 5 + 6 + 5$
(4) $8 - 5 + 8 - 5 + 8 - 5$
(5) $6 + 6 + 6 + 6 - 5 - 5$
(6) $7 + 3 + 7 + 7 + 7 - 4$

24. Compute the following:

(1) 70×80
(2) 80×90
(3) 500×6
(4) $400 \times 6\frac{1}{2}$
(5) 200×40
(6) $720 \div 60$
(7) $600 \div 30$
(8) $800 \div 40$
(9) $200 \div 5$
(10) $500 \div 20$
(11) $(60 + 40) \div 20$
(12) $(60 - 40) \div 20$
(13) $(6 + 4) \div (2 + 3)$
(14) $(12 - 8) \div (4 - 2)$
(15) $(8 + 12) \div (10 \div 2)$

25. Give the prime numbers between 1 and 25.

26. Give the prime factors of 12, 15, 7, 18, 16, 21.

27. Which is greater: $\frac{2}{3}$ or $\frac{3}{4}$? $\frac{3}{7}$ or $\frac{2}{5}$? $\frac{2}{3}$ or $\frac{5}{6}$?

28. Which would you rather own: a $\frac{2}{5}$ share or a $\frac{1}{3}$ share in a steamboat? $\frac{3}{7}$ or $\frac{2}{5}$?

29. State how many hundredths each of the following equals:

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{20}$ $\frac{4}{25}$ $\frac{8}{40}$ $\frac{2}{5}$ $\frac{3}{10}$ $\frac{4}{5}$ $\frac{7}{10}$

30. If a bushel contains $1\frac{1}{4}$ cu. ft., how many bushels will be contained in a box $3' \times 4' \times 5'$? $3' \times 3' \times 10'$? $2' \times 5' \times 7'$?

31. What is the cost of 2 qt. 1 pt. of strawberries at 5¢ a pint? At 12¢ a quart?

32. Count by $2\frac{1}{2}$'s from 10 to 50. By $2\frac{1}{3}$'s from $2\frac{1}{3}$ to 42. By $1\frac{2}{3}$'s from $1\frac{2}{3}$ to 30. By $6\frac{1}{4}$ from $6\frac{1}{4}$ to 100.

Let the pupil make up similar examples, or the teacher dictate them.

33. If your pulse beats 72 times in a minute and you take 18 breaths in a minute, how many beats does your pulse make while you are taking 1 breath?

34. Is a field 100 yd. long and 40 yd. wide more or less than an acre? A field 99 yd. × 40 yd.?

35. Is a field $200' \times 200'$ more or less than an acre? A field $198' \times 200'$?

36. Tell which of the following fields are more or less than an acre:

(*1*) $500' \times 100'$
(*2*) $502' \times 100'$
(*3*) $900' \times 50'$
(*4*) $400' \times 100'$
(*5*) $398' \times 100'$
(*6*) $1000' \times 50'$
(*7*) $1050' \times 50'$
(*8*) 100 yd. × 50 yd.
(*9*) 102 yd. × 50 yd.
(*10*) 90 yd. × 50 yd.
(*11*) 89 yd. × 50 yd.
(*12*) 90 yd. × 45 yd.

37. If I pay \$5 for the use of \$100 for a year, what ought I to pay for the use of \$300? Of \$400? \$500? \$1000? \$2000? \$3000?

38. If x stands for a missing number, find the value of x in

(*1*) $\$40 \times x = \$60$
(*2*) $\frac{x}{20} = 3$
(*3*) $x \div 3 = 15$
(*4*) $\frac{12}{x} = 3$
(*5*) $60 \text{ ft.} - x = 40 \text{ ft.}$
(*6*) $\frac{x}{12} = 16$
(*7*) 30 times $x = 120$
(*8*) $12\frac{1}{2}$ times $x = 200$
(*9*) $\$200 = \$100 + x$

39. Count by $12\frac{1}{2}$ from $12\frac{1}{2}$ to 200. From 200 down to $12\frac{1}{2}$.

40. Count by $33\frac{1}{3}$ from $33\frac{1}{3}$ to 200. From 200 down to $33\frac{1}{3}$.

41. How many examples on p. 113 can you now work at sight?

42. Work other Exercises at sight, as directed by the teacher.

EXERCISE 119

Special Review: Girls' and Boys' Party

1. Helen and Gilbert are twins. On June 2, when they were eleven years old, they had a birthday party. Their mother gave them $3.50 with which to buy refreshments. If they invited 5 girls and 5 boys to the party, how much could they spend in entertaining each member of the party?

2. They planned to have 1 gallon and 1 quart of ice cream. If each boy and 3 of the girls were to have a second dish of ice cream, what fraction of a quart must be served at each helping? Is this more or less than $\frac{1}{4}$ of a quart?

3. Helen and Gilbert could buy their ice cream at $1.20 a gallon. To make it at home, they would need 3 qt. 1 pt. of cream at 24¢ per quart, 2 lb. of sugar at 6¢ per pound, and 20 lb. of ice at 40¢ per 100 lb. Which way of getting their ice cream would cost them less? How much less?

4. Helen and Gilbert found that they could buy at the baker's 1 large cake for 60¢ and 3 doz. small cakes for 15¢ a dozen. Their mother offered to help make the cake if they would buy $2\frac{1}{2}$ lb. of sugar at 6¢, 2 qt. of milk at 8¢, $1\frac{1}{3}$ doz. eggs at 27¢, 4 lb. of flour at 5¢, and one cocoanut for 12¢. Which would be the cheaper way for them to get their cake?

5. They could make lemonade with 9 lemons at 3 for 5¢ and 1 lb. of sugar, or they could make coffee with 1 lb. of coffee at 36¢, 1 pt. of cream at 12¢, and 1 lb. of sugar. Which would be cheaper? Which drink would be more wholesome for them?

6. Instead of buying 25 sandwiches at 5¢ each, they bought a chicken weighing 2 lb. 4 oz. at 20¢ a pound, 2 loaves of bread at 6¢ a loaf, and 6 oz. of butter at 36¢ a pound. How much did they save by making their own sandwiches?

7. If they bought all of their refreshments in the cheapest way, how much money did they spend in all?

8. From their own garden they used 3 doz. roses worth 15¢ a dozen, and 4 qt. of strawberries, worth 8¢ a quart. What were these worth together?

9. How much of their $3.50 did they have left?

10. Helen's mother allowed her $11 with which to buy a new dress for the party. Let each girl in the class tell how many yards of cloth would be needed, and give probable prices for some kind of goods, the trimmings for the dress, and the cost of making the dress.

11. In one of the games at the party, the children were to name the author of each of several lines of poetry. Margaret won the prize by getting 21 names right out of 25. How many hundredths, or what per cent, of the whole number is this?

12. If Margaret had 21 authors right, 2 other children had 20 authors right, 3 had 19, 3 had 18, 2 had 16, and 1 had 12 right, what was the average number they all had right?

13. If Helen was 4 ft. 11.3 in. tall on her last birthday, and 5 ft. 1.7 in. tall on this birthday, how much had she gained in height in the year?

14. Herbert lives 8 city blocks from the home of Helen and Gilbert. Each block with its crossing is 560 ft. long. What decimal fraction of a mile is the distance between the houses? If Herbert can walk a mile in 20 minutes, how long will it take him to walk home?

15. If Herbert is to be at home by six o'clock, at what time must he leave the party?

16. Make up and work examples concerning the way you would spend $5 for refreshments for a party of 20 boys and girls.

17. Write the name of the months on the blackboard. As the teacher points to the name of the month, let a pupil give the number of the month.

Thus, when the teacher points at April, let the pupil say "four" or "fourth month."

PART TWO

CHAPTER VIII

NUMBERS THROUGH BILLIONS; FUNDAMENTAL OPERATIONS

NUMERATION AND NOTATION THROUGH BILLIONS

EXERCISE 120

1. What is a unit? Give three examples of units.

2. What are some of the uses of units?

3. What is a ten? A hundred? A thousand? A million?

4. What is numeration? What is notation? Give an example of each.

5. Write a number of three periods and seven places.

6. How many periods are there in a number of 6 places? Of 10 places? 4 places?

7. Express in figures eighteen million three hundred sixty-four thousand nine hundred twenty-eight. How many letters are used in expressing this number in words? How many figures are used in expressing it in figures? The number of letters is how many times as great as the number of figures?

8. In 18,364,928, how many items of work does the figure 8 on the right do? The 2? Each other figure? The 8 to the left is how many times as great in value as the 8 to the right?

9. Why is our system of numeration called the decimal system?

10. Write a number containing four periods. Do you know the name of the left-hand period?

109. A Billion is a thousand millions.

Thus, the number 26,784,000,000 is read "twenty-six billion seven hundred eighty-four million."

Form a numeration and notation table like that on p. 11, but extending through billions.

110. Shape Value and Place Value of the Figures in a Number. In the number 6754, the figure 7 by its shape expresses seven units. By its position 7 expresses seven hundreds.

Hence, every figure in a number has two values, a shape value and a place value.

EXERCISE 121

Read:

1. 17,623,000,000
2. 14,007,594,007
3. 786,542,387,215
4. 500,000,700,800

5. The wealth of the United States is over $120,000,000,000.

6. At one time the national debt of the United States was $3,210,000,000.

7. The national debt of France is over 30,000,000,000 francs.

8. The total power of the streams of the United States is estimated at 140,000,000 horse power. If one horse power working night and day is equivalent to the manual labor of 20 men, the water power in the United States is equivalent to the manual labor of 2,800,000,000 men. Since less than $\frac{1}{100}$ of this water is now utilized, the water power utilized in the United States is equivalent to the manual labor of less than 28,000,000 men.

Write in figures:

9. Nineteen billion seven hundred million six hundred forty-two thousand eight hundred.

10. Three hundred eighty-four billion seven hundred thousand.

11. Seven hundred billion eight hundred forty-six thousand.

Rewrite the following, expressing all numbers in figures:

12. The entire population of the earth is estimated as one billion five hundred million people.

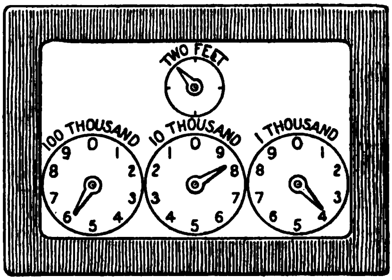
13. In the year 1800 the combined exports and imports of the United States had a value of one hundred sixty million two hundred twenty-four thousand dollars.

In the year 1900 the combined exports and imports of the United States were worth two billion fifty-five million one hundred fourteen thousand dollars.

14. In 7,625,203, what is the place value of each figure? What is the shape value of each?

15. Learn how to read the dial of the gas meter shown in the picture.

16. When the hand on the circle to the left points to a figure (as 5), how many times as much gas is indicated as when the hand in the middle circle points to the same figure?



Dial or Face of a Gas Meter

17. As when the hand in the circle to the right points to this figure?

18. In the Roman notation, placing a bar over a letter makes what change in its value? Using the letter M, write one million in the Roman notation. Write two millions.

19. Read $\overline{\text{M}}\,\overline{\text{D}}\,\text{CCC}$.

20. Beginning with any number and with 150 as a limit, practice counting by 2's both forward and backward. In like manner, practice counting by 3's. By each number up to 9.

ADDITION

EXERCISE 122

1. Work again Exs. 1–5 of Exercise 6 (p. 19).

2. What is addition? What is a sum? Addends? The sign of addition? Give an example of addition.

3. What is the use of addition? Give an example showing that addition is shorter and easier than counting one number on to another.

4. Add 7 horses + 5 cows by putting a more general word in the place of both "horses" and "cows." Give another example about robins and sparrows, where a general word makes an addition possible.

5. How do you check an addition? Apply this check to the example you gave in Ex. 2.

111. Addition by Groups of Figures whose sum is 10, 11, or 20. In adding a column of figures, it is often helpful to combine some of the figures of the column into small groups and add the sums of the groups thus formed. Groups of figures whose sums are 10, 11, or 20 are especially helpful in this way.

Ex. Add the adjoining column of figures.

The process is indicated by the figures annexed to the right of the given column of figures. The third column shows the way the addition is spoken.

Column	Groups	
9	10	58
1		
8		48
5	10	40
5		
3	11	30
8		
3	10	19
7		
9		9
58	*Sum*	

EXERCISE 123

Add both up and down, by 10's, 11's, and 20's, as far as possible:

1.	2.	3.	4.	5.	6.
6	5	1	18	$16.18	$64.10
4	6	4	31	24.02	45.55
8	9	5	30	31.91	35.50
7	7	3	31	14.09	73.62
3	3	7	31	42.05	8.48
8	5	7	30	10.10	62.07
2	3	2	17	21.62	4.94
5	2	9	45	35.35	91.37
4	9	4	34	15.08	26.76
1	5	6	71	18.05	75.17

Let the teacher dictate similar examples.

7. In the nine rooms of a certain school, the number of pupils was as follows: 31, 47, 32, 35, 46, 42, 44, 49, 38. How many pupils were in the school in all?

8. A farmer in Alabama raised twelve bales of cotton, the number of pounds in each being as follows: 491, 489, 516, 524, 479, 521, 514, 506, 510, 497, 504, 509. How many pounds did these weigh together? Can you find the average weight of the bales? What was the cotton worth at 13 cents a pound?

9. Make up and work an example similar to Ex. 8, concerning five wagonloads of coal.

10. Find the total area of all the states in the United States which have each an area of more than 50,000 square miles. (See p. 386.) Find the average area of these states. (What does the word *area* mean?)

11. Find the total area of the states which have each an area of less than 10,000 square miles. Find their average area.

12. Measure the length of the school yard or of some other yard. Also its width. Find the perimeter of the yard you have measured. (What does *perimeter* mean?)

13. Make a diagram of the yard you measured for Ex. 12. What can such a diagram be used for?

14. Practice oral addition by decades, as 46 + 7, 56 + 7, etc.

Subtraction

EXERCISE 124

1. Work again the examples in Exercise 10 (p. 27).

2. What is subtraction? A subtrahend? A minuend? A difference? Give an example of subtraction and in this example point out the meaning of each of the terms just defined.

3. What is the use of subtraction? Give an example showing that subtraction is shorter than counting one number off from another.

4. How do you check a subtraction? Apply this check to the example you have worked in Ex. 2.

Copy the next nine examples and time yourself to see if you can subtract and check them all in less than 5 minutes.

5. 7643
2192

6. $816.72
592.37

7. $495.67
278.63

8. $1000.00
594.87

9. $3050.00
1999.09

10. $7600.09
5437.25

11. $84,000.76
29,887.09

12. $9,672,348.72
3,154,972.09

13. $3,000,000.00
1,543,762.52

14. Denver is 5175 ft. above sea level. How much less than a mile is this? How much lower than the top of Mt. Washington? Lower than the top of Pike's Peak? (See p. 387.)

15. The area of the United States in the year 1800 was 827,944 sq. mi. How much had this area increased by the year 1910? (See p. 386.)

16. The area of North America is 6,446,000 square miles. What is the difference between this and the area of the United States?

17. In a recent year the United States life-saving service cost $2,249,375 and saved $9,286,260 worth of property. What was the gain in money? Can you tell what the United States life-saving service is? Is the saving of property its greatest value?

112. Mental Subtraction of Dates. It is useful to be able to determine mentally the number of years between two given dates.

Ex. 1. How many years are in the interval from 1620 to 1776?

From 1620 to 1700 is 80 years; from 1700 to 1776 is 76 years. 80 + 76 = 156 years.

Ex. 2. How many years are in the interval between 1492 and 1776?

From 1492 to 1500 is 8 years; from 1500 to 1776 is 276 years; 276 + 8 = 284 years.

EXERCISE 125

Oral Work

At sight find the interval between the following dates:

1. 1732, 1776
2. 1812, 1861
3. 1812, 1844
4. 1776, 1812
5. 1776, 1861
6. 1812, 1898
7. 1620, 1776
8. 1066, 1492
9. 1066, 1588
10. 1066, 1438
11. 1066, 1815
12. 1620, 1898

13. Find the interval between the present year and each of the dates named in the preceding examples.

An important event occurred in each of the years given in Exs. 1–12. How many of these events can you name?

14. Practice oral subtraction by decades, as 98 − 5, 88 − 5, 78 − 5, etc.

Addition and Subtraction Combined

113. The Sum Being Given, to Supply a Missing Addend.

Ex. Find what number must be added to the sum of $7625, $3870, $2163, and $1956 to make $18,926.

$7625	*Given*
3870	
2163	
1956	
3312	*Required*
$18,926	*Sum*

Instead of adding the first four numbers and subtracting their sum from $18,926, labor is saved by adding each column and supplying in it the figure necessary to make the column sum.

Thus, adding from the top we say "5, 8, 14, and 2 make 16," and write **2**. We proceed in like manner with the other columns.

EXERCISE 126

Supply the missing addend in each of the following:

1.	2.	3.	4.	5.
$416	$247	$486	$37.26	$327.16
231	516	127	15.17	
128	172		27.12	542.31
		321		761.37
$986	$1000	$1200	$84.21	$2000.00

6. The distance across the Atlantic Ocean by a certain steamship route is 2972 miles. A steamer has gone on successive days 482, 512, 493, 501, and 482 miles. How many miles has the steamer yet to go?

7. A merchant has agreed to pay a debt of $2000 on April 1. He now has in cash in different banks $327.40, $685.37, and $317. How much more money does he need to complete the $2000?

8. Make up and work an example similar to Ex. 7.

Supply the missing addend:

9.	**10.**	**11.**	**12.**	**13.**
$37.16	$100.00	$327.42	$	$387.54
9.15	97.54	5429.62	347.92	
819.13	2054.62	308.05	59.42	912.46
76.05	378.54	4275.92	387.92	589.27
82.00	392.06	372.48	164.37	123.45
38.00	1427.36	476.51	3756.48	314.16
19.72	461.50		2132.54	215.04
		375.08	123.92	172.80
$1500.00	$5376.42	$12,000.00	$7492.00	$5000.00

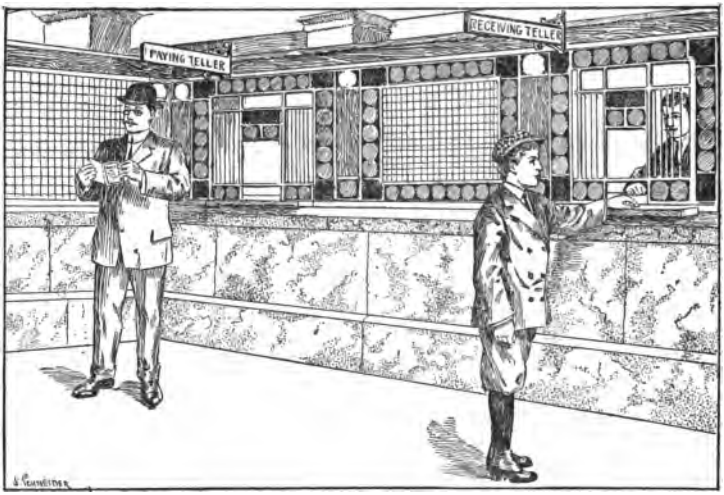
14. Work again Exercise 39 (p. 76).

114. Banking; Uses of Banks. A bank is a place where money and other valuables may be put for safe keeping. Banks also lend money to persons needing it, in return for certain payments called *interest* or *discount.*

In case a farmer raises a crop of 10,000 bushels of wheat, he may need $4000 immediately after harvesting the crop to pay his laborers and other expenses. He does not want to sell his wheat because he has reason to believe that by holding it three months he can get 20¢ a bushel more for it, or a total of $2000 extra. Under these circumstances a bank will often lend him money, to be repaid at a specified date, after he sells his crop.

If the farmer pays the bank $60 interest for the use of the $4000 for 3 mo., he gains $2000 – $60 or $1940 by the transaction. What does the bank gain?

Banks are also useful in other ways, some of which will be mentioned later.



EXERCISE 127

1. At the beginning of the week a man had $412.72 in a bank. At different times during the week he deposited in the bank $127.15, $202.32, $150, $106.23, $173.37, $323.15; and drew out $217.37, $123.15, $172.18, $212.15, $174.14, $153. How much money did he have in the bank at the end of the week?

2. On Sept. 1, Walter had $37.15 in the savings bank. During September he deposited $2.12, during October $1.75, November $2.22, and December $1.32. During December he drew out $3.75 with which to buy Christmas presents. On Jan. 1, he was credited with 73¢ interest. What balance did he have in the bank on Jan. 1?

3. In a certain city during the months from January to June, different schools deposited money in the school savings bank as follows:

	Jan.	Feb.	March	April	May	June
Washington School .	$36.72	$23.72	$17.63	$24.14	$14.18	$19.17
Adams School . . .	28.72	24.16	18.50	22.36	18.50	14.12
Jefferson School . .	51.23	23.72	14.16	12.42	23.14	15.17
Madison School . .	37.16	21.50	14.50	13.92	14.18	23.19
Monroe School . . .	42.00	17.00	16.50	31.14	11.18	19.14

Find the amount deposited by each school in the six months. Find the amount deposited by all the schools in each month. What short way is there of checking all your results?

4. Which school (in Ex. 3) deposited most? Which least? In which month was the largest sum deposited by all the schools together? Make up and answer two similar questions concerning the numbers given in Ex. 3.

5. When men need money, why do they borrow sometimes from their friends and not from the bank? Why sometimes from the bank and not from their friends?

6. If a man has many valuable papers to keep, what is the advantage of sending them to a bank instead of buying a safe and keeping these valuables in it at home?

7. Work Exercise 9 (p. 26) or similar examples in oral addition dictated by the teacher.

8. Work Exercise 12 (p. 31) or similar examples in oral subtraction dictated by the teacher.

MULTIPLICATION

EXERCISE 128

1. Work again the examples in Exercise 15 (p. 36).

2. Define multiplication, multiplicand, multiplier, product, and factors. Give an example which shows the meaning of each of these terms. What is the sign of multiplication?

3. What is the use of multiplication? Give an example to show that multiplication is shorter than addition. Which is more likely to contain a mistake: the long process of addition, or the short process of multiplication?

4. Give the rule for multiplying one number by another.

5. How do you check the accuracy of a multiplication? Apply this check to the example you have worked in Ex. 2.

EXERCISE 129

Multiply the following and check each result:

1. 6598 by 3712
2. 70,862 by 9273
3. $615.72 by 3713
4. 65,023 by 1209
5. Multiply each of the following by 1002:
   3861 $20.73 $159.72 $8007.54 $100.37 32,000
6. Multiply 3861 by 4090. By 9001, 8900, 4003, 60003, 1200.
7. How many feet are there in 7912 miles?
8. 1 acre contains 43,560 sq. ft. How many square feet are there in 8492 acres?
9. 1 cu. ft. contains 1728 cu. in.; how many cubic inches are there in 4876 cu. ft.? In 39,276 cu. ft.?

Read and multiply each of the following in the shorter way:

10. 8762 × 2¢
11. 95,872 × 8¢
12. 17¢ × 3807
13. 7416 × 22 in.
14. 13 oz. × 8916
15. 95,432 × 3 qt.
16. What is the cost of 8972 lb. of sugar at 4¢ a pound? Of 7965 lb. of rice at 7¢ a pound?
17. Practice the oral addition and subtraction of small fractions. (See Exercise 58, p. 101.)

115. In the **Multiplication of a Whole Number by a Mixed Number,** it will be found best to

Multiply first by the fractional part of the multiplier.

Ex. Multiply 85 by $12\frac{3}{4}$.

$$\begin{array}{rl} 85 & \\ \underline{12\frac{3}{4}} & \\ 4)\overline{255} & \\ 63\frac{3}{4} & (=\frac{3}{4}\text{ of }85)\\ 170 & (=2\times 85)\\ \underline{85} & (=10\times 85)\\ 1083\frac{3}{4} & \textit{Product}\end{array}$$

EXERCISE 130

Multiply each of the following:

1.	2.	3.	4.	5.
$\$34$	$\$35$	35 ft.	35 in.	235
$8\frac{1}{2}$	$8\frac{1}{2}$	$18\frac{1}{2}$	$18\frac{3}{4}$	$12\frac{1}{4}$
6.	**7.**	**8.**	**9.**	**10.**
$\$144$	145	88	66	81
$14\frac{3}{4}$	$14\frac{3}{4}$	$7\frac{1}{11}$	$15\frac{3}{8}$	$6\frac{7}{12}$

Find the cost of

11. 300 barrels of flour at $\$6\frac{3}{4}$ per barrel.
12. 27 yards of silk at $\$1\frac{1}{8}$ per yard.
13. 26 tons of coal at $\$6\frac{1}{4}$ per ton.
14. 24 baseballs at $\$1\frac{1}{3}$ for one ball.
15. 23 barrels of vinegar at $\$8\frac{3}{8}$ per barrel.
16. 320 suits of clothes at $\$14\frac{5}{8}$ per suit.
17. 288 acres of land at $\$32\frac{3}{4}$ per acre.

Multiply each of the following in the short way:

18. 24 times \$$33\frac{1}{3}$. Times \$$66\frac{2}{3}$. \$$12\frac{1}{2}$. $87\frac{1}{2}$¢.

19. 120 times $6\frac{1}{4}$¢. Times 50¢. 25¢. $37\frac{1}{2}$¢.

20. 237 by $12\frac{1}{2}$. By $33\frac{1}{3}$. By 25. By 50. By $66\frac{2}{3}$.

21. Make up and work two similar examples.

22. Multiply 348 by $1.66\frac{2}{3}$.

$1.66\frac{2}{3} = 1\frac{2}{3}$. Hence, we have

$$\begin{array}{r} 348 \\ \underline{1\frac{2}{3}} \\ 3\overline{)696} \\ \overline{232} \\ \underline{348} \\ 580 \end{array} \text{ \textit{Product}}$$

23. Multiply 240 by $1.33\frac{1}{3}$. By $1.12\frac{1}{2}$. By $1.66\frac{2}{3}$. By 1.25.

24. Multiply 144 by $2.33\frac{1}{3}$. By $5.12\frac{1}{2}$. By $12.66\frac{2}{3}$. By $8.37\frac{1}{2}$.

25. Find the cost of 214 yd. of cloth at \$$1.12\frac{1}{2}$ per yard. At \$$1.33\frac{1}{3}$ per yard.

26. Find the cost of 36 chairs at \$$3.37\frac{1}{2}$. At \$5.75. At \$$4.66\frac{2}{3}$.

27. Multiply at sight:

(1)	*(2)*	*(3)*	*(4)*	*(5)*
20	24	30	48	60
$1\frac{1}{4}$	$1\frac{3}{4}$	$2\frac{1}{5}$	$1\frac{1}{2}$	$2\frac{1}{3}$

(6)	*(7)*	*(8)*	*(9)*	*(10)*
72	48	24	24	48
$1\frac{2}{3}$	$33\frac{1}{3}$	$12\frac{1}{2}$	$66\frac{2}{3}$	$1.12\frac{1}{2}$

The teacher should dictate similar examples for further drill.

116. Approximations. In multiplying numbers, we may often avoid mistakes by making a preliminary estimate of the size of the product.

Ex. 1. Make a rough estimate of the product of 4271 by 302.

4000 × 300 = 1,200,000. Hence, the required product must exceed 1,200,000.

Ex. 2. How many figures are there in the product of 3162 by 4012?

3000 × 4000 = 12,000,000. Hence, the product will contain 8 figures.

Such estimates are also of value in practical life by enabling a person to determine mentally and rapidly whether the product of two given numbers falls below a required total amount.

Ex. 3. Will the product of 199 by 500 be less than 100,000?

200 × 500 = 100,000. Hence, the product of the given numbers will be less than 100,000.

EXERCISE 131

Sight Work

Give the product of the figure of largest place value in the multiplicand by the figure of largest place value in the multiplier:

1. 307 × 23
2. 201 × 72
3. 1105 × 31
4. 202 × 100
5. 806 × 50
6. 1000 × 22
7. $41.65 × 21
8. $207.72 × 100
9. $30.28 × 2137

How many figures are there in each of the following products?

10. 207 × 101
11. 317 × 206
12. 1005 × 35
13. 2116 × 51
14. 3175 × 101
15. 216 × 4103

16. What is the highest order of units in each of the products in Exs. 10–15?

State which of the following is greater: the indicated product, or the number to the right of it:

17.	99 × 7	900
18.	996 × 50	60,000
19.	101 × 9	800
20.	989 × 99	10,000
21.	876 × 8¢	$100
22.	2000 × 9¢	$200
23.	2000 × 9¢	$20
24.	1000 × 21¢	$200

25. Will 500 acres producing 21 bushels of wheat per acre yield more or less than 100,000 bushels?

26. A farmer has 960 bushels of corn which he expects to sell at the rate of 96¢ a bushel. He wants to know whether the receipts will enable him to pay a note of $1000. Can you tell him without doing any written work?

27. Will a ton of feed at 5¢ a pound cost $10 or $100?

28. If the United States averages 3000 miles in length and 1000 miles in width, about how many square miles are there in its area?

29. Which is greater: $\frac{1}{2}$ yd., or 1 ft. 9 in.? $1\frac{1}{2}$ gal., or 10 pt.?

30. New Jersey has an area of 7815 square miles. Is this area greater or less than that of a rectangle 40 miles × 200 miles? One 60 miles × 130 miles?

31. Palestine is approximately a rectangle 150 miles long and 45 miles wide. Is its area greater or less than that of New Jersey?

32. Is the area of Texas greater than that of a square each side of which is 500 miles?

33. Which crop will pay better: 300 bushels of wheat at $1.02 a bushel, or 400 bushels of corn at 70¢ a bushel?

34. Practice the oral multiplication and division of small fractions by 2, 3, 4, 5, etc.

DIVISION

EXERCISE 132

1. Work again the examples in Exercise 23 (p. 49).

2. Define division, dividend, divisor, quotient, remainder. Give an example which shows the meaning of each of these terms.

3. What is the sign of division? Are there any other ways in which division can be indicated?

4. What is the use of division? Give an example showing that division is shorter than successive subtractions.

5. Make up and work an example which shows that division is a short way of finding how many times a given number (the divisor) may be subtracted from another given number (the dividend).

6. What five steps are repeated in long division? Give an example of long division and show how these steps are repeated.

7. How do you check the accuracy of a division? Apply this test to the example you worked in Ex. 6.

EXERCISE 133

Divide and check each result:

1. $843,336 by $7208
2. $2,442,069 by 4307
3. 1,050,003 by 4321
4. 688,950 by 9186
5. 725,508 by 8637
6. 769,986 by 7938
7. 4,711,775 by 9715
8. 17,042,197 by 5071
9. 234,262,788 by 3502
10. 214,307,272 by 55,007
11. 576,267,915 by 7503
12. 2,672,105 by 8761
13. 3,878,043 by 7649
14. 6,284,208 by 8876
15. 37,800 by 1400
16. 170,146 by 2700
17. 1,663,200 by 5400
18. 9,010,027 by 17,000
19. 133,200 by 3000
20. 21,941,700 by 6710

21. One barrel of flour weighs 196 pounds. How many barrels are there in 40,180 pounds?

22. If 1 schoolhouse accommodates 1725 children, how many schoolhouses of the same size are needed for 100,000 children?

23. Rhode Island contains 1250 square miles. How many times as large is New York State? Texas?

24. 1 cubic foot contains 1728 cubic inches. How many cubic feet are there in 125,000 cubic inches?

25. If 1 cubic foot of iron weighs 480 pounds, how many cubic feet of iron are there in a ton? Place your hands in different positions to show how large this number of cubic feet is.

26. 1 mile contains 5280 feet. How many miles are there in 60,000 feet?

27. If a cannon sends a shot 23,000 yards, how many miles is this? Can you name some place at that distance from your home?

28. If the distance from New York to Chicago by a certain railroad is 912 miles, how many round trips between the two places will a locomotive make in traveling 100,000 miles?

29. If a good-sized horse weighs 1100 pounds and an elephant weighs $3\frac{1}{2}$ tons, the elephant is how many times as heavy as the horse?

30. The quotient is 567 and the dividend is 2,442,069. Find the divisor.

31. In a recent year the United States life-saving service had 281 stations. At these stations in this year $9,286,260 worth of property and 6608 lives were saved. How much was this, on the average, for each station?

32. If there were 237,000,000 miles of railroad track in the United States in 1910, how many times would this stretch from the Atlantic to the Pacific Ocean (3000 miles)? Assuming that the United States is 1000 miles wide from north to south, how far apart would these tracks be if they were equidistant from one another?

33. Practice the rapid addition of columns of figures as on p. 227. Let the teacher or pupils write original examples on the blackboard. Tell which of these examples can be added by grouping.

117. Approximations. In dividing one number by another, we may often avoid mistakes by making a preliminary estimate of the quotient.

Ex. 1. Make a rough estimate of the quotient in dividing 98,654 by 327.

90,000 ÷ 300 = 300. Hence, the first figure of the answer will be hundreds, or the answer will contain three figures.

Ex. 2. Will 10,000 ÷ 201 be greater or less than 500?

10,000 ÷ 200 = 50. Hence, the required quotient will be less than 50 and, of course, less than 500.

EXERCISE 134

Sight Work

In each of the following, state the first figure of the quotient and its place value:

1. 9764 ÷ 301
2. 97,648 ÷ 304
3. 235,981 ÷ 208
4. 47,654 ÷ 2160
5. $445,987 ÷ 199
6. $3799.46 ÷ 203
7. $81,923.72 ÷ 2030
8. $15,432.15 ÷ 6124
9. Will 10,000 ÷ 501 be greater or less than 20?
10. Will $2000 ÷ 39 be greater or less than $50?
11. Sound travels 1090 feet in 1 second. About how many seconds will it take to travel a mile?
12. The Mississippi River (3160 miles) is about how many times as long as the Delaware River (300 miles)?
13. As the Susquehanna (617 miles)? As the Ohio (950 miles)?

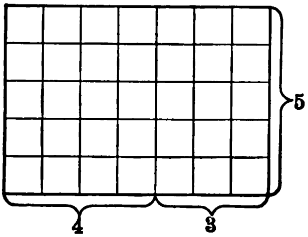
14. About how many acres will be needed to raise 4000 bushels of wheat, if 19 bushels are raised on 1 acre?

15. Practice weighing articles which could be sent by mail and give the amount of postage required by each article.

Combination of Operations

EXERCISE 135

1. Make a parenthesis mark. What is its meaning in arithmetic? What is its use?



2. On squared paper show that

$$4 \times 5 + 3 \times 5 = (4 + 3) \times 5,$$
$$= 7 \times 5.$$

Also that

$$7 \times 3 + 2 \times 3 = 9 \times 3.$$

Also that

$$6 \times 5 - 4 \times 5 = 2 \times 5.$$

3. Make up and work an example similar to Ex. 2.

4. On squared paper show that

$$3 \times 7 + 5 \times 7 + 4 \times 7 = 12 \times 7.$$

5. If a dealer buys coal at \$5.25 a ton, and sells it at \$6.75 a ton, what does he gain on 1742 tons?

6. Compute $376 \times 15 - 376 \times 13$ by doing the multiplication first. Now compute $376 \times (15 - 13)$. Compare the amount of work in the two processes.

7. In a city of 780,000 people, the birth rate for a certain year was 29 per thousand and the death rate 17 per thousand. Find, in the shortest way, the gain in population during the year.

8. What is cancellation? Give an example showing how the use of cancellation saves work.

9. If a ton of merchandise on the average occupies 40 cu. ft., how many tons will a box car hold which is 36 ft. long, 8 ft. wide, and 8 ft. high?

10. How many tons of hay, worth $12 a ton, must be given in exchange for 16 loads of coal at $9 a load?

11. 1 bu. of wheat weighs 60 lb. Find the cost of 12,650 lb. of wheat at 96 cents a bushel.

12. 1 bu. of oats weighs 32 lb. Find the cost of 7864 lb. of oats at 40 cents a bushel.

13. 1 bu. of corn weighs 56 lb. Find the cost of 32,746 lb. of corn at 72 cents a bushel.

14. Find the cost of 876 eggs at 26 cents a dozen.

Find the value of

15. $\$276 - (\$182 + \$16) + \$172$

16. $\$1000 - (\$200 - \$156) - \$272$

17. $12 + 64 \div 32$

18. $12 + 64 \div 32 \times 5$

19. $200 - (62 + 17\frac{1}{4})$

20. $\frac{5}{4}$ of $100 \times 2\frac{1}{5}$

21. $\frac{5}{4}$ of $(100 - 2\frac{1}{5})$

22. $\frac{5}{4}$ of $100 - 2\frac{1}{5}$

23. A boy can run 100 yards in 10 seconds. If he could run for an hour at the same rate, how many miles would he go in the hour?

24. Make up and work an example similar to Ex. 15. To Ex. 17. To Ex. 20.

EXERCISE 136

ORAL WORK

Give at sight the value of

1. $12 \times 16¢ - 2 \times 16¢$
2. $12\frac{1}{2} \times 50 + 37\frac{1}{2} \times 50$
3. $8 \times 6 + 12 \times 6 - 10 \times 6$
4. $100 - (40 + 20)$
5. $100 - (40 - 20)$
6. $\frac{18 \times 12}{6}$
7. $\frac{18 \times 12}{24}$
8. $\frac{12 \times 10}{5 \times 3}$
9. $\frac{10 - (5 + 1)}{4}$
10. $100 - (72 + 12)$
11. $100 - (72 - 12)$
12. $12 \times 8 \div 6$
13. $(12 - 8) \div 6$
14. $\frac{24 \times 9}{18}$
15. Work again Exercises 103 and 105 (pp. 181 and 186).

EXERCISE 137

1. Walter and Harry work a garden together during the summer and make $24 profits. Walter is older and stronger than Harry and receives twice as large a share of the profits. How much does each get?

$$\begin{aligned}
&\text{1 share} &&= \text{Harry's part of the \$24}\\
&\text{2 shares} &&= \text{Walter's part of the \$24}\\
&\text{1 share} + \text{2 shares} &&= \$24\\
&\text{or 3 shares} &&= \$24\\
&\text{1 share} &&= \tfrac{1}{3} \text{ of } \$24 = \$8, \text{ Harry's part}\\
&\text{2 shares} &&= 2 \times \$8 = \$16, \text{ Walter's part}
\end{aligned}\Bigg\} \textit{ Ans.}$$

2. May and Ada made $2.80 from the sale of popcorn balls. May, who supplied the popcorn and molasses, is to receive twice as much of the money as Ada. What does each receive?

3. A man and a boy together caught 320 fish. If the man is to have 3 times as many fish as the boy, how many does each get?

4. The sum of two numbers is 180, and one number is twice as large as the other. Find the numbers.

5. Find the numbers in Ex. 4 if one number is 3 times as large as the other. 4 times. 5 times.

6. A newsboy sold $\frac{5}{6}$ of his papers and had 20 left. How many did he have at first?

7. Helen has 4 times as many picture post cards as Florence. Together they have 120 cards. How many cards has each girl?

8. The cost of building a macadam road was $18,000. The county paid a certain part of the cost, and the state paid twice as much. How much money did each pay?

9. David, William, and Harry gathered chestnuts together. David and William received equal shares, but Harry, who climbed the trees, received twice as much as each of the others. If they gathered 3 quarts and 2 pints, how much did each receive?

10. How many of the examples in this Exercise can you work at sight?

11. Practice the multiplication of small decimal fractions by a whole number by the use of ring drills. Let the pupils take turns in making up the rings to be used.

EXERCISE 138

REVIEW

1. Divide each of the following by 7:

(*1*) 21,423 in. (*2*) $3100 (*3*) 5280 (*4*) 942,765

Divide these numbers also by 8. By 9. By 5. By 6.

2. Find the average population of all the states which border on your state.

3. In 77,777, the value of the left-hand 7 is how many times as great as that of the right-hand 7?

If you should make the right-hand 7 a figure $\frac{1}{4}$ inch high, and give the left-hand 7 a size in proportion to its value, how high would this left-hand 7 be?

4. By how much does the sum of $7250 and $1654 exceed their difference?

5. On Monday Mr. Harris had $672.54 in a bank. During the week he put in $172.50, took out $57.80, took out $67.43, put in $201.30, took out $315, and put in $17.82. How much did he have in the bank at the end of the week?

6. Find the total area of 7 lots, each 100 ft. square. Find the area of a lot 200 ft. by 300 ft. Find the difference between the two results.

7. A boy hires a motor boat at $1 per hour and uses it from 9 A. M. to 1.15 P. M. How much must he pay?

8. If 1 man using a machine for planting corn can do as much work as 3 men without a machine, and the wages of a man are $1.50 a day, how much does a farmer save in men's wages in 12 days by the use of a machine?

9. By keeping an account of his expenses and income for each crop, a farmer finds that he has made $30 per acre on land on which alfalfa was raised and fed to hogs, and only $2 per acre on land planted with wheat. What will he gain per year by raising alfalfa instead of wheat on 50 acres of land?

10. One year a farmer who had been neglecting his apple orchard, got on the average 1 peck from each of 300 trees. The next year he pruned the orchard at a cost of $12. He also sprayed it to protect it from insects, at an expense of $24.50. His crop

was then $2\frac{1}{2}$ bushels per tree. What was his gain, if apples were worth 40 cents a bushel?

11. If in the United States 13,000,000 bales of cotton averaging 500 pounds per bale are raised on 32,000,000 acres, how many pounds of cotton are raised per acre?

12. Find the altitude of the highest point in your state. How much higher or lower is this than Mt. Washington? (See p. 387.)

13. If the greatest depth of the Atlantic Ocean is 27,366 ft., how much less is this than the height of Mt. Everest?

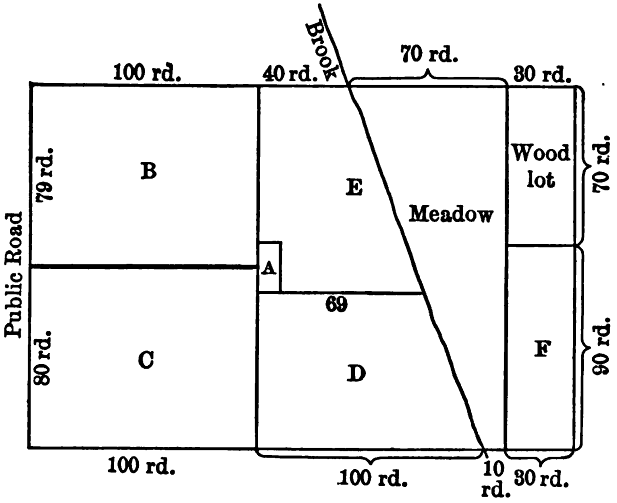
14. At sight, practice the division of small decimal fractions by 2, 4, 5 by the use of blackboard drills.

EXERCISE 139

INDUSTRY REVIEW: GENERAL FARMING

1. Above is a picture of the buildings on the farm of Mr. Ayres. On p. 248 is a map of this farm. The part A is occupied by the farm buildings. If this plot of ground is 20 rods long and 10 rods wide, how many acres does it contain?

2. If the private road which leads from A to the public road is 1 rod wide and 100 rods long, what part of an acre is it?



3. Find the number of acres in the fields marked B, C, and F, and in the wood lot.

Later we shall learn methods of finding the number of acres in E and in the meadow.

4. Find the length of the fence around the fields B and C together.

5. In one year Mr. Ayres sold 1240 bu. of corn at 78¢ a bushel, 972 bu. of potatoes at 46¢ a bushel, and 312 bu. of wheat at 96¢ a bushel. What did he receive for these crops?

6. He sold also 75 tons of hay at $14 a ton and $315 worth of cattle. What did he receive for these?

7. How much did he receive in all from his farm?

8. If his expenses for the year were $400 for fertilizer, $1050 for labor, and $200 for new machinery and repairs in old machinery, what were his farming expenses? What were his profits?

9. If Mrs. Ayres sold $80.40 worth of butter, $76 worth of milk, and $120.50 worth of poultry, how much did she receive?

10. Make a map of the farm like that given on p. 248 but with every line 4 times as long as in the book.

11. Mr. Ayres gave his 12-year-old son Howard a young colt. Howard took care of the colt for 2 years and spent $20.75 for its feed. At the end of that time he was offered $110 for the colt. How much would he have made if he had sold the colt?

12. Mr. Ayres allowed Howard and his sister Clara to plant a small piece of ground with vegetables and flowers. They paid $6.80 for fertilizer and $2.50 for seed. What was their outlay?

13. They sold 40 cabbages at 6¢, 3½ bu. of peas at 30¢ a peck, 12 bunches of beets at 8¢ a bunch, 40 heads of lettuce at 2¢, 40 doz. ears of corn at 10¢ a dozen, and flowers for $3.20. How much did they receive?

14. What were their profits? How much did each make, if they divided the profits equally?

15. Mr. Ayres used part of the profits from the farm to send his older son Robert to an agricultural college to learn to manage the farm more profitably. When Robert came home, he showed his father how to keep a set of accounts in order to find out which crop was most profitable.

The account showed that the wheat crop paid a profit of $4 per acre, while the potato crop paid $46.50 per acre. How much would Mr. Ayres make by planting a field of 20 acres with potatoes instead of wheat?

16. Robert showed his parents that by feeding the cows differently they could make $14 more per year from each cow. How much would be the increased profit on 15 cows?

Robert showed his father and mother many other useful facts which he had learned. Some of these we shall study later.

17. Draw a diagram of a plot of ground 120 ft. × 80 ft., and show how you would divide it and plant it with different crops.

18. Select a leading crop in your neighborhood. Find out the items of cost (seed, fertilizers, labor of planting, cultivating, and harvesting) in raising such a crop on a 10-acre field. Also the probable yield per acre, the price received, and hence the profit.

19. Practice making change. The telegraph or telephone game may be played in connection with this. (See Ex. 16, p. 86.)

CHAPTER IX

FACTORING; COMMON FRACTIONS

FACTORING

EXERCISE 140

1. What are the factors of a number? Give an example of a number and its factors.

2. Define a prime number, composite number, prime factor, even number, and odd number. Give two examples of each.

3. How does a knowledge of how to factor numbers sometimes save labor in dividing one number by another? Give an example.

4. How does the knowledge that a given number is prime prevent the waste of labor? For example, the knowledge that 17 is prime in $\frac{25 \times 42}{17}$.

5. Find the prime factors of each of the following numbers: 18, 48, 96, 63, 144.

6. Tell which of the following numbers are prime and which composite: 24, 23, 50, 49, 48, 47, 46.

118. A **square** is a number composed of two equal factors.

Thus, 9 is a square, since $9 = 3 \times 3$.

An **exponent** is a small figure placed above and to the right of a factor, to show how many times the factor is taken.

Thus, 3×3 may be written 3^2; also $7 \times 7 \times 7 \times 7$ may be written 7^4. In these examples 2 and 4 are exponents.

A **cube** is a number composed of three equal factors.

Thus, 8 is a cube, since $8 = 2 \times 2 \times 2 = 2^3$.

In what shorter way could you write $5 \times 5 \times 5 \times 5$? About how much shorter is your way of writing this number?

119. Advantages in a Knowledge of Square Numbers. (1) A knowledge of square numbers sometimes saves labor in factoring.

Ex. Find all the exact divisors of 48.

The first square number larger than 48 is 49, or 7^2. Hence, in trying divisors of 48, we need try none larger than 6, for we know that 7 is not a divisor of 48. Hence, $48 = 2 \times 24 = 3 \times 16 = 4 \times 12 = 6 \times 8$.

(2) Also a knowledge of square numbers helps us to visualize a given concrete number.

Ex. Visualize 82 square miles.

The square number nearest to 82 is 81, which is the square of 9. Think of a distance which is 9 miles, then of a square each side of which is 9 miles. 82 square miles will be slightly larger than this square.

EXERCISE 141

1. Make a list of the squares of all numbers from 1 to 25.

2. By use of your list of squares, tell what is the nearest square number to each of the following: 15, 37, 250, 192, 610, 120, 140.

3. Visualize each of the following by using the square number nearest to it:

37 sq. in.
65 sq. in.
142 sq. in
78 sq. ft.
350 sq. ft.
$15\frac{5}{7}$ sq. ft.
37.15 sq. ft.
$48\frac{3}{4}$ sq. mi.

4. Tell how you would visualize

26 sq. yd.
290 sq. yd.
$24\frac{1}{2}$ sq. yd.
16.53 sq. yd.
24 sq. mi.
17 sq. mi.
$16\frac{5}{8}$ sq. mi.
$15\frac{5}{9}$ sq. mi.

5. Make up and work an example similar to Ex. 4.

6. Reduce to square miles and visualize 5000 acres; 24,000 acres.

7. Compare the work of writing 10,000, with that of writing 10^4. 1,000,000 with 10^6.

8. In finding all the exact divisors of 65, what is the largest number that you need try? Answer the same for 63. For 82, 80, 120, 220.

9. How many of the examples in this Exercise can you work at sight?

10. Time yourself and see if you can add Exs. 11–15, on p. 23, in 2 minutes.

EXERCISE 142

1. Work again the examples in Exercise 36 (p. 70).

2. What is the short way of telling whether a given number, as 864, is divisible by 2? By 3? By 5? Give an example which illustrates each of these rules.

3. Find the prime factors of 100. Is 2×2 a divisor of 100? Is 100 always divisible by 4? Is 3×100, or 300, divisible by 4? Is 6700 divisible by 4?

4. Since $6748 = 6700 + 48$, is 6748 divisible by 4, if 48 is? Explain your answer.

5. Which is easier: to divide 6748 by 4, or to divide 48 by 4? Which process is more likely to be correct?

6. Find the prime factors of 1000. Is $2 \times 2 \times 2$ a divisor of 1000? Is 1000 always divisible by 8? Is 3×1000, or 3000, divisible by 8? Is 67,000 divisible by 8?

7. Since $67{,}424 = 67{,}000 + 424$, is 67,424 divisible by 8, if 424 is?

120. Tests of Divisibility by 4 and 8.

(1) *A number is divisible by* 4, *if the number expressed by its last two digits is divisible by* 4.

Thus, 742,368 is divisible by 4, since 68 is.

The reason for this is that 742,368 = 742,300 + 68; 100 is divisible by 4; hence, any number of hundreds is divisible by 4. Hence, 742,368 is divisible by 4, if 68 is.

(2) *A number is divisible by* 8, *if the number expressed by its last three digits is divisible by* 8.

Thus, 659,256 is divisible by 8, since 256 is. Show the reason for this.

EXERCISE 143

1. Tell which of the following numbers are divisible by 4:

120 130 140 2156 171 231 1728 31,416 97,605 123,456

Which of the above numbers are divisible by 8? By 3? By 5?

2. Using cancellation, find the value of

(*1*) $\frac{9 \times 18 \times 24}{12 \times 3 \times 36}$ (*3*) $\frac{12 \times 6 \times 25}{5 \times 36}$

(*2*) $\frac{75 \times 9 \times 96}{12 \times 15 \times 18}$ (*4*) $\frac{84 \times 8 \times 91 \times 20}{60 \times 32 \times 13}$

3. In determining whether 71 is a prime number, what is the highest divisor that it is necessary to try? What for 61, 51, 89?

4. Write a list of the prime numbers between 100 and 150.

5. Separate 36 into two factors in all possible ways. Which of the factors obtained are prime factors?

6. Treat in the same way 63, 108, 92.

7. On squared paper draw rectangles showing what is meant by each pair of factors for 36.

8. In each of the following find x, which expresses the omitted factor:

(*1*) $8 \times x = 32$	(*4*) $5 \times x \times 7 = 105$
(*2*) $2 \times 2 \times x = 12$	(*5*) $x \times 3^2 \times 4 = 108$
(*3*) $3^2 \times x = 36$	(*6*) $2 \times 3 \times x \times 7 = 252$

9. Which are more numerous: odd numbers, or even numbers? Prime numbers, or odd numbers? Are all prime numbers odd?

10. How many more symbols are there in $7 \times 7 \times 7 \times 7 \times 7 \times 7$ than in 7^6?

11. If 290 pupils stand so as to make a solid square, about how many pupils will there be on one side? Answer the same for 405 pupils. For 220. 85. 620.

12. How many of the examples in this Exercise can you work at sight?

13. Work again Exs. 34–45 on p. 30 and see if you can do them in less than 3 minutes.

G. C. D. AND L. C. M.

EXERCISE 144

1. Work again the examples in Exercise 40 (p. 77).

2. Define common factor, common divisor, and greatest common divisor. Give an example of each.

3. Work again Exs. 4, 7, 20, and 21, in Exercise 41 (p. 78).

4. What is the G. C. D. of 18 qt. and 24 qt.? What is the largest measure that can be used exactly in measuring 18 qt. and 24 qt.?

5. What is the largest measure for 4 gal. and 3 gal. 2 qt.? For 8 bu. and 6 bu. 4 qt.?

6. Work again Exs. 1–8 in Exercise 42 (p. 80).

7. Define multiple of a number, common multiple of two or more numbers, and least common multiple. Give an example of each.

8. In Exercise 43 (p. 82) work again Exs. 5–9, 19, 20.

9. Find the least common multiple of the denominators of the following fractions: $\frac{5}{6}$, $\frac{3}{8}$, $\frac{1}{10}$. Add these fractions.

Now add them using $6 \times 8 \times 10$, or 480, as the common denominator. How many more figures have you used in the second process than in the first?

10. Make up an example similar to Ex. 9, showing the advantage in the use of the L. C. M.

11. Work again Exs. 19, 20 on p. 82.

12. Time yourself and see if you can work Exs. 6–9 on p. 25 in 3 minutes.

COMMON FRACTIONS

EXERCISE 145

1. Work again the examples in Exercise 46 (p. 87).

2. Define fraction, denominator, numerator, and terms of a fraction. Give an example of each.

3. Define proper fraction, improper fraction, mixed number, and integer. Give an example of each.

4. Give an example of a mixed number, and reduce this mixed number to an improper fraction with full explanation of the change.

Show why at one time it is an advantage to use a mixed number and why at another time it is better to use an improper fraction.

5. Give the rule for reducing a mixed number to an improper fraction. Make up three examples and work these by the rule.

6. Give an example of an improper fraction and reduce it to a mixed number, giving an explanation. What is the rule for reducing an improper fraction to a mixed number? Make and work three examples of this kind.

7. Give an example of reducing a fraction to its lowest terms. Also set down a small fraction and change it to higher terms.

8. What is the advantage of reducing a fraction to its lowest terms? Give an example.

9. What is the advantage of changing a fraction to higher terms? Give an example showing the use of this process in adding fractions. Give another example showing the use of this process in comparing the values of two fractions.

10. Define similar fractions. Give an example of similar fractions. Do similar fractions have the same fractional unit? What is this fractional unit in your example?

EXERCISE 146

1. Work again the examples in Exercise 57 (p. 100).

2. What are the three steps in the addition of two fractions like $\frac{2}{3}$ and $\frac{1}{2}$?

Give an example in the addition of fractions where the first step is not used. Give an example where the last step is not used.

3. State the rule for the addition of fractions.

4. What are the uses of the addition of fractions? Give two examples showing these uses.

5. Make up and work three examples in the addition of mixed numbers.

6. Subtract $\frac{1}{6}$ from $\frac{2}{3}$. What three steps are there in this process?

Give an example of subtraction where the first step is not used. Give another example in which the last step is not used.

7. Make up and work an example in the subtraction of mixed numbers, in which the fraction in the minuend is less than the fraction in the subtrahend. Give the rule by which you proceed in such cases.

Simplify:

8. $\frac{2}{3} + \frac{5}{6} - \frac{1}{2}$

9. $\frac{8}{48} - \frac{5}{16} + \frac{7}{24}$

10. $\frac{3}{10} + \frac{1}{100} - \frac{1}{10}$

11. $4\frac{1}{2} + 7\frac{1}{2} - 3\frac{3}{4}$

12. $(\frac{8}{20} + \frac{5}{24}) - (\frac{1}{16} + \frac{1}{8})$

13. $(2\frac{1}{2} + 3\frac{3}{4}) - (4\frac{1}{8} - 3\frac{1}{16})$

14. On squared paper mark off a square each side of which is 10 spaces, and show what is meant by the addition of $\frac{8}{20}$ and $\frac{4}{25}$.

15. Work again the examples in Exercise 63 (p. 109).

16. Give the rule for multiplying a fraction by a whole number. Make up and work three examples of this process.

17. What is the advantage of multiplying a fraction by a whole number? Show this in one of your examples in Ex. 16.

18. Work again Exs. 15–18 in Exercise 64 (p. 110).

19. Work again Exs. 1–5 in Exercise 65 (p. 111).

20. Give the rule for multiplying a fraction by a fraction. Make up and work three examples of this process.

21. Work again Exs. 18–28 of Exercise 66 (p. 113).

22. $\frac{3}{5} \times \frac{4}{9} \times \frac{15}{16} = ?$

23. $\frac{14}{15} \times \frac{20}{21} \times \frac{9}{16} = ?$

24. Find $\frac{5}{7}$ of $\frac{4}{15}$ of $\frac{28}{29}$.

25. Find $\frac{5}{6}$ of $\frac{9}{10}$ of $\frac{5}{8}$.

26. Work Exercise 77 (p. 128).

27. Time yourself and see if you can work Exs. 7–10 on p. 53 in three minutes.

121. Multiplication of Mixed Numbers. In multiplying one mixed number by another, it is usually best to

Reduce each mixed number to an improper fraction and multiply as with fractions.

Ex. Multiply $2\frac{3}{4}$ by $5\frac{1}{3}$.

$$2\tfrac{3}{4} = \tfrac{11}{4}$$
$$5\tfrac{1}{3} = \tfrac{16}{3}$$
$$\frac{11}{\cancel{4}} \times \frac{\overset{4}{\cancel{16}}}{3} = \frac{44}{3} = 14\tfrac{2}{3}\ \textit{Product}$$

$2\frac{3}{4}$ reduced to an improper fraction is $\frac{11}{4}$; $5\frac{1}{3}$ is $\frac{16}{3}$. The product of $\frac{11}{4}$ and $\frac{16}{3}$ is $\frac{44}{3}$, or $14\frac{2}{3}$.

The process of multiplying a whole number by a mixed number has already been indicated in Art. 115, p. 235.

EXERCISE 147

Find the value of

1. $2\frac{1}{3} \times 1\frac{2}{7}$
2. $5\frac{1}{4} \times 1\frac{3}{7}$
3. $\frac{4}{11} \times 5\frac{1}{2}$
4. $\frac{3}{8}$ of $9\frac{1}{3}$
5. $7\frac{1}{2} \times 3\frac{1}{5}$
6. $2\frac{1}{4} \times 3\frac{1}{5}$
7. $12\frac{3}{4} \times 1\frac{1}{17}$
8. $7\frac{1}{3} \times 3\frac{1}{11}$
9. $5\frac{2}{3} \times 8\frac{1}{4}$
10. $\frac{3}{4}$ of $7\frac{1}{3}$
11. $\frac{9}{10}$ of $37\frac{1}{2}$
12. $\frac{11}{12}$ of $9\frac{3}{5}$
13. $\frac{1}{40}$ of $7\frac{3}{11}$
14. $21\frac{1}{4} \times 9\frac{1}{5}$
15. $7\frac{3}{16} \times 3\frac{1}{5}$

16. Find the area of the top of a table which is $7\frac{1}{2}$ ft. long and $3\frac{1}{4}$ ft. wide.

17. What is the cost of $14\frac{1}{2}$ lb. of iron wire at $5\frac{1}{4}$¢ a pound?

18. Find $\frac{4}{5}$ of the area of the table of Ex. 16.

19. How many square yards are there in a piece of carpet $\frac{3}{4}$ yd. wide and $12\frac{3}{8}$ yd. long?

20. Walter picked $15\frac{3}{4}$ bu. of cranberries in a week, and Roscoe can pick $\frac{2}{3}$ as many. How much would Roscoe make in a week, by picking cranberries at 40¢ a bushel?

21. How many pounds of butter can be made from 960 lb. of milk, if $\frac{1}{24}$ of milk is butter fat, and butter fat makes $1\frac{1}{16}$ times its own weight in butter?

22. Find the number of cubic feet in a bin whose dimensions are $12\frac{1}{2}' \times 3\frac{1}{2}' \times 2\frac{3}{4}'$.

23. A butcher bought $26\frac{1}{2}$ lb. of beef at $9\frac{3}{8}$¢ a pound and sold it at an average price of $14\frac{1}{2}$¢ a pound. What was his profit? Indicate the work by use of the parenthesis and do the work in the shortest way.

24. To look well, a door should be $\frac{2}{5}$ as wide as it is high. If a given door is 6 ft. 5 in. high, how wide should it be?

25. If gold is worth $\$18\frac{3}{4}$ an ounce, what are $2\frac{1}{2}$ oz. worth?

26. Practice the rapid addition of columns of numbers as on p. 227.

DIVISION OF FRACTIONS

EXERCISE 148

1. Work again the examples in Exercise 67 (p. 114).

2. Give the rule for dividing a fraction by a whole number. Make up and work three examples of this process.

3. Work Exs. 20–24 of Exercise 68 (p. 115).

122. Shortest Way of Dividing a Fraction by a Fraction.

Ex. Divide $\$\frac{6}{5}$ by $\$\frac{3}{10}$.

$\$\frac{1}{10}$ is contained in 1 dollar 10 times. Hence, $\$\frac{3}{10}$ (since it is 3 times as large as $\$\frac{1}{10}$) is contained in 1 dollar $\frac{1}{3}$ of 10 times, or $\frac{10}{3}$ times.

If $\$\frac{3}{10}$ is contained in 1 dollar $\frac{10}{3}$ times, it is contained in $\frac{6}{5}$ dollar $\frac{6}{5} \times \frac{10}{3}$ times, or 4 times. Or, in brief,

$$\$\frac{6}{5} \div \$\frac{3}{10} = \frac{\overset{2}{\cancel{6}}}{\cancel{5}} \times \frac{\overset{2}{\cancel{10}}}{\cancel{3}} = 4 \textit{ Quotient}$$

Hence, to divide one fraction by another, we have the following convenient rule:

Invert the divisor and multiply.

EXERCISE 149

1. 4 in. $\div \frac{1}{5}$ in. = ? Show this on a foot rule. 4 in. $\div \frac{2}{5}$ in. = ?

2. 9 in. $\div \frac{1}{10}$ in. = ? 9 in. $\div \frac{3}{10}$ in. = ? Show this on the foot rule.

3. $\$\frac{1}{10}$ is contained in \$1 how many times? Hence, $\$\frac{7}{10}$ is contained in \$1 how many times? $\$\frac{7}{10}$ is contained in $\$\frac{3}{4}$ how many times?

Divide $\$\frac{3}{4}$ by $\$\frac{7}{10}$ in the shortest way.

4. $\frac{1}{10}$ of an inch is contained in 1 inch how many times? $\frac{3}{10}$ of an inch is contained in 1 inch how many times? $\frac{3}{10}$ in. is contained in $\frac{6}{5}$ in. how many times?

Divide $\frac{6}{5}$ in. by $\frac{3}{10}$ in. in the shortest way.

5. Make up and work an example similar to Ex. 4.

6. Divide 64 by $\frac{1}{8}$.

Divide:

7. $\frac{7}{10}$ by $\frac{7}{12}$

8. $\frac{3}{8}$ by $\frac{9}{4}$

9. $\frac{4}{7}$ by $\frac{4}{5}$

10. $\frac{5}{6}$ by $\frac{1}{4}$

11. $\frac{9}{10}$ by $\frac{15}{16}$

12. $\frac{11}{12}$ by $\frac{55}{72}$

Divide:

13. $\frac{48}{49}$ by $\frac{16}{63}$

14. $\frac{4}{9}$ by $\frac{50}{27}$

15. $\frac{15}{56}$ by $\frac{60}{49}$

16. $\frac{64}{81}$ by $\frac{56}{27}$

17. $\frac{27}{28}$ by $\frac{108}{77}$

18. $\frac{169}{144}$ by $\frac{148}{96}$

19. $\frac{91}{140}$ by $\frac{169}{196}$

20. $\frac{576}{625}$ by $\frac{96}{250}$

21. 516 by $\frac{86}{7}$

22. Divide $\frac{9}{10}$ by $\frac{15}{16}$ by the method of Art. 68 (p. 117). Now divide by the method of Art. 122 (p. 261). How many figures are used in each of the processes? How much shorter is the second process than the first?

23. $\frac{3}{5}$ of a ton of coal is put into bags each containing $\frac{1}{20}$ of a ton. How many bags will there be?

24. How many strips of carpet $\frac{3}{4}$ yd. wide are needed in a room 21 ft. wide?

25. The area of a rectangle is 18 sq. yd. and its width is $2\frac{3}{4}$ yd.; find the length of the rectangle.

26. A man who owns $\frac{5}{6}$ of a sailing vessel can divide his share into how many shares each equal to $\frac{1}{12}$ of the vessel?

27. If 1 turn of a screw carries the screw forward $\frac{1}{10}$ in., how many turns are necessary in order to advance it $\frac{3}{5}$ in.?

28. If a boy grows $\frac{1}{8}$ in. per month, how long will it take him to grow $1\frac{1}{2}$ in.?

29. How many slices of bread, each $\frac{3}{8}$ in. thick, can be made from a loaf of bread $10\frac{1}{2}$ in. long?

30. If 100 ft. of wire is cut into pieces each $2\frac{1}{2}$ ft. long, how many pieces will be obtained?

31. If a family use $\frac{3}{8}$ of a barrel of flour in a week, how long will 6 barrels of flour last them?

32. Work again Exercises 113 and 115 (pp. 199 and 203).

EXERCISE 150

ORAL WORK

Divide at sight:

1. $\frac{1}{3}$ by $\frac{1}{2}$

2. $\frac{2}{3}$ by $\frac{1}{2}$

3. $\frac{1}{4}$ by $\frac{1}{2}$

4. $\frac{2}{5}$ by $\frac{2}{3}$

5. 3 by $\frac{1}{3}$

6. $\frac{4}{9}$ by $\frac{2}{3}$

7. $\frac{5}{6}$ by $\frac{5}{2}$

8. $\frac{3}{4}$ by $\frac{1}{2}$

9. $1\frac{1}{2}$ by $\frac{1}{2}$

10 $\frac{4}{9}$ by $\frac{1}{3}$

11. $\frac{3}{7}$ by $\frac{3}{5}$

12. $2\frac{1}{3}$ by $\frac{2}{3}$

13. How many of Exs. 1–30 in Exercise 149 (p. 261) can you work at sight?

14. As the teacher points, state the meaning of each abbreviation, and repeat the table or tables which contain the unit named.

pt.	qt.	mi.	da.
yd.	sq.in.	gal.	ft.
A.	rd.	mo.	pk.
T.	cu.yd.	lb.	oz.

123. Division of Mixed Numbers. In dividing one mixed number by another,

First reduce the mixed numbers to improper fractions, then divide as usual.

Ex. 1. Divide $40\frac{1}{2}$ by $2\frac{3}{4}$.

$$40\frac{1}{2} \div 2\frac{3}{4} = \frac{81}{2} \div \frac{11}{4}$$
$$= \frac{81}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{11} = \frac{162}{11}$$
$$= 14\frac{8}{11} \text{ Quotient}$$

$40\frac{1}{2} = \frac{81}{2}$ and $2\frac{3}{4} = \frac{11}{4}$. Hence, to divide $40\frac{1}{2}$ by $2\frac{3}{4}$ is the same as to divide $\frac{81}{2}$ by $\frac{11}{4}$. Hence, the quotient is $\frac{81}{2} \times \frac{4}{11}$, or $14\frac{8}{11}$.

If the divisor is an integer and the dividend is a large mixed number, it is better to proceed first as in short division of integers.

Ex. 2. Divide $142\frac{1}{2}$ by 6.

$$6\overline{)142\frac{1}{2}}\;4\frac{1}{2}$$
$$23\frac{4\frac{1}{2}}{6} = 23\frac{3}{4} \text{ Quotient}$$

Dividing 6 into $142\frac{1}{2}$, we obtain 23 for a quotient, with a remainder of $4\frac{1}{2}$. $4\frac{1}{2} \div 6 = \frac{9}{2} \div 6 = \frac{3}{4}$. Hence, the entire quotient is $23\frac{3}{4}$.

EXERCISE 151

Divide:

1. $2\frac{1}{4}$ by $3\frac{3}{8}$
2. $4\frac{1}{5}$ by $5\frac{1}{5}$
3. $4\frac{2}{3}$ by $3\frac{1}{2}$
4. $9\frac{3}{4}$ by $4\frac{1}{3}$
5. $21\frac{1}{3}$ by $3\frac{5}{9}$
6. $18\frac{1}{5}$ by $3\frac{9}{10}$
7. $95\frac{1}{3}$ by 4
8. $87\frac{3}{5}$ by 8
9. $103\frac{1}{2}$ by 6
10. $76\frac{2}{3}$ by $10\frac{10}{27}$
11. $4\frac{1}{8}$ by $19\frac{4}{5}$
12. $62\frac{1}{2}$ by 50
13. $7\frac{3}{11}$ by $8\frac{8}{9}$
14. $4\frac{5}{6}$ by $43\frac{1}{2}$
15. $37\frac{1}{2}$ by 5

16. Divide $7\frac{1}{2}$ inches by $1\frac{1}{4}$ inches. Show the meaning of this on your foot rule.

17. Subtract $1\frac{2}{3}$ from $13\frac{1}{3}$ as many times as possible. Now divide $13\frac{1}{3}$ by $1\frac{2}{3}$. Compare the amount of work in the two processes.

18. How many baskets, each containing $\frac{3}{8}$ bu. of apples, can be filled from a barrel which contains $4\frac{1}{2}$ bu.? From 8 barrels?

19. A rod contains $5\frac{1}{2}$ yd. How many rods are there in 1000 yd.?

20. If the rows of corn in a field are $3\frac{1}{2}$ ft. apart, how many rows will there be in a field 800 ft. wide?

21. In three successive years Henry grew $1\frac{1}{4}$ in., $1\frac{3}{4}$ in., and $2\frac{1}{2}$ in. Find his average growth per year.

22. If a horse eats $1\frac{1}{2}$ pk. of oats per day, how long will 10 bu. of oats last him?

23. If a ruffle is to be $3\frac{1}{2}$ in. wide, and $\frac{1}{2}$ in. is allowed on one side of the ruffle for a hem and $\frac{3}{8}$ in. on the other side for a seam, how many strips of ruffling can be cut from 35 in. of cloth?

24. If a boy grows $1\frac{1}{4}$ in. per year, how long will it take him to grow $7\frac{1}{2}$ in.?

25. How many quarter-pound candy boxes can be filled from a box of candy containing $10\frac{1}{2}$ lb.?

26. If the weight of an elephant is $3\frac{1}{2}$ tons and the weight of a whale 60. tons, the whale is how many times as heavy as the elephant?

27. If $7\frac{1}{2}$ inches of snow when melted makes water 1 inch deep, a foot of snow is equal to how many inches of rainfall?

28. The product of two numbers is $24\frac{7}{8}$. One number is $4\frac{1}{2}$. Find the other number.

29. If the record for the 100-yard dash is $9\frac{3}{5}$ seconds, what is the number of feet covered in 1 second?

30. A recent swimming record for 100 yards was $55\frac{2}{5}$ seconds. According to this, a man can run how many times as fast as he can swim? (See Ex. 29.)

31. Practice the oral multiplication of numbers by $12\frac{1}{2}$, $33\frac{1}{3}$, and other simple aliquot parts of 100 by use of blackboard drills. (See Exercise 73, p. 121.)

COMPLEX FRACTIONS

124. A Complex Fraction is one which has a fraction in its numerator, or in its denominator, or in both. To simplify a complex fraction,

Simplify the numerator and the denominator;

Divide the simplified numerator by the simplified denominator.

Ex. 1. Reduce $\dfrac{3\frac{2}{3}}{4\frac{5}{6}}$ to a simple fraction.

$$\frac{3\frac{2}{3}}{4\frac{5}{6}} = \frac{11}{3} \div \frac{29}{6} = \frac{11}{\cancel{3}} \times \frac{\overset{2}{\cancel{6}}}{29} = \frac{22}{29} \textit{ Ans.}$$

Ex. 2. Simplify $\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{3}}$

$$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{3}}=\frac{\frac{3}{6}+\frac{4}{6}}{\frac{9}{12}-\frac{4}{12}}$$

$$=\frac{\frac{7}{6}}{\frac{5}{12}}=\frac{7}{\cancel{6}}\times\frac{\overset{2}{\cancel{12}}}{5}=\frac{14}{5}=2\frac{4}{5}\ \textit{Ans.}$$

We first find the sum of the fractions in the numerator and the difference of those in the denominator. We then divide the simplified numerator, $\frac{7}{6}$, by the simplified denominator, $\frac{5}{12}$, obtaining $\frac{14}{5}$, or $2\frac{4}{5}$, as a result.

EXERCISE 152

Simplify:

1. $\dfrac{3\frac{1}{2}}{4\frac{2}{3}}$

2. $\dfrac{5\frac{1}{5}}{3\frac{1}{4}}$

3. $\dfrac{\frac{2}{3}\text{ of }\frac{6}{7}}{\frac{1}{2}\text{ of }3\frac{1}{3}}$

4. $\dfrac{\frac{2}{3}+\frac{1}{6}}{\frac{3}{4}+\frac{5}{6}}$

5. $\dfrac{1\frac{1}{6}-\frac{3}{4}}{1\frac{5}{12}+\frac{5}{6}}$

6. $\dfrac{\frac{3}{5}\text{ of }6\frac{2}{3}}{\frac{4}{7}\text{ of }2\frac{3}{4}}$

7. $\dfrac{\frac{1}{3}+\frac{3}{4}+\frac{1}{6}}{\frac{3}{5}+1\frac{2}{3}+\frac{9}{10}}$

8. $\dfrac{2\frac{1}{2}+1\frac{2}{3}}{4\frac{7}{8}-2\frac{11}{12}}$

9. $\dfrac{\frac{3}{4}\text{ of }\frac{4}{7}\text{ of }5\frac{5}{6}}{\frac{3}{5}\text{ of }\frac{10}{27}\text{ of }7\frac{1}{5}}$

10. Divide the sum of $4\frac{2}{3}$ and $1\frac{5}{6}$ by their difference.

11. If a merchant buys 2550 lb. of sugar at $4\frac{1}{6}$¢ a pound and sells it at $4\frac{2}{3}$¢ a pound, how much does he make?

12. If he buys 300 doz. eggs at $22\frac{1}{2}$¢ a dozen and sells them at 25¢ a dozen, how much does he make?

13. If a man buys 2724 lb. of cotton at $12\frac{1}{2}$¢ and sells it at $13\frac{1}{4}$¢, how much does he make?

14. Work again Exercise 125 (p. 229) or similar examples dictated by the teacher.

15. Practice the game of drawing to scale (see p. 394).

EXERCISE 153

ANALYSIS WITH FRACTIONS

1. Work again every example divisible by 3 in Exercises 74 and 75 (pp. 123, 125).

2. If $\frac{4}{5}$ of an acre of land costs $120, what will 1 acre cost?

3. In skimming milk by hand, only $\frac{4}{5}$ of the cream is obtained. By skimming milk in this way, during a certain week, a farmer obtained 135 lb. of cream. How many pounds would he have obtained if he had used a separator, which extracts practically all the cream?

4. If Archie grows $1\frac{1}{2}$ inches in $\frac{3}{4}$ of a year, how much will he grow in 1 year? In $1\frac{1}{2}$ years?

5. If Anna pays 60¢ for $\frac{3}{4}$ of a yard of cloth, how much must she pay for 1 yard? For $1\frac{1}{4}$ yards?

6. If John walks $2\frac{1}{4}$ miles in $\frac{3}{4}$ of an hour, how far will he walk in 1 hour? In $3\frac{1}{2}$ hours?

7. $48\frac{2}{7}$ is $\frac{4}{5}$ of what number?

8. Two boys together own $\frac{5}{12}$ of a sailboat and one of these boys owns 4 times as large a share as the other. How much does each own?

9. A man and a boy together spaded $\frac{9}{10}$ of a garden. If the man spaded twice as much as the boy, what part did each spade?

10. In a certain state the cost of building a macadam road is shared by the town, county, and state. The town pays $\frac{1}{6}$, the county $\frac{1}{6}$, and the state the remainder. If the town pays $1200 for a certain road, what does the county pay? The state?

11. Practice making change. Let the pupil put on the blackboard a table of purchases and of sums paid as on p. 33.

EXERCISE 154

REVIEW OF FRACTIONS

Simplify:

1. $\frac{2}{3} + \frac{1}{2} - \frac{5}{6}$

2. $5\frac{1}{2} - 2\frac{3}{4} - (4\frac{1}{12} - 2\frac{1}{8})$

3. Divide 250 by $37\frac{1}{2}$. By $33\frac{1}{3}$. $12\frac{1}{2}$. $62\frac{1}{2}$.

4. What number added to $3\frac{5}{6}$ gives $5\frac{2}{3}$? What number subtracted from $8\frac{7}{12}$ gives $5\frac{2}{5}$?

5. What fraction multiplied by $\frac{4}{5}$ gives $\frac{6}{7}$? What fraction divided by $\frac{2}{7}$ gives $\frac{1}{4}$?

6. What part of 10 is $2\frac{1}{2}$? What part of 100 is $3\frac{1}{3}$? $6\frac{1}{4}$?

7. $12\frac{1}{2}$ is what part of $37\frac{1}{2}$? $37\frac{1}{2}$ of $87\frac{1}{2}$? $33\frac{1}{3}$ of $66\frac{2}{3}$?

8. If a boy at the age of 1 year is 23 in. tall and at 17 years is 5 ft. 10 in. tall, what has been his average growth per year?

9. If Texas is $212\frac{1}{2}$ times as large as Rhode Island, and New York is $39\frac{1}{3}$ times as large as Rhode Island, then Texas is how many times as large as New York?

10. If the fare to Atlantic City is $10 for a father and mother together, what will be the fare for 3 children, for each of whom half fare is paid?

11. On a field of 9 acres a farmer had successive cuttings of alfalfa as follows: $12\frac{3}{4}$ tons, $8\frac{1}{2}$ tons, $7\frac{3}{4}$ tons, and $5\frac{1}{8}$ tons. What was the yield per acre for the season?

12. If 3,500,000 pounds of sugar are bought at $4\frac{1}{4}$¢ and sold at $4\frac{3}{8}$¢, find the profit in the shortest way. What would be the long way of computing the profit?

13. If skimming cream from milk by hand collects $\frac{4}{5}$ of the cream and a farmer collected 180 lb. of cream in this way in one week, how many pounds of cream did he lose? What was this worth at 18 cents a pound? What would the loss amount to in 20 weeks, at the same rate?

14. One fourth of the area of New York exceeds the area of New Jersey by how much?

15. A torpedo boat has gone $37\frac{1}{10}$ knots in one hour. How many miles is this, if 1 knot = $1\frac{1}{7}$ mi.?

16. One motor boat went 20 miles in 45 minutes and another went 25 miles in 55 minutes. Which boat traveled faster?

17. It is estimated that, if the house fly were kept from food, deaths in cities in summer would be $\frac{1}{3}$ less. If the deaths in the summer months are at the rate of 12 per 1000 people, how many lives might be saved in this way in a city of 1,000,000 inhabitants in one summer?

18. Practice approximate multiplication and division.

Let the teacher put on the blackboard examples similar to those on p. 237 and p. 241.

EXERCISE 155

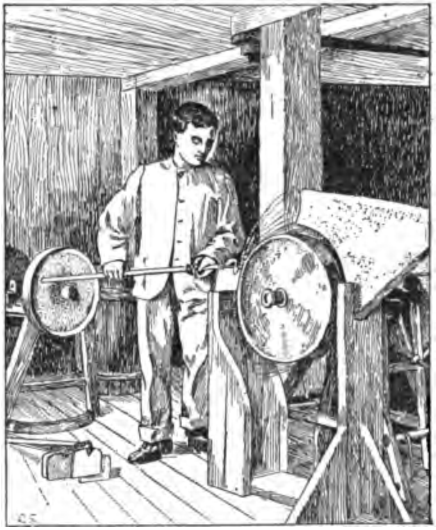
INDUSTRY REVIEW: MANUFACTURING HOES

1. The steel from which hoes are made is bought in long bars, at $40 per ton. What will 3200 lb. of such steel cost?

2. How many dozen hoes will this steel make, if $17\frac{1}{2}$ oz. will make 1 hoe?

3. What will be the cost of the steel used in making 1 dozen hoes?

4. In order to make the metal part of a hoe, a bar of steel is rolled out in a flat plate and part of the plate is cut out with a die. (Can you find out what a die is?) If it costs $300 to make 6000 hoe blades out of steel bars, what is the cost of making 1 dozen hoe blades?



5. If $620 covers the cost of handles for 6000 hoes, together with the labor of putting on the handles, labeling them, polishing, and finishing the hoes, what is the cost of the handles and this labor for 1 dozen hoes?

6. If the other items in the cost of manufacturing 6000 hoes are $70 for fuel, $80 for office work, and $130 for insurance, repairs, and miscellaneous expenses, what is the total cost of making 1 dozen hoes?

7. If a carload of 1200 dozen hoes is sold to a wholesale dealer (called a jobber) for $3720, what is the profit to the manufacturer on 1 dozen hoes?

8. If the carload of hoes weighs 52,400 lb. and the freight rate for the distance shipped is $.72 per 100 lb., how much freight does the jobber pay?

9. What is the total cost of the carload of hoes to the jobber after the freight is paid?

10. If the jobber sells the hoes at $5.20 per dozen, how much does he make per dozen?

11. Howard Ayres (see p. 249) bought a hoe for 50 cents and used it 4 years in cultivating $300 worth of vegetables. What was the cost of the hoe per dollar of vegetables raised?

12. If Howard should hire the village blacksmith to make the metal part of a hoe, it would take the blacksmith 3 hours at 50¢ an hour to do this. It would also take a carpenter 3 hours at 40¢ an hour to supply the handle. What would one hoe cost if made that way? One dozen hoes?

13. Find out an important manufacture in your neighborhood, as the manufacture of some kind of clothing, hardware, or building material. Find out the unit used in manufacturing this article, as 1 yard for muslin, or 1 barrel for cement. Find the average cost of making one of these units and find the price received for one. You can then learn the profit on one unit.

Make up and work other examples on this subject.

The teacher may help the pupil to collect his facts and to arrange them in order. The pupil should make his own examples from these facts.

CHAPTER X

DECIMAL FRACTIONS TO MILLIONTHS; BILLS AND ACCOUNTS

NUMERATION AND NOTATION THROUGH MILLIONTHS

EXERCISE 156

1. Work again the examples in Exercise 79 (p. 133).

2. Define a decimal fraction. Give an example of a decimal fraction of one place.

3. What is a decimal point? Give an example and from the example show the uses of a decimal point.

4. On squared paper draw a diagram showing the meaning of .3. Of .7. Of .47. Of .256.

5. What is the advantage of a decimal fraction over a common fraction in expressing denominators? Give an example which shows this.

6. What part of a cent is a mill? What part of a dime is a mill? What part of a dollar is a mill?

7. In 56.775, which 7 has the greater value? The value of this 7 is how many times as great as that of the other 7?

8. In 56.775, state the place value and shape value of each figure.

9. What is the rule for reading a decimal fraction? Give an example.

10. What is the rule for writing a decimal fraction? Give an example.

125. Decimal Fractions to the Fourth and Fifth Places are sometimes needed to express small parts of a unit.

Thus, if 20,000 lines to the inch are marked on glass, the distance between two adjacent lines will be .00005 inch. This is read "five hundred-thousandths of an inch."

EXERCISE 157

1. Make a numeration and notation table like that given on p. 137 but including two more places in the decimal part of the table (ten-thousandths and hundred-thousandths).

Read:

2. 3.205, .7623, 17.8215, 9.0726, 18.0003, .0008, .0105.

3. 1.92165, .70623, 19.00017, 676.01005, .00005.

Read the following, expressing the figures in words:

4. The circumference of a circle is 3.1416 times as long as the diameter.

5. More exactly, the circumference is 3.141592 times the diameter.

Write the following as decimal fractions:

6. Eighty-nine thousandths.

7. One hundred twenty-eight ten-thousandths.

8. Six thousand five hundred seventy-six ten-thousandths.

9. Nine ten-thousandths.

10. Six hundred-thousandths.

11. Seventeen thousand five hundred eighty-four hundred-thousandths.

12. Explain the difference between "six hundred and twelve ten-thousandths" and "six hundred twelve ten-thousandths." Write each of these in figures.

13. Rewrite, using figures: The exact length of the year is three hundred sixty-five and twenty-four thousand two hundred twenty-three hundred-thousandths days.

14. Which would you rather have: 3 tenths, or 3 ten-thousandths of a gold mine? Why?

15. Tell at sight how many figures are necessary to write 17 thousandths. 17 ten-thousandths. 596 hundred-thousandths.

16. In 86.5923, if the decimal point is moved one place to the right, what change is made in the value of each digit? What change, if the decimal place is moved 2 places to the right? One place to the left?

17. What change is made in the value of each figure in 86.5923, if one zero is inserted between the decimal point and the 5? If two zeros are inserted there?

18. Make up and answer an example of your own similar to Ex. 16. Also one similar to Ex. 17.

19. In 4.157, change the decimal point so that 7 will represent tenths.

20. How many examples in Exercise 70 (p. 117) can you now work at sight?

PROCESSES WITH DECIMALS

I. *Addition and Subtraction of Decimals*

EXERCISE 158

1. How do you arrange decimal fractions in order to add or subtract them to best advantage? Having arranged them, where do you begin to add or subtract?

Make up and work an example in addition of decimal fractions. Also one in subtraction of decimal fractions.

2. Add the following, grouping the digits by 10's, 11's, or 20's, as far as possible (see Art. 111, p. 226):

3.	**4.**	**5.**	**6.**
$35.52	$744.44	$41.72	3.7261
477.41	637.52	27.8652	17.00728
294.13	18.25	56.26	3.62
83.33	282.34	37.3538	.07362
63.71	11.78	72.457	15.29514
57.04	423.15	15.252	8.0358
22.24	740.50	23.50	9.74976
15.81	857.48	34.506	23.61259

Find the value of

7. .0897 − .032 **9.** 3.1416 − 2.9761 **11.** 2½ − 1.625

8. 5.128 − 1.0076 **10.** 39.37 − .0893 **12.** 2150.42 − 1.728

Supply the missing addend in each of the following:

13.	**14.**	**15.**	**16.**	**17.**
$54.72	$17.62	6.218	146.27	193.82
613.87	73.73	131.51	37.16	
91.40	46.45	72.623		416.72
52.37	208.87	27.72	54.32	518.54
			716.84	2167.82
$1082.37	$1000.00	272.865	1250.16	3000.00

18. Add the following both by rows and by columns, obtaining the complete sum in each way:

(1)	*(2)*	*(3)*	*(4)*	*(5)*
62.17	5.128	3.1416	.30103	.47715
51.32	3.141	2.150	.47712	.90309
39.37	2.471	1.6214	.21504	.29002
81.08	1.057	2.8376	.69897	.49715
29.76	4.866	2.2046	.77815	.1728
30.20	1.414	3.1622	.8451	.6288

19. Mr. Walters had a tract of land containing 12 acres. He has sold at different times 1.72 A., 2.375 A., .0825 A., and 3.175 A. How can he tell how much he has left, without going to the expense of having it surveyed? Find out the remainder for him.

20. The English shilling is worth $.2434, the French franc $.193, and the German mark $.2385. Find the difference between the value of each pair of the coins. Also find the difference between each of them and the United States quarter-dollar.

21. If a gallon of water weighs 8.355 lb. and a gallon of milk 8.622 lb., find the difference in their weight. What is the difference in weight between 10 gallons of each? 100 gal.? .1 gal.?

22. Compute in the shortest way $926 \times 1.0735 + 926 \times 2.0565$.

23. Compute in the shortest way $528 \times 3.7285 - 528 \times 1.7265$.

24. On squared paper, using a square 10×10, show the meaning of .745 and of .225. Also of the addition and subtraction of these numbers.

25. Add at sight:

(1)	*(2)*	*(3)*	*(4)*	*(5)*
$1.27	$.97	$2.45	$1.16	$1.23
.32	.34	1.13	.42	.57

26. Also subtract at sight the numbers in Ex. 25.

27. Work again Exercise 150 (p. 263) or similar examples dictated by the teacher.

II. *Multiplication of Decimals*

EXERCISE 159

1. In multiplying one decimal fraction by another, how do you determine where to place the decimal point in the product? Why? Make up an example and work it in such a way as to show why you place the decimal in the product as you do.

2. State the rule for multiplying one decimal by another.

Read and find the product in each of the following, checking your work for each example:

3. 2.712 in. × 123
4. .0082 ft. × 120.5
5. 7.75 × .0192 A.
6. 1.52 × 3.162 lb.
7. .0055 sec. × 1200
8. 2.15042 × .0018

9. On squared paper show the meaning of $\frac{2}{10}$ of .2. Hence, of $(.2)^2$. Also of $(.3)^2$. $(.5)^2$.

10. What is the easiest way to multiply a decimal fraction by 10? By 100? By 1000? Give an example of each of these processes.

11. What is the easiest way to multiply a decimal fraction by .1? By .01? By .001? Give an example of each of these processes.

12. Multiply 1875.439 by each number mentioned in Ex. 10. Then multiply by each number in Ex. 11.

13. Multiply 27.5 by .1. By .0001 100.1 .001 1000

14. Multiply 150.027 by 10. By .001 1000 .1 .01 100

15. One franc is worth $.193. What are 122.5 francs worth?

16. If 1 gal. of milk weighs 8.622 lb., how much will 6.35 gal. weigh?

17. What is the cost of 2745 cu. ft. of gas at $.80 per thousand cubic feet?

18. 1 cu. ft. of water weighs 62.5 lb. If ice weighs .92 as much as water, what does 1 cu. ft. of ice weigh?

19. The freight between two places on a certain railroad is $.26 per hundred pounds. What will be the rate on 800 lb.?

20. If shingles cost $16.75 a thousand, what will be the cost of 32,600 shingles?

21. If the tax on $1 is $.012, what will be the tax on $650?

Find the cost of

22. 12,500 bricks at $7.25 per thousand.

23. 278 stamped envelopes at $21.40 per thousand.

24. 1870 oranges at $1.24 per hundred.

25. Practice the multiplication of small decimal fractions at sight by means of blackboard drills.

III. *Division of Decimals*

EXERCISE 160

1. In dividing a number by a decimal fraction, what change do you make in the decimal points in divisor and dividend? Make up an example and work it in such a way as to show the method and the reasons for moving the decimal points. (See example in Art. 83, p. 151.)

2. Give the rule for dividing one decimal fraction by another.

Find the value of each of the following:

3. 7.38 in. ÷ .12

4. 30 in. ÷ .015 in.

5. 1.2225 ÷ .035

6. 9.075 ft. ÷ .0121 ft.

7. .14688 ÷ .108

8. 3.1416 ÷ .08

9. What is the easiest way to divide a decimal fraction by 10? By 100? By 1000? By 10,000? Give two examples of each of these processes.

10. What is the easiest way to divide a decimal by .1? By .01? By .001? By .0001? Give one example of each of these processes.

11. Divide 7926.592 by each number mentioned in Ex. 9. In Ex. 10.

12. Divide 39.37 by 10. By .001 1000 .1 .01 100

13. Divide .009 by 9. By .09 .9 90

Divide each of the following to three decimal places:

14. 1.7 by 11

15. .013 by 17

16. .048 by 1.9

17. 380 by 2.3

18. .001 by .07

19. .0068 by 30

20. 7 by 300

21. 9 by .0011

22. 70.1 by .0202

23. Divide 30.24 by 3.6. After having done so, write at sight the quotient of 30.24 by .36. Also by .036. By 36. By 360.

24. How many books, at $1.35 each, can be bought for $286.20?

25. If 1 turn of a screw advances the screw .172 in., how many turns must be made to advance the screw .852 in.? 2.5 in.? 5 in.?

26. If one bushel contains 2150.42 cu. in., how many bushels can be put into a bin 10′ × 10′ × 10′?

27. One mark of German money is worth $.238. How many marks are necessary to make $100?

28. If it costs $162,700 to build 9.28 miles of railroad, what does it cost to build 1 mile?

29. The area of a lot is 271.36 sq. ft. and its width is 32.6 ft. Find the length of the lot.

30. 1 quart of milk weighs 2.15 pounds and a cow gave 5228.8 pounds in 1 year. How many quarts was this?

31. If the diameter of the earth is 7918 miles and that of the moon is 2162 miles, the diameter of the earth is how many times as great as the diameter of the moon? (Find to 3 decimal places.)

32. Practice the division of decimals at sight by the use of blackboard drills. (See Ex. 37, p. 151.)

COMMON AND DECIMAL FRACTIONS

EXERCISE 161

1. Write a decimal fraction and change it to an equivalent common fraction.

2. State in your own words the method of changing a decimal fraction to a common fraction.

3. Change $.56\frac{1}{4}$ to a common fraction.

$$\text{Write}\quad .56\tfrac{1}{4} = \frac{56\frac{1}{4}}{100} = \frac{\;\frac{225}{4}\;}{100} = \frac{225}{400}\text{ etc.}$$

$$\text{Or write } .56\tfrac{1}{4} = .5625 = \frac{5625}{10{,}000}\text{ etc.}$$

Change each of the following to a common fraction:

4. .125 **6.** .375 **8.** .0006 **10.** 6.4125

5. .275 **7.** .006 **9.** .0015 **11.** .0375

12. Write a common fraction and change it to the form of a decimal fraction.

13. State in your own words the method of changing a common fraction to a decimal fraction. Give two examples of this process.

Change to an equivalent decimal fraction:

14. $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ $\frac{7}{16}$ $\frac{8}{25}$ $\frac{17}{20}$ **15.** $\frac{19}{32}$ $1\frac{5}{16}$ $4\frac{18}{25}$ $3\frac{27}{80}$

Find correctly to four decimal places the value of

16. $\frac{1}{6}$ $\frac{2}{7}$ $\frac{5}{9}$ $\frac{7}{11}$ **17.** $\frac{7}{18}$ $\frac{15}{17}$ $3\frac{1}{9}$ $12\frac{5}{6}$

18. On squared paper show that $.55 = \frac{11}{20}$. Also that $.12 = \frac{3}{25}$.

19. Make up and work two similar examples.

20. What decimal part of a pound is 7 ounces?

21. Reduce to simplest form $2.5 + 1\frac{3}{5}$.

22. Also $3.8 + 2\frac{1}{2} - 4.07$.

23. Give an example of a common fraction which is easier to write and to use than the equivalent decimal fraction. Give two other examples.

24. Give an example of a decimal fraction which is easier to write and to use than the equivalent common fraction.

25. Work again Exercise 97 (p. 167).

126. Aliquot Parts of One Dollar.

Using the method of Ex. 3, p. 280, show that

$.06\frac{1}{4} = \frac{1}{16}$
$.08\frac{1}{3} = \frac{1}{12}$
$.12\frac{1}{2} = \frac{1}{8}$
$.16\frac{2}{3} = \frac{1}{6}$
$.25 = \frac{1}{4}$
$.33\frac{1}{3} = \frac{1}{3}$
$.37\frac{1}{2} = \frac{3}{8}$
$.50 = \frac{1}{2}$
$.62\frac{1}{2} = \frac{5}{8}$
$.66\frac{2}{3} = \frac{2}{3}$
$.75 = \frac{3}{4}$
$.87\frac{1}{2} = \frac{7}{8}$

Ex. 1. Find the cost of 36 yd. of muslin at 12½¢ a yard.

$$36 \times \$.12\tfrac{1}{2} = 36 \times \$\tfrac{1}{8} = \$4.50 \text{ } \textit{Ans.}$$

Ex. 2. At 12½¢ a yard, how many yards of muslin can be bought for $5?

$$\$5 \div \$.12\tfrac{1}{2} = \$5 \div \$\tfrac{1}{8} = 5 \times 8 = 40 \text{ } \textit{Ans.}$$

EXERCISE 162

Multiply:

1. 48 by $.12\frac{1}{2}$. By $37\frac{1}{2}$. $.25$ $.50$ $.62\frac{1}{2}$ $.87\frac{1}{2}$

2. 2400 by $.33\frac{1}{3}$. By $.37\frac{1}{2}$. $.25$ $.62\frac{1}{2}$ $.87\frac{1}{2}$ $.06\frac{1}{4}$

3. Multiply 96 by .66⅔. By .62½. .33⅓ .12½ .08⅓

4. Make up and work an example similar to Ex. 3.

Find the cost of each of the following:

5. 27 lb. @ $.25
6. 48 lb. @ $.37½
7. 70 qt. @ $.33⅓
8. 94 ft. @ $.66⅔
9. 68 bu. @ $.87½
10. 95 pt. @ $.12½
11. 24 doz. @ $.06¼
12. 83 gal. @ $.75
13. 46 pt. @ $.33⅓
14. 82 yd. @ $.62½
15. 30 pk. @ $.37½
16. 39 bu. @ $.75

Divide each of the following:

17. 3600 by .33⅓. By .37½. .25 .62½ .87½ .06¼
18. 960 by .12½. By .08⅓. 37½ 66⅔ .25
19. 72 by .62½. By .66⅔. .50 .75 .25
20. $164.25 by $.37½.
21. $141.25 by $.25.
22. $234.375 by $.62½.
23. $2.875 by .06¼.

Tell how many of the following can be bought:

24. Yards of muslin at 12½¢, for $2.50. For $7.50.
25. Pounds of butter at 33⅓¢, for $5.50. For $12.
26. Collars at 16⅔¢, for $3. For $4.50.
27. Gallons of oil at 12½¢, for $2.
28. Baseballs at 66⅔¢, for $4.
29. Quarts of canned corn at 12½¢, for $1.75.
30. Bushels of wheat at 87½¢, for $140.
31. Dozens of oranges at 33⅓¢, for $2.
32. Quarts of milk at 6¼¢, for $1.50.

33. How many penknives at $37\frac{1}{2}$¢ can be bought for $6?

34. Find the cost of 30 yards of calico at $12\frac{1}{2}$¢.

35. At $12\frac{1}{2}$¢ a yard, how many yards of calico can be bought for $30?

36. Find the cost of 32 lb. of rice at $6\frac{1}{4}$¢ a pound.

37. At $6\frac{1}{4}$¢ a pound, how many pounds of rice can be bought for $32?

38. Find the cost of 80 bushels of potatoes at $62\frac{1}{2}$¢ a bushel.

39. At $62\frac{1}{2}$¢ a bushel, how many bushels of potatoes can be bought for $80?

40. At $16\frac{2}{3}$¢ a can, how many cans of peaches can be bought for $5?

41. At 2 for a quarter, how many collars can be bought for $1.25? For $1.75?

42. At 3 for a quarter, how many quarts of strawberries can be bought for $1.50? For 75¢? For $2.50?

43. What is the cost of 2 dozen melons at the rate of 3 melons for $1?

44. Make up and work an example similar to Ex. 28. To Ex. 29. To Ex. 36.

45. Make up also an example like Ex. 37. Ex. 42.

46. How many examples in this Exercise can you work at sight?

EXERCISE 163

ORAL WORK

1. Change each of the following to the form of a decimal fraction:

$\frac{1}{2}\quad \frac{1}{3}\quad \frac{2}{3}\quad \frac{1}{4}\quad \frac{3}{4}\quad \frac{1}{5}\quad \frac{2}{5}\quad \frac{3}{5}\quad \frac{1}{6}\quad \frac{1}{8}\quad \frac{3}{8}\quad \frac{5}{8}\quad \frac{1}{16}$

2. Change each of the following to the form of a common fraction:

.4 .6 .24 .16 .25 .5 .35 .125

3. Find the value of

(*1*) $\frac{1}{2} + .25$
(*2*) $\frac{3}{5} + .1$
(*3*) $\frac{1}{3} + .33\frac{1}{3}$
(*4*) $.75 + \frac{1}{20} + .05$
(*5*) $2\frac{1}{8} + .125 + .375$
(*6*) $.66\frac{2}{3} + \frac{1}{3} + \frac{1}{6}$

4. Find the value of

(*1*) $.5 - \frac{1}{4}$
(*2*) $.3 - \frac{1}{5}$
(*3*) $\frac{3}{4} - .25$
(*4*) $2.66\frac{2}{3} - 1\frac{1}{3}$
(*5*) $4.87\frac{1}{2} - \frac{3}{8}$
(*6*) $1.75 - \frac{5}{8}$

5. How many of the examples in Exercise 162 (p. 281) can you work at sight?

6. Practice making change for the postage on different articles.

EXERCISE 164

Write each of the following in a brief form by use of the parenthesis:

1. $9.65 \times 740 + 8.15 \times 740$. Also $9.65 \times 740 - 8.15 \times 740$.

2. $7.805 \div 15 + 5.965 \div 15$. Also $7.805 \div 15 - 5.965 \div 15$.

3. \$376 less the sum of \$15.97, \$127.18, and \$37.14.

4. The sum of 8.762 and 5.192 divided by their difference.

5. The product of the sum and the difference of 8.762 and 5.192.

In Exs. 6 and 7 indicate the work by use of the parenthesis and then compute in the short way:

6. If a merchant buys 825 suits of clothes at \$7.35 per suit and sells them at \$8.45 per suit, how much does he make?

7. If a merchant buys 1250 pairs of shoes at \$2.62$\frac{1}{2}$ per pair and sells them at \$2.87$\frac{1}{2}$ per pair, how much does he make?

8. Simplify by cancellation:

(*1*) $\frac{32 \times 1.26}{.12}$ (*2*) $\frac{1.5 \times 8.4}{.24 \times 2.5}$

9. If 2.65 acres of land cost $350.50, what will 3.45 acres cost?

10. Work again Exercises 105 and 115 (pp. 186 and 203).

EXERCISE 165

ORAL WORK

Give at sight the value of

1. $12 \times .25 + 12 \times .05$
2. $12 \times .25 - 12 \times .05$
3. $16 \times .37\frac{1}{2} - 16 \times .25$
4. $4 - (.37\frac{1}{2} + .125)$
5. $4 - (.37\frac{1}{2} - .12\frac{1}{2})$
6. $24 \times .62\frac{1}{2} - 36 \times (.37\frac{1}{2} + .12\frac{1}{2})$
7. $\frac{.635 - .135}{2}$
8. $\frac{.125 + .37\frac{1}{2}}{.375 - .12\frac{1}{2}}$
9. Practice the rapid addition of columns of figures.

EXERCISE 166

ANALYSIS WITH DECIMAL FRACTIONS

1. Work the odd-numbered examples in Exercise 90 (p. 154).

2. If .27 of a square piece of ground costs $540, what will the whole square cost? (Diagram this on squared paper, using a square 10 × 10.)

In explaining your work say:

I wish to find out what the whole square of land costs. I will first find out what .01 of the land costs and then what 100 hundredths, or the entire square, costs.

3. Make up and work an example similar to Ex. 2.

4. If .3 of a pie is worth 12¢, what is the whole pie worth? (Diagram this, using a circle divided into tenths by lines drawn from the center.)

5. Make up and work an example similar to Ex. 4.

6. A man pays a tax of $48.24. This is .012 of the value of his property. What is his property worth?

7. A man borrowed a certain sum of money, and paid .085 of the sum, or $25.50, for its use for one year. What sum did he borrow?

8. If .35 of a sailing vessel is worth $7000, what is .65 of it worth?

9. If .28 of a piece of land is worth $7430, what is .75 of it worth? (Diagram this.)

10. Make up and work an example similar to Ex. 9.

11. If a boy grows 3.6 inches in 1.2 years, how much will he grow in 1.8 years?

12. If 3.5 inches of snow when melted makes .5 inches of water, how much water will 4.5 inches of snow make when melted?

13. Mrs. X sold .35 of her chickens and had 130 chickens left. How many chickens did she sell?

14. After selling .7 of his farm, a farmer has 27 acres left. How many acres did he have at first?

15. A baseball nine lost .3 of its games and won the remainder, which was 28 games. How many games did it play?

16. Make up and work an example similar to Ex. 15.

17. A county pays .3 of the cost of a macadam road and the state pays the rest, which is $1470. What does the county pay?

18. The sum of two numbers is 3.24 and one number is twice as large as the other. Find the numbers.

19. The sum of two numbers is .231 and one number is 6 times as large as the other. Find the numbers.

20. Two boys make $24.48 picking berries. One boy sells the berries and receives twice as much as the other boy. How much does each boy make?

21. A man and a boy together spade .9 of a garden and the man spades twice as much as the boy. What part of the garden does the boy spade?

22. If 3.4 tons of coal cost $28.50, what will 1.7 tons cost?

SUG. Show by comparison that 3.4 is twice 1.7.

23. In skimming cream from milk by hand, .82 of the cream is obtained. During a certain month a farmer obtained thus 492 lb. of cream. How many pounds would he have obtained if he had used a mechanical separator, which extracts practically all the cream from the milk?

24. Copy a part of a railroad time-table on the blackboard, and practice determining at sight the intervals between the times given on the time-table.

BILLS, ACCOUNTS, RECEIPTS, CHECKS

EXERCISE 167

1. What is a bill?

2. Make a bill for the following: On July 3, 1912, Mr. Harold Williams bought of Stucher Bros., Philadelphia, 6 lb. sugar @ 6¢, 4 lb. coffee @ 34¢, 6 cans corn @ 13¢, 1½ pk. potatoes @ 18¢.

3. Point out the necessary parts of the bill in Ex. 2.

4. What are the advantages in using bills?

127. A Receipt for a Bill is a written statement that the bill has been paid.

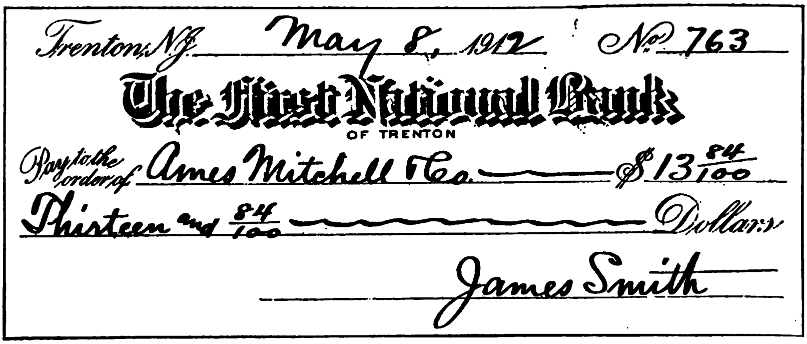
Thus, the bill on page 162 might have been receipted by writing the following on the bill:

Received payment,
AMES, MITCHELL AND Co.,
Per R. S.

The method of receipting a bill may vary in some details. If a person receipts his own bill, he signs his own name. If a bill is receipted by a clerk for a company, the clerk writes "per" followed by his own name or initials. The date of receipting the bill may or may not be written. Sometimes a receipt is written not on the bill, but on another paper. In this case the sum paid and the name of the person paying it must appear in the receipt.

128. Paying a Bill by Check. If a person has money deposited in a bank, it is often convenient to pay a bill by check.

Thus, the bill on page 162 might have been paid by Mr. Smith, by a check like the following:



Trenton, N.J. May 8, 1912 No. 763

The First National Bank
OF TRENTON

Pay to the order of Ames Mitchell & Co. $13 $\frac{84}{100}$

Thirteen and $\frac{84}{100}$ Dollars

James Smith

A **check** is a written order made on a bank by a depositor in the bank, ordering the payment of a specified sum of money to a person or business firm named in the check.

The necessary parts of a check are *date, place, name* of the person to whom the check is to be paid, *signature* of maker (i. e. the person who issues the check), name of *bank,* and *amount* of the check, usually stated both in figures and in words.

129. The Advantages of Payment by Check are that (1) a bill may be paid by mail. This saves time and diminishes the risk of losing money.

(2) The check, after it is paid by the bank, is returned to the maker and forms a receipt, or evidence that the bill has been paid.

EXERCISE 168

1. Work again Exs. 1–5 of Exercise 95 (p. 163) and receipt each bill by acting as a clerk for the seller of the bill of goods named in each example.

2. Work again Exs. 6–9 (pp. 164–165) and for each example write a check by which a school friend could pay his bill. Then receipt each bill in your own name.

3. Let each boy suppose himself a dealer in coal, hay, lime, and grain, and make out three bills payable by other pupils. Let those pupils pay by check and let the maker of the bills receipt the bills.

Let each girl suppose herself the proprietor of a dry-goods store and do the same.

4. Why is it important to file and keep all receipted bills?

5. Examine a number of receipted bills at home or in some store, and make a list of the different ways in which each bill is receipted.

6. Let the teacher dictate a list of checks in which some essential part is lacking, and let the pupil supply what is lacking.

7. Work again Exercise 141 (p. 252).

130. Accounts. If a person frequently buys articles of the same company, it is a convenience for him to have an account with this company.

An **account** is a record of various articles bought and of sums of money paid.

At some convenient date, as the first of the month, a *statement* of the account is sent to the buyer. The sending of this statement usually means that the payment of the balance is desired.

What are the advantages in keeping an account at a store? The disadvantages?

The following is an example of an account:

TRENTON, N. J., March 1, 1912.

Mr. Marcus Harper,
15 Washington St., Trenton.
To C. L. Green Co.,
Dealers in Coal, Lumber, Lime, and Grain
217 East State St.

				Dr.		Cr.	
Feb.	1	Balance due		217	42		
"	3	3 tons Coal	$6.50	19	50		
"	5	Cash				150	00
"	12	1 load Kindling Wood		2	00		
		1000 feet Yellow Pine		46	00		
"	21	Cash				50	00
"	28	Cash to balance				84	92
				284	92	284	92

Accounts, however, may be kept in many different ways. Sometimes the Dr. items are recorded on each left-hand page of a book, and the corresponding Cr. items on the opposite right-hand page. Sometimes each account is kept on a separate card. Every good business man has a system of keeping accounts which is adapted to his own business.

131. Boys' and Girls' Accounts. A convenient form in which a boy or a girl may keep an account of personal expenses and receipts is shown in the following.

Note that in this form of account the cash on hand is shown in the middle column, not merely at the end of a certain time, as a month, but at all times.

Personal Account of Walter Howard

1912	Expenses		Balance on hand	Receipts	
Jan. 1	Balance on hand		17 12		
" 1	Presents	1 24	15 88		
" 2	Pair of Gloves	80	15 08		
" 5			15 73	Shoveling snow	65
" 8	½ doz. Oranges	18	15 55		

EXERCISE 169

1. During the month of May, Mr. Dudley Marsh bought the following groceries of Huntly and Co., and paid the following sums:

Bought May 2, 8 lb. sugar @ 6½¢, 3 lb. crackers @ 12¢. May 7, 1 bbl. flour $7.75, 2 lb. coffee @ 42¢, 2 gal. vinegar @ 40¢. Paid May 15, $5. Bought May 20, groceries $16.24. Paid May 23, $10.

Make out an account (dated June 1) for these items similar to that given on page 290.

2. Make up a similar account for the month of June, using probable quantities and prices.

3. Make up an account for the purchases of a family at a dry-goods store during the month of March.

4. Let each boy make up an account of the probable receipts and expenditures of a boy in the month of July. Also in December.

Let each girl do the same for a girl's probable receipts and expenditures.

5. Of what benefit is it to a boy or a girl to keep an exact account of receipts and expenditures?

6. Practice the multiplication and division of small fractions by small whole numbers. Let the pupils put on the blackboard lists of fractions for drill.

EXERCISE 170

REVIEW

1. On squared paper show the meaning of .08. 5% .004 $.00\frac{1}{8}$

2. What is the short way of getting the sum of 3.1416 + 3.1416 + 3.1416 + 3.1416?

3. Simplify $\frac{.06 \times .03}{7.5}$. Also $\frac{1.5 \times .24}{3.6}$.

4. The product of two numbers is .0006 and one of the numbers is .04. Find the other number.

5. If the dividend is .0124 and the quotient is 3.2, find the divisor.

6. Find the value of x in each of the following:

(*1*) $1.2 + x = 3.65$

(*2*) $x - 39.37 = 62.5$

(*3*) $\frac{2.31}{x} = 3.3$

(*4*) $30.25 = x - 11.35$

(*5*) $\frac{2.56}{x} = .16$

(*6*) $\frac{x}{1.8} = .7$

7. Remove the zero in each of the following and state what change is made in the value of the decimal fraction:

.015 .240 .108 .2805 0.316 .0316

8. If one yard of cloth costs 87½¢, how many yards can be bought for $21?

9. Mr. Walters and his partner make $6900 in business in one year. Mr. Walters, who manages the store, is to receive 30% more of the profits than his partner. How much does each receive?

10. What would be gained by buying 1000 bu. of corn at 61½¢ per bushel and selling at 73¢?

11. If 1 lb. of butter contains 15.6 oz. of fat, how many ounces of fat do 3 lb. of butter contain? How many pounds of fat?

12. Anna and Mary earn $12 by sewing. Mary furnishes the thread and the sewing machine and is to receive .4 more than Anna. How much money does each receive?

13. Mt. Everest is 29,002 feet high. How many miles and what decimal of a mile (to 3 places) is this?

14. .38 of a farm is in wheat, .435 in grass, and the rest is orchard. What decimal part of the farm is orchard?

15. From a book which costs him $2.12 a farmer gained information about methods of farming which enabled him to increase his profits the next year by $560. What did he make in one year by the purchase of the book?

16. A torpedo boat has traveled 37.1 knots in one hour. How many miles is this, if 1 knot is 1.15 miles?

17. If the height of Mt. Washington is .445 of the height of Pike's Peak and Mt. Washington is 6290 ft. high, find the height of Pike's Peak.

18. How many examples in this Exercise can you work at sight?

19. Practice the oral multiplication of numbers by the aliquot parts of 100.

EXERCISE 171

INDUSTRY REVIEW: LUMBER AND FORESTRY

1. Mr. Hamilton bought $20\frac{3}{4}$ acres of woodland at $60 per acre. What did the woodland cost him?

2. He cut off the trees and sold oak lumber worth $1275, pine lumber worth $1975.50, and hickory worth $172. He also sold 123 cords of firewood at $2.50 a cord, and 14,300 lb. of bark for tanning purposes at $10 a ton. What were his total receipts?

3. He paid out $672.75 for the labor of cutting the trees, $237.40 for hauling the logs to the mills, $262.80 for the sawing.

The land, after the timber was cut off, was worth $22 an acre. What was his net profit on the whole transaction?

By "net profit" is meant the excess of what is received over what is paid out in a given transaction.

4. Mr. Davis owns a cranberry bog and makes his own barrels. He buys the staves and hoops, but cuts the heading for the barrels on his own land. He needs 30,000 feet of boards for heading each year, and on an average one pine tree makes 200 feet. How many trees must he cut each year?

5. When the first settlers came to the United States, it is estimated that .62 of the land was covered with forest. How many square miles was this? How many states as large as New York would this forest area make?

6. In a recent year it was estimated that only .28 of the United States was covered with forest. How many states as large as New York would this make?

7. What bad effects will follow, if too large a part of our forest land is cleared? What is the purpose of Arbor Day?

8. If each pupil in your school should plant a tree, how many acres would these trees cover, 100 sq. ft. being allowed for each tree?

9. In the year 1910, there were 17,061,000 pupils enrolled in the public schools of the United States. If each pupil should plant 10 trees on Arbor Day, how many acres would this make, allowing 100 sq. ft. for each tree? How would the ground covered compare with the area of New York State?

10. Can you find out the number of children enrolled in the public schools of your own state last year, and make a calculation similar to that called for in Ex. 9?

11. If you live in the country (or visit in the country), find out the two most important kinds of trees cut for lumber in your neighborhood. Find out the price of 1000 feet of the lumber made from each. Find the price of other useful parts of these trees. Then make up and work examples concerning these facts.

12. How many examples in Exercise 153 (p. 267) can you now work at sight?

CHAPTER XI

COMPOUND NUMBERS

EXERCISE 172

1. What is a unit? Give three examples of units.

2. What is a compound number? Give two examples of a compound number. What is a simple number? Give two examples of a simple number.

3. Work again Exercise 99 (p. 172).

4. In what cases is it better to use a compound number than the equivalent simple number? Give an example.

5. In what cases is it better to use a simple number than the corresponding compound number? Give an example.

6. Recite the table of weight.

7. Work the odd-numbered examples in Exercise 100 (p. 175).

8. Also work the odd-numbered examples in Exercise 101 (p. 177).

9. How many of the examples in Exercise 100 can you work at sight?

10. What is an angle? A right angle? An acute angle? An obtuse angle? Draw an angle of each kind.

11. Repeat the table of measures of length. How many feet in a mile? How many yards?

12. In measuring the length of a room, why does it save labor to use a long tape-measure instead of a foot rule? Which method of measurement is more likely to be accurate?

13. Work the odd-numbered examples in Exercise 102 (p. 178).

This and similar parts of this review may be shortened or enlarged at the discretion of the teacher.

14. In measuring an area, how many lines do you measure? How many in measuring a volume?

15. Define surface, square, rectangle, and area. Draw a square whose area is 9 sq. in. A rectangle whose area is 18 sq. in.

16. What is the short way of finding the area of a rectangle? What is the long way?

17. Recite the table for square measure.

18. Work the odd-numbered examples in Exercise 104 (p. 183). How many of these can you work at sight?

19. Draw an irregular figure which can be cut up into rectangles (as in Art. 102, p. 188). Find the area of this figure.

20. Define solid, cube, rectangular solid, and volume.

21. Using inch cubes, let one pupil build a cube whose volume is 27 cu. in. Let another pupil build a box-shaped solid whose volume is 36 cu. in.

22. What is the short way of getting the volume of a box-shaped solid?

23. Recite the table for cubic measure.

24. Work the odd-numbered examples in Exercise 108 (p. 191). How many of these can you work at sight?

25. Recite the table for dry measure.

26. How are the units in dry measure connected with those in cubic measure?

27. Work the odd-numbered examples in Exercise 110 (p. 195). How many can you work at sight?

28. Recite the table for liquid measure. How are the units in liquid measure connected with those in cubic measure?

29. Work the odd-numbered examples in Exercise 112 (p. 197). How many can you work at sight?

30. Recite the table of measures of time.

31. Name the months of the year in order, with the number of days in each month. Repeat the rhyming lines which give the same facts.

32. How many days and what fraction of a day are there in a year? What is a leap year? Name three years which are leap years. Also three which are not leap years.

33. Work again the odd-numbered examples in Exercise 114 (p. 201).

34. One peck is what decimal part of a bushel? One peck is what per cent of a bushel?

35. One pint is what decimal part of a gallon? One pint is what per cent of a gallon?

36. Practice the rapid addition of columns of figures like those given on p. 227. Let the pupils put on the blackboard examples adapted to this drill.

MEASURES OF VALUE

132. A Unit of Money is a certain amount of gold used to measure the values of things.

Instead of gold, silver and copper are sometimes used in a limited way to measure value.

Coin is metal that has been weighed and stamped by the government for use as money; as a silver dollar, or a gold eagle, or an English sovereign.

Paper money consists of printed promises to pay a certain amount of coin to the bearer.

The fundamental unit of United States money is the gold dollar, which weighs 25.8 grains, $\frac{9}{10}$ of this being gold.

Repeat the table of United States money.

133. In **English Money** the fundamental unit is 1 pound. The value of the pound in United States money is $4.8665.

TABLE

4 farthings (far.) = 1 penny (d)
12 pence = 1 shilling (s)
20 shillings = 1 pound (£)
21 shillings = 1 guinea (G)

EXERCISE 173

Reduce:

1. £2 10s to shillings.
2. 4s 9d to pence.
3. $5\frac{3}{4}$s to pence.
4. £$4\frac{1}{2}$ to pence.

Change to higher denominations:

5. 100d
6. 35s
7. 1000d
8. 100s
9. 400 far.
10. 1000 far

Find in United States money the value of

11. £10
12. £13
13. £5 6s
14. £6.8
15. £5.8
16. £3.45

Find in English pounds and decimal of a pound the value of

17. $486.65
18. $1000
19. $583.98
20. $15,000

21. A suit of clothes bought in London cost £3 85s; what was the cost in United States money?

22. A dozen pairs of gloves bought in London cost £1 45s 3d; what was the cost of 1 pair in United States money?

23. What is the unit of money in France? How much is this unit of money worth in United States money?

24. What is the unit of money in Germany? How much is this unit worth in United States money?

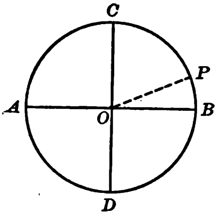
25. Practice the oral addition and subtraction of 2 small three-figured numbers representing money, as $2.21 and $1.08.

CIRCULAR MEASURE

134. **A plane surface** is a flat surface, as the surface of the blackboard. The surface of a baseball is not plane, but curved.

A **plane figure** is a figure on a plane surface. The square and the rectangle are plane figures. A cube is not a plane figure.

A **circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within, called the center.



The **circumference** is the line which bounds a circle.

A **quadrant** is one of the four equal parts into which a circumference may be divided.

Thus, if CPB is the fourth part of the circumference ACBD, it is called a quadrant.

The angle COB is a right angle. (What is a right angle?)

An **arc** is any part of a circumference, as PB.

135. A circumference is divided into 360 equal parts, called degrees. How many degrees are there in a quadrant?

An angle at the center, as POB in the diagram on p. 301, is regarded as containing the same number of degrees as the corresponding arc, PB. How many degrees are there in a right angle?

A **perpendicular** is a line which makes a right angle with a given line. Thus, CO is perpendicular to OB.

TABLE

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
360 degrees = 1 circumference.

A degree is sometimes divided into tenths and hundredths (*i.e.*, decimally) instead of into minutes and seconds.

EXERCISE 174

1. Name two surfaces that are plane. Name two that are not plane.

Reduce:

2. 3° 27′ to minutes.
3. 47° 31′ to minutes.
4. 10′ 35″ to seconds.
5. 6° 20′ to seconds.
6. 4° 12′ 30″ to seconds.
7. 12° 25″ to seconds.

Change to higher denominations:

8. 210′
9. 415″

10. Each angle of a square contains how many degrees?

11. How many degrees in a right angle? In 2 right angles? In 3 right angles? In $\frac{3}{4}$? In $\frac{1}{3}$? In $1\frac{2}{3}$?

12. Find:

$\frac{1}{10}$ of 360° $\frac{2}{3}$ of 180° $\frac{3}{5}$ of 180° $\frac{3}{4}$ of a right angle

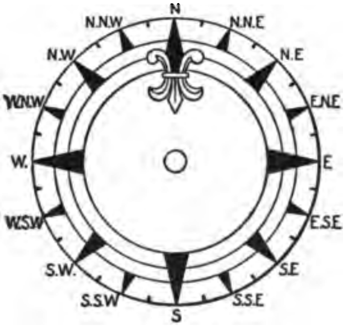
$\frac{3}{4}$ of 360° $\frac{3}{4}$ of 180° $\frac{7}{12}$ of 180° $\frac{2}{3}$ of a quadrant

13. The earth turns completely on its axis once in 24 hours. Through how many degrees does it turn in 1 hour? In 2 hours? 5 hours? $1\frac{1}{2}$ hours?

14. Through how many degrees does the earth turn in 20 minutes? 45 minutes? 15 minutes?

15. How many of the examples in this Exercise can you work at sight?

136. Points of the Compass. In stating the direction of one object from another on the surface of the earth, the direction in which the wind is blowing, etc., we use the points of the compass. The most important of these are given on the adjoining diagram.



N. N. E. is read "north north east."

E. N. E. is read "east north east."

On the diagram each quadrant, as that from N. to E., is divided into 8 equal parts.

EXERCISE 175

1. How many degrees are there between N. and N. N. E.? Between N. and E. N. E.? N. N. E. and E. N. E.?

2. How many degrees between N. E. and S. E.? Between E. S. E. and W.? S. E. and N. N. W.?

3. If the wind shifts from E. to N. E., through how many degrees does it change its direction?

4. Through how many degrees does the wind change, if it shifts from E. N. E. to N. W.? To W. N. W.? To N. N. W.?

5. Make up and work two examples similar to Ex. 3.

6. A ship starts from a certain point and sails 10 miles north and then 10 miles east. What direction will the ship then be from its starting point? What, if it sails 10 miles west and then 10 miles south?

7. Make up and work an example similar to Ex. 6.

8. Your home is in what direction from the schoolhouse? The post office is in what direction from your home? The schoolhouse from the post office?

9. Name three objects and state the direction of each object from each of the others.

10. Name the points of the compass as given on the diagram on p. 303, beginning at N. and going around through E.

11. Stand facing the north. Now turn to the left till you face the west. Through how many degrees have you turned?

Turn again till you face the south. Through how many degrees have you turned in all?

Turn till you face the east. The north. Through how many degrees have you turned in all?

12. Draw a diagram like that on p. 303.

13. How many of the examples in this Exercise can you work at sight?

14. Work again Exercise 34 (p. 66).

137. Miscellaneous Units.

NUMBERS	NUMBER OF SHEETS OF PAPER
12 units = 1 dozen (doz.)	24 sheets = 1 quire (qr.)
12 dozen = 1 gross (gr.)	20 quires = 1 ream (rm.)
12 gross = 1 great gross	2 reams = 1 bundle
20 units = 1 score	5 bundles = 1 bale

Writing paper (as well as other kinds of paper) is also often bought by 100 or 1000 sheets, or by the pound, instead of by the quire. Envelopes are usually bought by the 100 or 1000.

EXERCISE 176

1. How many units in 7 dozen? In a gross? In 6 score?

2. How many dozen in 60 units? In a great gross? In 15 score?

3. How many quires in a bundle? In a bale?

4. How many sheets in a ream? In 15 quires? In a bundle?

5. How many years are there in threescore years and ten? In fourscore years? In 6 score and a dozen?

6. How many sheets of paper in 4 reams? In 10 quires?

7. If I pay 50¢ for a ream of typewriter paper, how many sheets do I get for 1 cent?

8. 6 sheets is what decimal part of a quire? 6 sheets is what per cent of a quire?

9. Which is cheaper: to pay 30 cents a dozen for eggs, or 45 cents a score?

10. How many of the examples in this Exercise can you work at sight?

11. Practice subtracting dates as on p. 229. Let the pupils put the dates with the corresponding events on the blackboard.

138. Other Measures and Tables. Various other units and tables are used for special purposes.

For example, *Apothecaries' Measure* is used by druggists in selling drugs; *Troy Weight* is used in weighing gold, silver, and jewels; and *Surveyors' Measure* is used in measuring land.

Most countries use a system of weights and measures called the *Metric System.*

PROCESSES WITH COMPOUND NUMBERS

139. I. Reduction Descending.

Ex. Reduce 3 bu. 2 pk. 5 qt. to quarts.

3 bu. = 3 × 4 pk. = 12 pk.
12 pk. + 2 pk. = 14 pk.
14 pk. = 14 × 8 qt. = 112 qt.
112 qt. + 5 qt. = 117 qt. *Ans.*

or

$$\begin{array}{rl} & \text{3 bu. 2 pk. 5 qt.} \\ & \underline{4} \\ 12 & \text{(pk.)} \\ \underline{2} & \\ 14 & \text{(pk.)} \\ \underline{8} & \\ 112 & \text{(qt.)} \\ \underline{5} & \\ 117 & \text{qt. } \textit{Ans.} \end{array}$$

140. II. Reduction Ascending.

Ex. Change 117 qt. to higher units.

117 qt. = $\frac{117}{8}$ pk. = 14 pk. + 3 qt.
14 pk. = $\frac{14}{4}$ bu. = 3 bu. + 2 pk
3 bu. 2 pk. 3 qt. *Ans.*

or

8)117 (qt.)
4)14 (pk.) + 3 (qt.)
3 (bu.) + 2 (pk.)
3 bu. 2 pk. 3 qt. *Ans.*

EXERCISE 177

Reduce to the lowest unit given in each example:

1. 8 yd. 2 ft. 10 in.
2. 5 bu. 2 pk. 6 qt.
3. 7 wk. 9 hr.
4. 8 sq. yd. 6 sq. ft.

5. 21° 14′ 38″

6. 6 cu. ft. 278 cu. in.

7. 3 T. 7 cwt. 80 lb.

8. 20 rd. 3 yd. 2 ft.

9. 30 A. 84 sq. rd.

10. 24 da. 9 hr. 18 min.

11. 13 gal. 1 pt.

12. 1 cu. yd. 8 cu. ft.

13. 6 sq. rd. 5 sq. yd. 7 sq. ft.

14. 20 sq. rd. 1 sq. yd. 3 sq. ft.

Change to higher units:

15. 17,560 lb.

16. 12,836 cu. in.

17. 1463 pt. (liquid)

18. 38,715 sec.

19. 8888 sq. in.

20. 7777 hr.

21. 4587 pt. (dry)

22. 4869 sq. yd.

23. 3009 oz.

24. 2106 pt. (liquid)

25. 4819 pt. (dry)

26. 5897 in.

27. 3360 sheets

28. 7865 ft.

29. If I waste 5 minutes a day, how many minutes do I waste in a year? Reduce this result to higher units. Which is easier to picture to yourself: the answer expressed in minutes, or in higher units?

30. If you sleep 8 hours a day on the average, how many hours do you sleep in a month of 30 days?

31. A locomotive weighs 156,800 lb. In order to grasp this number more readily, express it in higher units.

32. Divide 8 bu. 3 pk. by 1 bu. 3 pk. In order to divide these numbers readily, how must you reduce them?

33. From the ground floor of a house to the attic there are 54 steps, each 5 inches high. What is the height of the attic above the ground floor in feet and inches?

EXERCISE 178

Oral Work

Reduce at sight:

1. 2 ft. 9 in.
2. 2 lb. 4 oz.
3. 2 pk. 5 qt.
4. 3 sq. yd. 5 sq. ft.
5. 3 yd. 2 ft.
6. 2 hr. 12 min.
7. 3 T. 125 lb.
8. 5 mo. 12 da.
9. 3 qt. 2 pt.
10. 4 ft. 3 in.
11. 3 mi. 100 rd.
12. 1 cu. yd. 5 cu. ft.

At sight, change to higher units:

13. 80 in.
14. 20 pt. (dry)
15. 100 oz.
16. 25 qt. (liquid)
17. 200 min.
18. 38 da.
19. 14 ft.
20. 29 sq. ft.
21. 29 cu. ft.

22. Practice the oral multiplication and division of one small fraction by another.

For this purpose, let the teacher put on the blackboard an arrangement of small fractions similar to that given on p. 128 (Ex. 1).

141. III. Addition of Compound Numbers. In the addition of compound numbers,

Set similar units in the same column;
Begin at the right and add each column;
Simplify the sum of each column by reduction ascending.

Ex. Add 2 bu. 3 pk. 5 qt., 3 bu. 1 pk. 3 qt., and 5 bu. 2 pk. 7 qt.

bu.	pk.	qt.	
2	3	5	
3	1	3	
5	2	7	
11	3	7	*Sum*

The sum of the quarts is 15 qt., which, reduced to pecks, is 1 pk. 7 qt. We set down the 7 qt. and add the 1 pk. with the column of pecks.

The sum of the pecks is 7 pk., or 1 bu. 3 pk. We set down 3 pk. and add the 1 bu. with column of bushels.

The sum of the bushels is 11.

Hence, the entire sum is 11 bu. 3 pk. 7 qt.

EXERCISE 179

Add:

1. 3 bu. 3 pk. 1 qt.; 2 bu. 3 qt. 1 pt.; and 8 bu. 2 pk. 3 qt. 1 pt.

2. 3 gal. 1 qt. 1 pt.; 5 gal. 3 qt.; and 4 gal. 2 qt. 1 pt.

3. 7 cu. yd. 10 cu. ft. 386 cu. in.; 9 cu. yd. 432 cu. in.; and 8 cu. yd. 1217 cu. in.

4. 3 T. 18 cwt. 40 lb.; 15 cwt. 35 lb.; and 2 T. 70 lb.

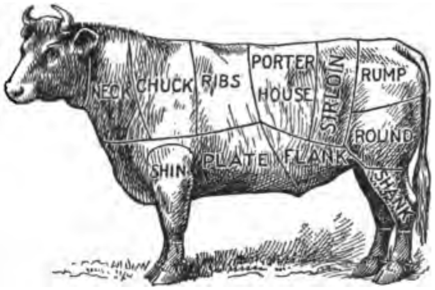
5. A milkman has in different cans 12 qt. 1 pt.; 27 qt. 1 pt.; 28 qt.; and 13 qt. 1 pt. How can he tell how much he has in all, without measuring his milk? Do this for him.

6. On a camping trip, Harry carried 12 lb. 4 oz. in his pack, Walter carried 15 lb. 8 oz., and James 14 lb. 5 oz. How much did they carry all together?

7. If it is high water at New York at 5.12 P. M. on a given day, and high water at New Haven comes 3 hr. 5 min. later, when will it be high water at New Haven?

8. The diagram shows the parts into which it is customary to divide a beef. If the shank weighs 23 lb. 8 oz., the round 125 lb. 8 oz., and the rump 57 lb. 4 oz., how much does the hind quarter of the beef weigh?

9. The sirloin, porterhouse, and ribs are regarded as the choice cuts of a beef. If the sirloin of a beef weighs 59 lb. 8 oz., the porterhouse 57 lb. 12 oz., and the ribs 69 lb. 12 oz., what do the choice cuts weigh together?



10. A boy, 4 ft. $7\frac{1}{2}$ in. in height, gains 1 ft. $3\frac{3}{4}$ in. in five years. What is his height at the end of the five years?

11. Add 7 ft. 9 in. and 8 ft. 6 in. Now add 7.75 ft. and 8.5 ft. Which process is easier? Why?

12. Add 7 yd. 2 ft. 6 in. to 4 yd. 2 ft. 9 in.

13. Add £3 4s. 9d., £12 13s. 7d., and £9 8s. 11d. Now add $37.19, $14.13, and $21.03. Which process is easier? Why?

14. Practice the oral multiplication of one small decimal fraction by another.

142. IV. Subtraction of Compound Numbers.

Ex. Subtract 2 bu. 3 pk. 4 qt. from 5 bu. 2 pk. 7 qt.

bu.	pk.	qt.	
5	2	7	
2	3	4	
2	3	3	*Difference*

4 qt. from 7 qt. leaves 3 qt. 3 pk. cannot be subtracted from 2 pk. Hence, we borrow 1 bu. (or 4 pk.) from the 5 bu. and add it to the 2 pk., giving 6 pk. 3 pk. from 6 pk. leaves 3 pk.

2 bu. from 4 bu. leaves 2 bu.

Hence, the difference is 2 bu. 3 pk. 3 qt.

143. Difference in Time between Two Dates. In finding the interval of time between two dates, 30 days are usually reckoned as one month, and 12 months as one year.

Ex. How many years, months, and days are there between Oct. 19, 1910, and June 6, 1912?

yr.	mo.	da.	
1912	6	6	
1910	10	19	
2	7	17	*Difference in time*

Since January is the 1st month in the year, June is the 6th month and October the 10th month.

Let the pupil supply the rest of the explanation.

EXERCISE 180

Subtract:

1. 4 yd. 2 ft. 10 in. from 10 yd. 3 ft. 5 in.
2. 17° 9′ 38″ from 28° 3′ 41″
3. 38° 42′ 17″ from 180°
4. 7 A. 85 sq. rd. 9 sq. yd. from 80 A. 19 sq. rd.
5. 85 bu. 3 pk. 7 qt. from 100 bu. 5 qt.
6. 80 gal. 2 qt. from 273 gal.

Find the difference in time between the following pairs of dates:

7. Oct. 12, 1492, and Dec. 21, 1620.
8. Dec. 21, 1620, and April 19, 1775.
9. April 19, 1775, and Oct. 19, 1781.
10. Oct. 19, 1781, and April 30, 1789.
11. Apr. 30, 1789, and Jan. 8, 1815.
12. Jan. 8, 1815, and May 1, 1898.

13. Abraham Lincoln was born Feb. 12, 1809; and died April 15, 1865. Find his age at death.

14. From a hind quarter of a beef which weighed 210 lb., a butcher has cut off the shank weighing 26 lb. 12 oz. and the rump weighing 28 lb. 8 oz. How much of the quarter is left (or is "in the round")?

15. A piece of rope is 27 yd. 1 ft. 8 in. long. From it a piece 12 yd. 2 ft. 9 in. long is cut off. How can I tell the length of what is left, without measuring it?

16. From a roll of calico 60 yd. long, lengths of 34 yd. 8 in. and 15 yd. 9 in. have been cut off. Show how to find how much is left, without taking the trouble of measuring it.

17. From a round of beef weighing 87 lb. 8 oz., a butcher has sold to different customers 2 lb. 4 oz., 5 lb. 12 oz., 4 lb. 4 oz., and 1 lb. 4 oz. How much of the round is left?

18. A boy's height increases in one year from 4′ 10.8″ to 5′ 1.7″. What is the amount of the increase?

19. If it is high water at New York City at 7.12 A. M. on a certain day, and high water comes 3 hr. 23 min. earlier at Providence, R. I., at what hour is it high water at Providence?

20. Make up and work an example similar to Ex. 18. To Ex. 19.

21. If a girl's weight increases from 66 lb. 9 oz. to 82 lb. 5 oz. in two years, what is her gain in weight?

22. Find the time between each pair of the following dates: Oct. 12, 1492, Dec. 21, 1620, July 4, 1776, May 1, 1898. What important event happened on each of these dates?

23. At New York City on June 21, the sun rises at 4.27 A. M. and sets at 7.36 P. M. How long is the day?

If on Dec. 21 the sun rises at 7.21 and sets at 4.35, how long is the day? Why are these dates important?

24. If a railroad employee who is now 63 yr. 7 mo. 18 da. old is to retire on a pension when he is 70 years of age, how long has he yet to serve?

25. Harold can walk from the farm to the city in 2 hr. 12 min. He can ride the same distance on a bicycle in 47 min. How much time does he save by riding on the bicycle? How much time would he save on 20 trips?

26. If the last frost in spring comes on May 10 and the earliest frost in autumn comes on Oct. 2, how many months and days are there in the interval?

27. Walter raised 10 bu. of peanuts and has sold 1 bu. 1 pk. and 2 bu. 1 pk. How can he tell the amount of peanuts he has left, without measuring them? Do this for him.

28. Practice the oral division of one small decimal fraction by another by means of blackboard drills.

144. V. Multiplication of Compound Numbers.

Ex. Multiply 2 bu. 3 pk. 5 qt. by 7.

bu.	pk.	qt.	
2	3	5	
		7	
20	1	3	*Product*

We write the multiplier under the lowest unit of the multiplicand. 7 × 5 qt. = 35 qt., or 4 pk. 3 qt. We set down the 3 qt. and add the 4 pk. to the next product.

7 × 3 pk. = 21 pk.; and 21 pk. + 4 pk. = 25 pk., or 6 bu. and 1 pk. We set down 1 pk. and add 6 bu. to the next product.

7 × 2 bu. = 14 bu.; and 14 bu. + 6 bu. = 20 bu.

Hence, the product is 20 bu. 1 pk. 3 qt.

EXERCISE 181

Multiply:

1. 3 da. 7 hr. 25 min. by 6.
2. 7 bu. 3 pk. 6 qt. by 5.
3. 3 yd. 5 in. by 4.
4. 5 T. 65 lb. by 7.
5. Multiply 5 sq. yd. 6 sq. ft. 7 sq. in. by 8.

6. Multiply 8 bu. 4 pk. by 17. Now multiply 8.5 bu. by 17. Compare the amount of work in the two processes.

7. A certain lighthouse uses 2 gal. 3 qt. of oil each day. How much oil will it use in a month of 30 days? Multiply 2.75 gal. by 30.

Find the volume of a bin 24 ft. 9 in. long, 4 ft. 6 in. high, and 3 ft. 6 in. wide.

9. Find the area of a rectangle 10 ft. 6 in. long and 4 ft. 9 in. wide: (1) By reducing each dimension to inches and reducing the product back to square feet: (2) By using 10.5 ft. and 4.75 ft. as the dimensions of the rectangle. Compare the work in the two processes.

10. If I save 1 hr. 12 min. each day by prompt performance of duties, how much time do I save in 300 days? What is this worth at 40¢ an hour? Is the money value the only value of the time which is saved?

11. What is the length in yards of 32 steam pipes, each of which is 19 ft. 9 in. long?

12. The circumference of a bicycle wheel is 13 ft. 8 in. Express in miles, rods, yards, and feet the distance covered by 2764 revolutions of the wheel.

13. Let the pupils write on the blackboard a list of objects which are likely to cost $12\frac{1}{2}$¢ each. Also similar lists for $16\frac{2}{3}$¢, 25¢, $33\frac{1}{3}$¢, and $37\frac{1}{2}$¢. Let the pupil state the cost as the teacher points at the name of an article and gives a number (for example, the teacher points at "cans of corn" and says 15).

14. Work again Exercise 165 (p. 285).

145. VI. Division of Compound Numbers. The division of a compound number may be of two kinds:

(1) The division of a compound number into a number of equal parts, or

(2) The division of a compound number by another compound number.

Which of these is partition and which is measurement? (See p. 58.)

Ex. 1. Divide 52 gal. 3 qt. 1 pt. into 9 equal parts.

$$\begin{array}{rrrl} \text{gal.} & \text{qt.} & \text{pt.} & \\ 9)\underline{52} & \underline{3} & \underline{1} & \\ 5 & 3 & 1 & \textit{Quotient} \end{array}$$

We write the divisor to the left, and divide the highest unit first. If 52 gal. is divided into 9 equal parts, we get 5 gal. in each part, with a remainder of 7 gal.

7 gal. = 28 qt., and 28 qt. + 3 qt. = 31 qt. 9 into 31 qt. gives 3 qt., with a remainder of 4 qt.

4 qt. = 8 pt., and 8 pt. + 1 pt. = 9 pt. 9 into 9 pt. gives 1 pt.

Hence, we have 5 gal. 3 qt. 1 pt. as the quotient.

Ex. 2. Divide 6 rd. 1 yd. 2 ft. 3 in. by 3 yd. 2 ft. 7 in.

Reducing each of the two compound numbers to the same lowest unit, we have

$$\begin{array}{lcr} \text{6 rd. 1 yd. 2 ft. 3 in.} & = & \text{1251 in.} \\ \text{3 yd. 2 ft. 7 in.} & = & \text{139 in.} \end{array}$$

$$\begin{array}{rl} 9 & \textit{Quotient} \\ 139)\overline{1251} & \\ \underline{1251} & \end{array}$$

EXERCISE 182

Divide:

1. 76 gal. 3 qt. 1 pt. by 5.
2. 102 bu. 1 pk. 4 qt. by 7.
3. 64 yd. 1 ft. 9 in. by 5.
4. 129 wk. 3 da. 22 hr. by 12.
5. 256 mi. 265 rd. by 9.
6. 157 bu. 2 pk. by 8 bu. 3 pk.
7. 76 gal. 2 qt. by 4 gal. 1 qt.
8. 33° 45′ by 15.
9. Divide 42° 15′ 40″ by 15.

10. In a mile relay race, the time of the different runners was 4 min. 36 sec., 4 min. 52 sec., 4 min. 29 sec., and 4 min. 28 sec. What was the average time of the runners?

11. Divide 6 gal. 1 qt. 1.4 pt. by 5. Now divide 6.425 gal. by 5. Compare the work in the two processes.

12. A wire fence 212 yd. 1 ft. long is supported by posts 2 ft. 4 in. apart. How many posts are required?

13. If 10 chickens weigh 37 lb. 12 oz., what is their average weight? What is their average value at 22¢ a pound?

14. If an aeroplane goes 78 mi. in 1 hr. 8 min., how many miles and yards is this per minute?

15. Take 10 of your ordinary steps in a straight line. Now measure the distance you have walked. How long is your average pace?

16. Pace the length and breadth of the school yard, counting the number of your paces in each distance. How long is the school yard? How wide? Make a diagram of the school yard.

17. Count the number of steps you take in going home to-night. Then determine the distance from your home to the schoolhouse.

18. How many examples on pp. 103–104 can you work at sight?

19. Practice the rapid addition of columns of figures.

EXERCISE 183

REVIEW

1. How many cubic inches are there in 20 gallons?

2. A stationer bought 100 reams of paper at $2.50 a ream and sold it at 20 cents a quire. What was his gain?

3. A boy bought 6 bu. of peanuts at $1.20 a bushel. He then roasted them and sold them at 10 cents a quart. What was his profit?

4. Change 14,000 ft. to higher units.

5. What will $\frac{3}{4}$ of a mile of wire fence cost at 8 cents a foot?

6. Change 78,175 sec. to higher units.

7. How many cubic inches in a peck? In a dry quart?

8. My grocer charges me 20 cents for a half peck of potatoes. What should he charge for a bushel?

9. Which is cheaper: buying oranges at 32 cents a dozen, or 55 cents a score? How much is saved on 600 oranges by buying them the cheaper way?

10. 12 days = $\frac{12}{30}$ mo. = .4 mo.
Hence, 5 mo. 12 da. = $\frac{5.4}{12}$ yr. = .45 yr.
Perform the divisions required in the above work.

Reduce the following to decimal parts of a year:

11. 3 mo. 15 da.
12. 5 mo. 18 da.
13. 7 mo. 6 da.
14. 9 mo. 12 da.
15. 10 mo. 15 da.
16. 2 mo. 19 da.

17. .45 yr. = 12 × .45 mo. = 5.4 mo.
.4 mo. = 30 × .4 da. = 12 da.
Hence, .45 yr. = 5 mo. 12 da.
Perform the multiplications required in the above work.

Reduce the following to months and days:

18. .75 yr.
19. .15 yr.
20. .6 yr.
21. .06 yr.
22. .725 yr.
23. .93 yr.

24. In a recent year the record for the running long jump was 24 ft. 7¼ in. Martin can jump 18 ft. 8¾ in. How much farther would he have to jump in order to equal this record?

25. The amateur record for the 25-mile run, in a recent year, was 2 hr. 52 min. 24 sec. What was the average time per mile?

26. How long is the interval from July 14, 1789, to June 18, 1815?

EXERCISE 178

ORAL WORK

Reduce at sight:

1. 2 ft. 9 in.
2. 2 lb. 4 oz.
3. 2 pk. 5 qt.
4. 3 sq. yd. 5 sq. ft.
5. 3 yd. 2 ft.
6. 2 hr. 12 min.
7. 3 T. 125 lb.
8. 5 mo. 12 da.
9. 3 qt. 2 pt.
10. 4 ft. 3 in.
11. 3 mi. 100 rd.
12. 1 cu. yd. 5 cu. ft.

At sight, change to higher units:

13. 80 in.
14. 20 pt. (dry)
15. 100 oz.
16. 25 qt. (liquid)
17. 200 min.
18. 38 da.
19. 14 ft.
20. 29 sq. ft.
21. 29 cu. ft.

22. Practice the oral multiplication and division of one small fraction by another.

For this purpose, let the teacher put on the blackboard an arrangement of small fractions similar to that given on p. 128 (Ex. 1).

141. III. Addition of Compound Numbers. In the addition of compound numbers,

Set similar units in the same column;
Begin at the right and add each column;
Simplify the sum of each column by reduction ascending.

Ex. Add 2 bu. 3 pk. 5 qt., 3 bu. 1 pk. 3 qt., and 5 bu. 2 pk. 7 qt.

bu.	pk.	qt.	
2	3	5	
3	1	3	
5	2	7	
11	3	7	*Sum*

The sum of the quarts is 15 qt., which, reduced to pecks, is 1 pk. 7 qt. We set down the 7 qt. and add the 1 pk. with the column of pecks.

The sum of the pecks is 7 pk., or 1 bu. 3 pk. We set down 3 pk. and add the 1 bu. with column of bushels.

The sum of the bushels is 11.

Hence, the entire sum is 11 bu. 3 pk. 7 qt.

EXERCISE 179

Add:

1. 3 bu. 3 pk. 1 qt.; 2 bu. 3 qt. 1 pt.; and 8 bu. 2 pk. 3 qt. 1 pt.

2. 3 gal. 1 qt. 1 pt.; 5 gal. 3 qt.; and 4 gal. 2 qt. 1 pt.

3. 7 cu. yd. 10 cu. ft. 386 cu. in.; 9 cu. yd. 432 cu. in.; and 8 cu. yd. 1217 cu. in.

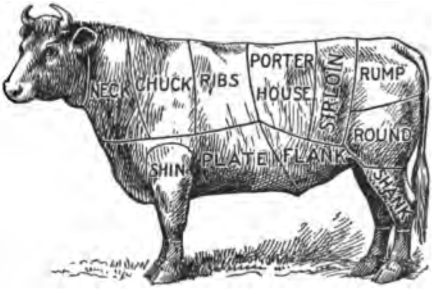
4. 3 T. 18 cwt. 40 lb.; 15 cwt. 35 lb.; and 2 T. 70 lb.

5. A milkman has in different cans 12 qt. 1 pt.; 27 qt. 1 pt.; 28 qt.; and 13 qt. 1 pt. How can he tell how much he has in all, without measuring his milk? Do this for him.

6. On a camping trip, Harry carried 12 lb. 4 oz. in his pack, Walter carried 15 lb. 8 oz., and James 14 lb. 5 oz. How much did they carry all together?

7. If it is high water at New York at 5.12 P. M. on a given day, and high water at New Haven comes 3 hr. 5 min. later, when will it be high water at New Haven?

8. The diagram shows the parts into which it is customary to divide a beef. If the shank weighs 23 lb. 8 oz., the round 125 lb. 8 oz., and the rump 57 lb. 4 oz., how much does the hind quarter of the beef weigh?



9. The sirloin, porterhouse, and ribs are regarded as the choice cuts of a beef. If the sirloin of a beef weighs 59 lb. 8 oz., the porterhouse 57 lb. 12 oz., and the ribs 69 lb. 12 oz., what do the choice cuts weigh together?

146. Measurement is the process of finding how many times a given object contains another given object or quantity of the same kind, taken as a unit.

Thus, in measuring a line we find how many times a foot, or other unit of length, is contained in the line.

Measurement of Areas

147. The Running Foot; The Pace. A lot of land is frequently bought and sold by the number of linear feet in the front of the lot and not by its actual area. In this case the lot is said to be bought or sold by the *running foot.*

Ex. 1. What is the cost of a city lot having a front of 62 ft., at $140 per running foot?

$$62 \times \$140 = \$8680 \textit{ Ans.}$$

How is labor saved by estimating the value of a piece of land by the running foot?

In making a quick, off-hand estimate of the size of a piece of land, the *pace* (= 3 ft.) is frequently used.

One acre = 4840 sq. yd., which is a little less than 4900 sq. yd. Hence,

One acre approximately equals a square each side of which is 70 *paces* (or yards).

Ex. 2. Mr. Stanley wishes to buy a certain rectangular piece of land, provided it contains two acres. On pacing off the land, he finds its length to be 108 paces and its width 35 paces. Will it pay him to hire a surveyor to measure the exact area of the piece of land?

$$\text{Area of land} = 108 \times 35 \text{ sq. yd.} = \frac{\overset{54}{\cancel{108}} \times \cancel{35}}{70 \times \underset{2}{\cancel{70}}} \text{ A.} = \frac{\overset{27}{\cancel{54}}}{\underset{35}{\cancel{70}}} \text{ A.} = \frac{27}{35} \text{ A. } \textit{Ans.}$$

Hence, the area is much less than 1 acre, and Mr. Stanley will not consider the proposition further.

Rough estimates are often made easy by remembering that 1 *square mile* = 640 *acres*.

Ex. 3. How many acres are there in a meadow which averages $\frac{1}{4}$ mile in width and is $\frac{1}{2}$ mile long?

Area of meadow $= \frac{1}{4} \times \frac{1}{2}$ sq. mi. $= \frac{1}{8}$ sq. mi. $\frac{1}{8} \times 640 = 80$ A.

EXERCISE 185

1. Find the cost of a city lot with a frontage of 120 ft. at $16 per running foot. Also at $12.75.

2. Find the cost of a city lot with a frontage of 28 ft. 8 in. at $32.50 per running foot.

3. If a lot has a frontage of 48 ft. and a depth of 120 ft., will it cost more at $.85 per square foot of area, or at $16 per running foot?

How many acres are there in each of the following fields? (Use cancellation.)

4. 250 ft. × 300 ft.
5. 600 ft. × 240 ft.
6. 100 yd. × 180 ft.
7. 500 yd. × 100 yd.

Approximately how many acres are there in a field whose dimensions are as follows? (Use cancellation.)

8. 140 paces × 70 paces
9. 140 paces × 35 paces
10. 630 yd. × 280 yd.
11. 700 yd. × 350 yd.
12. 700 yd. × 280 yd.
13. 147 yd. × 125 yd.
14. 95 yd. × 210 yd.
15. 91 yd. × 123 yd.
16. 300 yd. square.
17. 280 yd. square.

18. Is a field 120 paces × 40 paces greater or less than one acre?

19. Is a field 200 paces × 60 paces greater or less than half an acre? 40 paces × 20 paces greater or less than half an acre?

20. Make up and work three examples similar to Ex. 18.

21. How many acres are there in a field $\frac{1}{4}$ mile × $\frac{1}{4}$ mile?

22. How many acres in a meadow $\frac{3}{8}$ mile × $\frac{1}{2}$ mile? In a rectangular island 7 miles long and $\frac{1}{2}$ mile wide?

23. How many acres in a farm $1\frac{1}{2}$ mile × $\frac{3}{4}$ mile? 2 mi. × $\frac{1}{2}$ mi.? $\frac{5}{8}$ mi. × $\frac{1}{4}$ mi.?

24. Is a tract of land $\frac{3}{8}$ mile × $\frac{1}{4}$ mile greater or less than 100 acres?

25. On squared paper let 1 space = 10 yd. and draw a diagram to represent an acre. (Take 1 A. as 70 yd. × 70 yd.)

26. What is the area of a rectangle 24 ft. by 8 ft.? What is the area of a rectangle of double this length and width? On squared paper draw diagrams showing the two rectangles.

27. What is the area of a rectangle 18 ft. by 12 ft.? Increase each of these dimensions by one half of itself. By what fraction have you increased the area?

28. The national forestry reservation at Billings, North Dakota, contains 14,000 acres. Find in the shortest way the number of square miles in this reservation, and show how to visualize it.

29. Treat in the same way as in Ex. 28 the forestry reservation at Ocala, Florida, which contains approximately 200,000 acres.

30. How many of the examples in this Exercise can you work at sight?

31. Practice the oral subtraction of small two-figured numbers.

148. Townships and Sections. In the eastern part of the United States, land was first divided by the original settlers without any regular system. In the Western and Southern states, however, most of the land was originally owned by the government, and it has been divided according to a systematic plan, and disposed of in this form to settlers.

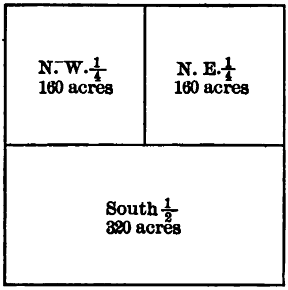
By this plan, land is first divided into townships. A *township* is a square each side of which is 6 miles; hence a township contains 36 square miles. The sides of townships run east and west, and north and south.

Each township is subdivided into 36 sections. Each *section* contains 1 square mile, or 640 acres. The sections in a township are numbered according to a regular plan from 1 to 36.

A section is subdivided into *quarter-sections,* each containing 160 acres.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

A Township



A Section

EXERCISE 186

1. How many acres are there in a township? How many square miles?

2. How many acres in a section? In 2 sections? $3\frac{1}{2}$ sections? $\frac{1}{8}$ of a section?

3. How many rods of fence are necessary to inclose a section?

4. What will it cost to inclose a quarter-section with fence at 80¢ a rod?

5. What is the cost of a quarter-section at $22.50 per acre?

6. Make a diagram of the school grounds and buildings, and from this diagram make up problems concerning areas.

7. How many rectangles are formed by the walls of a room? How many by the walls and ceiling? How many by the walls, ceiling, and floor?

8. How many rectangles are formed by a roof with a single peak or ridge? Draw a picture to show this.

9. Work again Exercise 118 (p. 215).

10. Practice the oral addition of small mixed numbers.

149. Paving and Roofing. In paving, the unit of measurement is the square yard. In roofing, a unit of 100 sq. ft., called a *square,* is sometimes used.

EXERCISE 187

1. A concrete walk is to be 30 ft. long and 12 ft. wide. What will be the cost of making the walk at 90¢ a square yard?

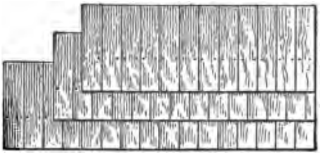
2. Find the cost of shingling a roof 48 ft. × 23 ft. at $8.45 for every 100 sq. ft.

3. Find the cost of paving a city street 3 mi. long and 40 ft. wide at 80¢ a square yard.

4. Find the cost of tinning a roof 24 ft. long and 18 ft. 6 in. wide at $1.10 a square yard.

5. If a roof is 32 ft. long, how many shingles 4 in. wide will be required to make a row? If each row is exposed 6 in., how many

rows will be required, if the roof is 14 ft. wide? How many shingles will be needed for the whole roof? What will they cost at $18 a thousand?



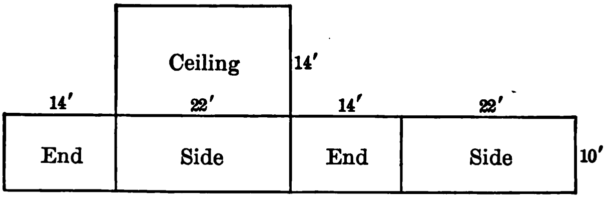
6. It takes 36,000 shingles, each 4 in. wide, to roof a certain house. How many shingles 3 in. wide would be needed?

7. Practice the oral subtraction of small mixed numbers.

150. In **Plastering and Painting,** the unit of measurement is the square yard. In computing the area of a wall, in some cases the area of doors and windows is deducted from the area of the wall. In other cases no deduction is made. No deduction is to be made in the following examples unless it is mentioned.

Ex. What will it cost to plaster the walls and ceiling of a room 22 ft. long, 14 ft. wide, and 10 ft. high at 33¢ a square yard?

The walls and ceiling may be indicated as follows:



The walls (that is, the ends and sides combined) make a rectangle 2 × 22 ft. + 2 × 14 ft., or 72 ft. long.

$$\begin{array}{lll} \text{Hence, area of walls} & = 72 \times 10 \text{ sq. ft.} = & 720 \text{ sq. ft.} \\ \text{Area of ceiling} & = 22 \times 14 \text{ sq. ft.} = & \underline{308 \text{ sq. ft.}} \\ & \text{Total area} = & 1028 \text{ sq. ft. or } \frac{1028}{9} \text{ sq. yd.} \end{array}$$

$$\text{Cost of plastering} = \frac{\$\overset{11}{\cancel{.33}} \times 1028}{\underset{3}{\cancel{9}}} = \$37.69 \textit{ Ans.}$$

EXERCISE 188

In working each of the following examples draw a diagram similar to that used in the example of Art. 150.

Find the cost of:

1. Plastering a room 20 ft. long, 14 ft. wide, and 9 ft. high at 30¢ a square yard.

2. Kalsomining a room 24 ft. long, 16 ft. wide, and 9 ft. high at 6¢ a square yard.

3. Painting the walls and ceiling of a room 18 ft. long, 12 ft. 6 in. wide, 8 ft. high at 12¢ a square yard.

4. Plastering a room 20 ft. long, 12 ft. 8 in. wide, 9 ft. 6 in. high at 36¢ a square yard.

5. Plastering a room 18 ft. 6 in. long, 12 ft. 6 in. wide, 8 ft. high at 36¢ a square yard, if 8 sq. yd. are deducted for doors and windows.

6. What is the cost of plastering a room 20 ft. long, 15 ft. wide, 10 ft. high at 36¢ a square yard, if we deduct 2 sq. yd. each for 3 doors and $1\frac{1}{2}$ sq. yd. each for 4 windows?

7. Find the cost of plastering in your neighborhood and make up and work three different examples concerning it.

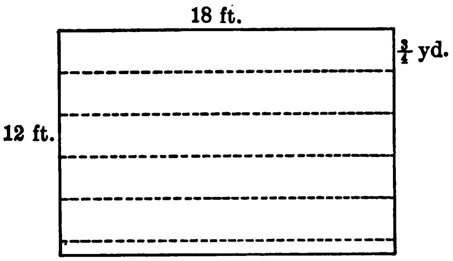
8. Practice the oral multiplication of small fractions both by small integers and by small fractions.

151. Carpeting. Carpet and matting are bought by the *running yard,* that is, according to the length of the material and without regard to its width. (See Art. 147, p. 320.)

To determine the number of yards of carpet needed to carpet a given room, it is necessary to

Determine the number of strips which the room will require; and to multiply the length of each strip by the number of strips.

A strip of part width counts the same as a whole strip. When the carpet is figured, in order to match the figures, strips of carpet must usually be made a little longer than the length of the room. The unused ends are cut off or folded under.



Ex. A room 12 ft. wide will require how many strips of carpet $\frac{3}{4}$ yd. wide and running lengthwise? If the room is 18 ft. long, how many yards of carpet will be needed?

12 ft. $= 4$ yd.

Number of strips $= 4 \div \frac{3}{4} = 4 \times \frac{4}{3} = 5\frac{1}{3}$, or 6 strips.

Length of 1 strip $= 18$ ft. $= 6$ yd.

Number of yd. of carpet $= 6 \times 6$ yd. $= 36$ yd. *Ans.*

EXERCISE 189

In working each of the following examples, draw a diagram like that used in the example of Art. 151.

1. How many strips of ingrain carpet 1 yd. wide are needed in a room 20 ft. wide?

2. How many strips of Brussels carpet $\frac{3}{4}$ yd. wide would be needed in the room of Ex. 1?

3. How many strips of matting are needed in a room 17 ft. 6 in. wide? If the room is 22 ft. long, how many yards of matting are needed?

4. If the room of Exs. 1 and 2 is 30 feet long, how many yards of ingrain carpet would be required? What would this cost at 80¢ a yard?

How many yards of Brussels carpet? What would this cost at $1.20 a yard?

5. How many yards of carpet 36 in. wide are needed for a room 19 ft. wide and 25 ft. 6 in. long?

6. How many yards of carpet 30 in. wide are needed for the room of Ex. 5?

7. Make up and work two examples concerning carpeting.

8. Practice the oral division of small fractions by other small fractions and by small integers.

152. Papering. The unit used for wall paper is the *roll.* A roll is 8 yards long and 18 inches (or $\frac{1}{2}$ yd.) wide. The double roll, 16 yards long and 18 inches wide, is also used at times.

A roll will cover from 30 to 34 sq. ft. of wall surface, the variation being due to losses in matching patterns, turning corners, and fitting door and window frames.

A part of a roll counts as a whole roll, since only entire rolls are sold.

EXERCISE 190

Draw a diagram showing the walls and ceiling in each of the following examples.

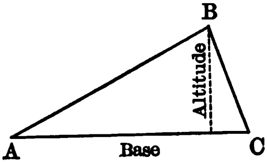
1. A room is 20 ft. long, 12 ft. wide, 8 ft. high. How many rolls of paper are needed for the walls and ceiling of the room, if one roll covers 32 sq. ft. of surface? What will this paper cost at 25¢ a roll? (What is the short way of multiplying by 25?)

2. A room is 25 ft. long, 12 ft. 6 in. wide, 8 ft. high. At $16\frac{2}{3}$¢ per roll, find the cost of paper for the walls and ceiling of the room, allowing a roll for every 30 sq. ft. of surface.

3. A room is 18 ft. long, 15 ft. wide, 10 ft. high. Paper costing 30¢ a roll is used for the walls, and paper costing 20¢ a roll for the ceiling. If one roll covers 32 sq. ft., what will be the cost of the paper for the entire room?

153. A triangle is a figure with three sides.

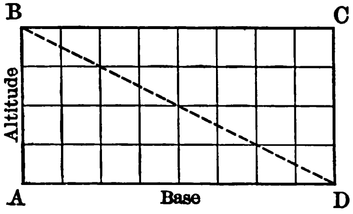
A triangle is named by letters placed at its corners, as ABC.



The **base** of a triangle is the side upon which it is supposed to stand, as AC.

The **altitude** of a triangle is the perpendicular drawn to the base from the opposite corner.

From squared paper cut out a rectangle ABCD 8 spaces long and 4 spaces wide. Draw a line from B to D. Cut the rectangle along the line BD. Place one of the triangles thus formed upon the other to show that one triangle exactly covers the other, and hence that the triangles are equal.

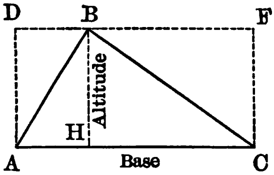


Hence, the area of one triangle (as ABD) $= \frac{1}{2}$ area of rectangle

$$= \frac{1}{2} \times 8 \times 4 = 16.$$

Verify this result for the given triangle ABD by counting the small square spaces which form its area. You find in the triangle ABD 12 unbroken small squares. Piecing together the parts of squares, you get 4 more small squares. This gives 16 small squares in all in the triangle.

Which is easier: to count the squares and parts of squares in ABD in this way, or to find the product of $\frac{1}{2} \times 8 \times 4$?



Draw any triangle ABC, and let BH be its altitude. Complete the rectangle having AC for its base and BH for its altitude, and thus obtain the rectangle ADFC.

$$\text{Then ABH} = \tfrac{1}{2}\text{ ADBH.}$$

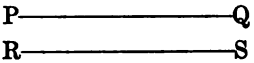
$$\text{Also HBC} = \tfrac{1}{2}\text{ HBFC.}$$

$$\text{Adding, ABC} = \tfrac{1}{2}\text{ ADFC.}$$

But the area of rectangle ADFC = length (or base) × width (or altitude). Hence, the area of the triangle ABC is one half this product.

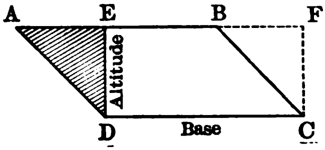
Hence, *area of any triangle* = 1 *sq. unit* × $\frac{1}{2}$ *base* × *altitude.*

154. Parallel lines are lines which do not meet however far they may be extended, as PQ and RS.



On your squared paper draw a pair of parallel lines.

A **parallelogram** is a four-sided figure whose opposite sides are parallel, as ABCD.



Draw the parallelogram ABCD. From it cut off the triangle AED and place this triangle in the position BFC. Show that the parallelogram equals the rectangle EFCD, which has the same base and altitude as the parallelogram. Hence, we find the area of a parallelogram just as we find the area of a rectangle.

Hence, *area of parallelogram = 1 sq. unit × base × altitude.*

EXERCISE 191

Find the area of each of the following triangles:

	Base	Altitude		Base	Altitude
1.	10 ft.	10 ft.	5.	$2\frac{1}{2}$ ft.	16 in.
2.	16 in.	8 in.	6.	5 yd.	3 ft.
3.	2 ft. 8 in.	18 in.	7.	1 mile	$\frac{1}{2}$ mile
4.	2 ft. 4 in.	1 ft. 6 in.	8.	$\frac{3}{4}$ mile	$\frac{3}{5}$ mile

9. In Exs. 7 and 8, find the area in acres also.

10. How many square feet are there in the gable end of a barn, if the gable is 20 ft. wide and 12 ft. high? (The gable is a triangle.)

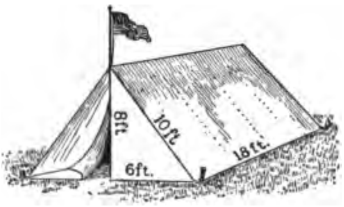
11. One side of a triangular piece of ground is 240 ft. long and the perpendicular to that side from the opposite corner is 186 ft. long. What is the piece of ground worth at $82 an acre?

Find the area of each of the following parallelograms:

	Base	Altitude		Base	Altitude
12.	30 in.	20 in.	**15.**	300 ft.	200 ft.
13.	12 ft.	18 in.	**16.**	124 ft. 6 in.	200 ft.
14.	14 ft.	8 ft. 4 in.	**17.**	24.62 ft.	16.84 ft.

18. In Exs. 15–17, find the area in acres.

19. Dudley wishes to make a tent like this for camping in the woods. How many triangles of canvas are there in the tent? How many parallelograms? What will the canvas for the tent cost him at 30¢ a square yard?



20. The delta of the Nile is approximately a triangle with a base of 100 miles and an altitude of 88 miles. Find the area of this triangle in square miles and in acres.

21. Above the delta, the fertile strip along the Nile is 500 miles long and 11 miles wide on the average. How many more acres are there in this strip than in the delta?

22. Make up and work an example similar to Ex. 11.

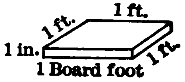
23. How does the amount of fertile land in Egypt (as shown in Exs. 20, 21) compare with the area of New Jersey? (See p. 386.)

24. Practice the oral multiplication of small decimal fractions by other small decimal fractions.

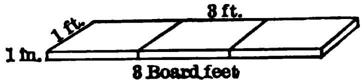
Measurement of Volumes

155. Board Measure. In measuring boards and lumber, the unit is the *board foot.* A board foot is a rectangular piece of wood 1 foot square and 1 inch thick. Preferably boards are cut 16 feet long.

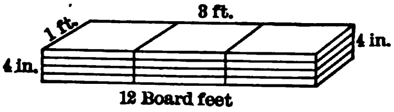
Large quantities of lumber are sold by the *hundred* or *thousand feet*. A thousand feet of lumber are often expressed by the letter *M*.



Boards less than 1 inch in thickness are estimated as if they were 1 inch thick.



The teacher should have on the desk a real board foot, and also, if possible, a piece of plank 1 foot square, and 2 or 3 inches thick.



Where no error is likely to result, the term "board foot" is contracted into "foot."

Thus, men often say "six thousand feet of white pine" instead of "six thousand board feet of white pine lumber."

To find the number of board feet in a given board or piece of lumber,

Multiply two of the dimensions expressed as feet by the other dimension expressed as inches.

In computations concerning board feet, labor is often saved by the use of cancellation.

Ex. How many board feet are there in a plank 16 ft. long, 10 in. wide, and 3 in. thick?

$$\text{The width} = 10 \text{ in.} = \tfrac{10}{12} \text{ ft.}$$

$$\text{No. board feet} = \frac{\overset{4}{\cancel{16}} \times 10 \times \cancel{3}}{\underset{\cancel{4}}{\cancel{12}}} = 40 \text{ board feet} \quad \textit{Ans.}$$

EXERCISE 192

1. How many board feet are there in 1 cu. ft.? Draw a diagram showing this.

2. How many board feet in 2 cu. ft.? In $2\frac{1}{2}$ cu. ft.?

3. How many board feet in a board 5 ft. long, 1 ft. wide, and 1 in. thick? Draw a diagram.

4. How many board feet are there, if the board in Ex. 3 is 2 in. thick? 3 in. thick? Draw a diagram for each.

Find the number of board feet in a piece of lumber measuring

5. $16' \times 10'' \times 1''$

6. $16' \times 8'' \times 1''$

7. $16' \times 8'' \times 2''$

8. $8' \times 15'' \times 3''$

9. $8\frac{1}{2}' \times 1' \times 1''$

10. $12\frac{3}{4}' \times 8'' \times 2''$

11. $10.5' \times 1.5' \times 2''$

12. $8\frac{1}{2}' \times 1\frac{1}{4}' \times 2''$

13. Which is easier: to compute the number of board feet in a plank $14' \times 2' \times 3''$ by line measurements and multiplication as in the above examples, or to mark the plank with lines to show board feet in it, and count the number of board feet?

14. What is the cost of 12,400 ft. of hemlock at $24 per M?

Compute the cost of the following, using cancellation wherever possible:

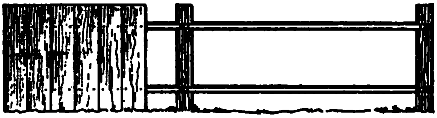
	Number of Pieces	Dimensions	Cost per M
15.	10	$8' \times 4'' \times 2''$	$24
16.	20	$16' \times 10'' \times 1''$	$30
17.	40	$16' \times 10'' \times 3''$	$60
18.	342	$16' \times 8'' \times 1''$	$21.75

Make out a receipted bill for each of Exs. 19 and 20, supplying a place and date:

19. Walter Bowers bought of the Henry R. Kennedy Co., 750 ft. of hemlock at $40 per M; 1865 ft. of yellow pine at $28 per M; 650 ft. of cypress at $42 per M.

20. George Applegate bought of the Henry R. Kennedy Co., 18,600 ft. of cedar at $48; 20,450 ft. of yellow pine at $32.50; 15,400 ft. of white pine at $72; 18,500 shingles at $18 per M.

21. How many feet of lumber are there in the boards used in building a fence 5 ft. high around a garden 60 ft. × 40 ft.?



22. How many feet of lumber in the scantling of the fence in Ex. 21, if the scantling is 2″ × 2″ and two pieces are used all the way round?

23. How many posts are used (in Ex. 21), if the posts are 10 ft. apart?

24. Find the cost of the lumber used in Ex. 21, at the current prices in your neighborhood.

25. If white pine costs $60 per M, what will a board containing 16 ft. cost? A plank containing 40 ft.? 45 ft.?

26. The board walk at Asbury Park and Ocean Grove at one time was 2 miles long, 30 feet wide, and was made of white pine plank 2 inches thick. What was the cost of the plank in it at $60 per M?

27. How many of the examples in this Exercise can you work at sight?

28. How many examples in Exercise 162 (p. 281) can you now work at sight?

156. Capacity of Bins. In measuring the capacity of an ordinary bin, the bushel is used as the unit. One bushel contains 2150.42 cu. in. Hence, to find the number of bushels a bin will hold,

Divide the number of cubic inches in the bin by 2150.42.

Ex. 1. A bin 20 ft. long, 8 ft. wide, and 4 ft. deep will contain how many bushels?

1 cu. ft. contains 1728 in.; hence,

$$\text{No. of bushels} = \frac{20 \times 8 \times 4 \times 1728}{2150.42} = 514.28 \text{ bu. } \textit{Ans.}$$

2150.42 cu. in. = $1\frac{1}{4}$ cu. ft. nearly. In many cases a sufficiently accurate computation of the number of bushels in a bin is obtained by dividing the number of cubic feet in the bin by $1\frac{1}{4}$; that is, by multiplying it by $\frac{4}{5}$. Hence,

The approximate number of bushels in a bin = $\frac{4}{5}$ *the number of cubic feet in the bin.*

Ex. 2. Approximately how many bushels will be contained in a bin 15′ × 8′ × 6′?

$$\text{Approx. no. bu.} = \frac{\overset{3}{\cancel{15}} \times 8 \times 6 \times 4}{\cancel{5}} = 576 \text{ bu. } \textit{Ans.}$$

EXERCISE 193

1. A box 10″ × 8″ × 4″ will hold what fraction of a bushel?

2. How many bushels of oats can be put into a bin 15′ × 7′ × 4′? (Accurate computation.)

3. Answer Ex. 2 by approximate computation.

4. Find the approximate number of bushels of potatoes in a wagon body 12′ × 4′ × 2′.

5. Find approximately the number of bushels in a bin 20′ × 3′ × 4′.

6. Also in a bin 30′ × 6′ × $2\frac{1}{2}$′.

7. A bushel of shelled corn weighs 56 pounds. Can 2 tons of corn be put into a bin 12′ × 4′ × 3′?

8. A bushel of oats weighs 32 pounds. Can 40 tons of oats be shipped in a car which is 8′ × 8′ × 40′?

9. A heap of corn in the ear is estimated as 40 ft. long, 10 ft. wide, 3 ft. deep. How many bushels does the heap contain, if $1\frac{1}{2}$ cu. ft. of corn in the ear equals 1 bu. of shelled corn?

10. If we allow $1\frac{1}{2}$ cu. ft. for each bushel, a building 40′ × 40′ × 90′ will hold how many bushels of wheat?

11. How many examples in Exercise 166 (p. 285) can you now work at sight?

157. Capacity of Cisterns. A gallon contains 231 cu. in. Hence, to find the number of gallons a cistern will hold,

Divide the number of cubic inches in the cistern by 231.

Ex. How many cubic inches in a cistern 14 ft. × 4 ft. × 18 in.?

Multiplying by 12 to change feet to inches, we have

$$\text{No. gallons} = \frac{\overset{2}{\cancel{14}} \times \overset{4}{\cancel{12}} \times 4 \times 12 \times 18}{\underset{\cancel{33}\ \ 11}{\cancel{231}}} = 628\tfrac{4}{11}\ \textit{Ans.}$$

It is also often convenient to use 1 cubic foot as approximately equivalent to the following gallons:

1 cu. ft. = $7\frac{1}{2}$ gal.

EXERCISE 194

In working the following examples, find the exact number of gallons, unless otherwise directed.

Find how many gallons there are in a cistern

1. Containing 120 cu. ft.
2. Containing 8 cu. yd.
3. 12′ × 4′ × 2″
4. 12′ 6″ × 4′ 6″ × 2″
5. 8.5′ × 4.2′ × 2.5′
6. 8′ 6″ × 4.5′ × 2.75′

7. Find the approximate number of gallons of water in a gymnasium swimming pool 70 ft. long and 25 ft. wide, if the water is 6 ft. deep.

8. How long will it take to pump the required amount of water into the tank in Ex. 7, if we pump in 20 gallons per minute?

9. If we put 1000 gallons of water into a tank 20 ft. long and 8 ft. wide, how deep will the water be?

10. An engine pumps 1200 cu. ft. of water out of a mine in an hour. Approximately how many gallons is this per minute?

11. How many examples in Exercise 153 (p. 267) can you now work at sight?

158. Excavations; Masonry. In moving dirt, as in making excavations or embankments, the unit is one cubic yard. A cubic yard is also termed a *load*.

In the measurement of masonry, no uniform method is followed. Sometimes masonry is estimated by the cubic foot; at other times by the *perch*, which is $24\frac{3}{4}$ cubic feet. Some masons compute the contents of a wall by taking the entire length of the outside wall and multiplying it by the thickness and height. They thus make no allowance for corners. Other masons make allowance for corners.

Loose stone is measured by the *cord*, or 99 cu. ft. (= 4 perches).

An ordinary brick is $8'' \times 4'' \times 2''$. From 21 to 23 bricks are used in making a cubic foot of wall (according to the amount of mortar used).

EXERCISE 195

1. What is the cost of excavating a cellar $18' \times 24' \times 6'$ at 30 cents a load?

2. If the diagram in Ex. 1, p. 188, represents the outline of a cellar which is to be 5 ft. deep, how many loads of dirt would be removed from the cellar?

3. What is the length of wall around this cellar in Ex. 2?

4. If this wall (Ex. 3) is to be 18 in. thick and is to rise 2 ft. above the ground, how many cubic feet will it contain? (No allowance is made for corners.)

5. How many bricks will be needed to build the wall (Exs. 2–4), if 22 bricks per cubic foot are used? What will these bricks cost at $9 per M?

6. How many cartloads of earth are needed to cover 2 ft. deep a lot 60′ × 40′?

7. At $6.25 per cord, what will be the cost of the stone used in building a wall 6 ft. high and 1 ft. 6 in. thick around the lot in Ex. 6? (No allowance for corners.)

8. See how many of the examples on pp. 261, 262 you can now work at sight.

EXERCISE 196

REVIEW

1. Find the area of a triangle with a base measuring 3 yd. and an altitude of 1 ft. 6 in. Draw a diagram of such a triangle to a convenient scale on squared paper.

2. A triangular flower bed is to have an area of 12 sq. yd. and a base of 8 yd. What will the altitude of the triangle measure?

3. In a room 18′ × 12′ × 8′, what will be the cost of plastering at $33\frac{1}{3}$¢ per sq. yd.? Of papering the walls, if 1 roll costing 25¢ is allowed for 32 sq. ft., and $4.50 is paid for labor?

4. If a ton of coal fills 36 cu. ft., how many tons will be contained in a bin 20′ × 6′ × 6′?

5. Postage on books is $\frac{1}{2}$¢ per oz. If I mail a book weighing 1 lb. 3 oz., what postage must I pay? What stamps shall I use, if I use the smallest number possible?

6. A single tennis court is 78′ × 27′; a standard swimming pool is 75′ × 30′. Which has the greater area? How much greater? On squared paper draw diagrams of the two, overlapping so as to show the difference in shape and size between the two.

7. 150 bushels of corn were bought at 75¢ a bushel. After being cracked at a cost of $1\frac{1}{2}$¢ a bushel, the corn was sold at the rate of $1.53 per hundred pounds. What was the gain?

8. How many pounds of cotton are there in 13,500,000 bales, if 1 bale averages 500 lb. in weight?

9. If a cow yields 8000 lb. of milk in a year, how many quarts per day is this, if 1 qt. of milk weighs 2.3 lb.?

10. A recent record for the mile run is 4 min. $15\frac{3}{5}$ sec. At this speed, what is the time for 100 yd.? How much greater is this than the record for 100 yd. ($9\frac{3}{5}$ sec.)?

11. A recent record for the running high jump is 6 ft. $6\frac{3}{4}$ in.; for the pole vault, 12 ft. $10\frac{1}{4}$ in. By how much does the latter exceed the former?

12. How many seconds will it take sound to travel one mile? (See p. 389.)

13. Count from 1 to 20 slowly so that the interval between each two numbers shall be 1 second. How far would the noise of an explosion travel through the air while you were counting?

14. A steamship traveling at an average rate of 13.5 miles per hour received a wireless message when it was 960 miles from New York. How many days and hours sooner did the passengers receive the news than if they had not learned it until they reached New York?

15. Four bales of cotton were raised on one acre. If these bales contained 498, 505, 495, and 503 pounds respectively, what was this cotton worth at 13¢ a pound?

16. A given locomotive weighs 410,000 lb. How many tons is this?

17. This locomotive (in Ex. 16) can pull 10,000 tons. How many freight cars would this be, if the cars averaged 40 tons in weight and the freight carried in each averaged 50 tons?

18. How many examples on pp. 267, 268 can you now work at sight?

EXERCISE 197

INDUSTRY REVIEW: COAL MINING

1. In West Virginia, a mine operator bought a tract of $27\frac{2}{3}$ acres of coal land for $40,000. What did it cost him per acre?

2. Underneath the surface he found a vein of coal 6 ft. 3 in. thick. How many tons of coal would be in one acre, allowing 40 cu. ft. for 1 ton? If in the process of mining $\frac{2}{5}$ of this coal were wasted or left in the mine as supporting columns, etc., how many tons would the operator mine from 1 acre?

3. He sold the coal taken from the mine for $2.10 per ton. What were his receipts per acre?

4. The cost of sinking a shaft and of building tunnels was $250 per acre. In mining the coal, 72¢ per ton was paid to the miner, 20¢ per ton was allowed for the cost of handling the coal, and 20¢

per ton was allowed for office expenses and superintendence. What was the net profit per acre?

5. If the operator employed 100 miners, each of whom mined 5 tons per day, how long would it take him to mine all the coal in 1 acre, allowing 25 working days to the month?

6. How many trainloads would this coal make, if each train consisted of 30 cars and each car carried 50 tons of coal?

7. If $\frac{1}{16}$ of the area of the United States is coal land, how many square miles of coal land is this? How many acres?

8. If the coal averages 3 feet in thickness in all our coal lands, how many tons of coal do they contain?

9. After all our coal is mined and burned, what have we to take its place?

10. 70 cu. ft. of water per minute falling over a dam 10 ft. high in a river, will make in a year as much electricity or heat as the burning of 50 tons of coal. 1000 cu. ft. of water falling over this dam will equal how many tons of coal? 50,000 cu. ft.? 1,000,000 cu. ft.?

11. A certain stream of water is 100 ft. wide, 5 ft. deep, and flows 1 ft. 5 in. per second. How many cubic feet of water does it send over a dam in a minute? If the dam is 10 ft. high, the water falling over it in one year will be equivalent to how many tons of coal? Is this source of power ever exhausted like a coal mine?

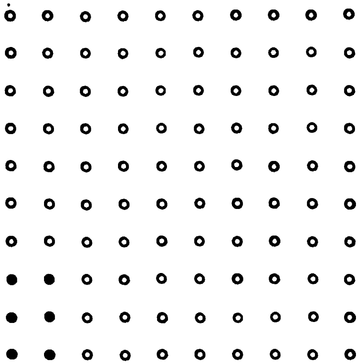
12. If there is a mine, quarry, or oil well which you can visit, find out some facts about the cost of the mine, of labor, etc., and about the receipts. Make examples concerning these facts.

CHAPTER XIII

PERCENTAGE

EXERCISE 198

1. Draw 100 small circles as in the diagram and shade 6 of them.



2. If you should make 200 small circles and shade 6 per hundred, how many would you shade in all?

How many would 6 per hundred be in 300? 400? 250?

3. A short way of writing "per hundred" is "per cent."

Cent is a short form of the Latin word *centum*, which means *one hundred.*

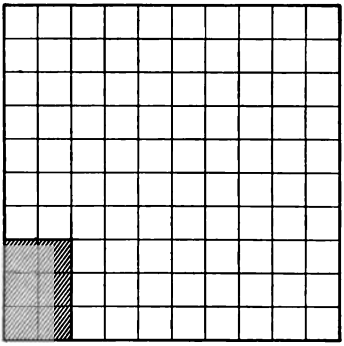
4. What is 6 per cent of 200? Of 300? Of 400? Of 450?

5. On squared paper mark off a square with 10 spaces on each side. (See diagram, p. 343.) What part of the whole figure is one small square? How many hundredths are there in the whole figure?

6. Shade 6 of the small squares. How many hundredths have you shaded? What per cent have you shaded? Is 6 per cent of the whole figure the same as 6 hundredths of the figure?

7. Draw similar diagrams on squared paper to show the meaning of 3 per cent. 8 per cent. 20 per cent. 70 per cent.

8. Mrs. Walters had 350 chickens and sold 8 out of every hundred; how many did she sell in all?



9. In working Ex. 8, it is natural for you to multiply the number of hundreds, or 3.5, by 8. Show that 350 × .08 gives the same result.

The reason for this is that

$$\frac{350}{100} \times 8 = \frac{350 \times 8}{100} = 350 \times \frac{8}{100}.$$

Which of these two processes leaves the number 350 unchanged? Which of the two numbers, 8 or .08, suggests to us that 100 is the basis of the computation?

10. For what two reasons, then, is it better to multiply 350 by .08, than 3.5 by 8?

11. Compute 8 per cent of 250. Also 4 per cent of 250. 6 per cent. 20 per cent.

12. $800 invested in real estate brings in $48 per year. What does each $100 of the $800 bring in?

13. $500 invested in bonds brings in $25 per year. What does each $100 of the $500 bring in?

14. Which of these two investments in Exs. 12 and 13 is more profitable?

15. Why do we now make definitions and rules?

159. **Percentage** is the process of computing with 100 as a basis.

The **rate** per cent is the number of hundredths taken.

The symbol for the words *per cent* is %.

Thus, 6% = $\frac{6}{100}$ = .06.

The **base** is the number of which a certain per cent is taken.

The **percentage** is the number obtained by taking a certain per cent of the base.

Thus, if a farmer has 300 chickens and sells 27 per cent of them, he sells 27 out of every hundred, or 3 × 27 (or 81) out of 3 hundred.

300 chickens = the base.
27% or .27 = the rate.
81 chickens = the percentage.

Be careful not to confuse the rate per cent (the number taken from one hundred) with the percentage (the number taken from all the hundreds).

The rate, base, and percentage are sometimes represented by r, b, and p respectively.

160. Uses of Percentage. The use of 100 as a basis of comparison makes it easy to compare different rates of gain (or loss). (See Exs. 12, 13, and 14, p. 343.) This advantage leads to other advantages which will be understood later.

161. I. Given the Base and Rate, to Find the Percentage.

Ex. 1. In a school of 225 pupils, 36% are boys. How many boys are there in the school?

$$\begin{array}{rl} 225 & \textit{Base} \\ \underline{.36} & \textit{Rate per cent} \\ 1350 & \\ \underline{675} & \\ 81.00 & \textit{Percentage} \qquad \text{81 boys } \textit{Ans.} \end{array}$$

Hence, to find the percentage,

Multiply the base by the rate expressed decimally.

Or use the symbols, $p = b \times r$.

EXERCISE 199

1. On squared paper mark off a square 10 spaces on a side. Shade 5 small spaces. What per cent of the whole square have you shaded? What %? Shade 3 more small spaces; what per cent of the large square have you now shaded?

2. Walter takes care of the chickens and receives 7 out of every hundred that are sold. In one year 400 were sold. How many did Walter receive?

3. Mary receives 28 eggs out of every hundred eggs that she finds. In one month she found 450 eggs. How many eggs did she receive?

4. If a lawyer is given $5 out of every hundred dollars that he collects, how many dollars would he receive for collecting $200? $300? $800? $850?

5. Find $\frac{3}{100}$ of $400. Find .03 of $400. Find 3% of $400.

6. Show that 860 × .06 gives the same result as 8.60 × 6. In working examples in percentage, for what two reasons is it better to use the first of these two forms?

7. Show that 8.75 × 6 = 875 × .06. That 10.40 × 5 = 1040 × .05. That $37.26 × 23 = $3726 × .23.

Find:

8. 4 per cent of 600. Of 8500. 3780 420 in.

9. 12% of $3700. Of $262. 3160 lb. 5280 ft.

10. 37% of 480. Of 819 gal. $31.40 4263

11. 80% of 6000. Of 9200. $10,000 $112.36

12. 23½% of $4000. Of 1760 yd. 4400 lb. $37.26

13. Make up and work an example similar to Exs. 9–12.

14. If I pay $2 tax on every $100 that I am worth, how much tax do I pay on $2250?

15. Mr. X bought a horse for $120 and, after training him for a year, sold him at an advance of 40%. What was the selling price?

16. $8500 is received for a crop of cotton, and the expense of raising the crop is 73% of this sum. How many hundredths of the crop is profit? How many %? How many dollars?

17. Mr. Wylie had an income of $4500. Of this, he gave 10% to general benevolence and 15% more to his mother. How many hundredths of his income did he give away in all? How many %? How many dollars?

18. A boy had 72 marbles and lost 25% of them. How many marbles did he have left?

19. A bushel of potatoes weighs 60 lb. If 78% of potatoes is water, how many pounds of water are there in a bushel of potatoes?

20. Draw a line 1 foot long and at one end make 50% of the line red, and at the other end make 25% of it blue. What per cent of the line is left uncolored?

21. The income of an unscientific farmer from his crops was $6450. Later, by planting crops in rotation, he increased his yield by 25%. How much money did he gain by using the new method?

22. Which would you rather have: 21% of $3876.42, or 17% of $4550.70?

23. How many examples in Exercise 185 (p. 323) can you now work at sight?

EXERCISE 200

Oral Work

Find:

1. 10% of 400. Of 350. 270 110
2. 3% of 700. Of 2000. 900 50
3. 20% of 80. Of 100. 400 60
4. 30% of 60. Of 600. 300 1000
5. About how much shorter is it to write 6% than 6 per hundred?
6. Read Exs. 1–4, using "hundredths" instead of %.
7. Out of 75 examples, a boy solved 80% correctly. How many examples did he solve correctly?
8. A man whose salary was $2000 received an increase of 5%. How many dollars was this?
9. 100% = how many hundredths? Find 100% of $250. Of $300. Of $100.
10. What is the difference between rate per cent and percentage? Show this by an example.
11. What does the letter *b* stand for in percentage? *r*? *p*? Show the value of *b*, *r*, *p* in Ex. 7. In Ex. 8.

162. Special Case. When the rate per cent is an aliquot part of 100 (see Art. 69, p. 121), the process of computing the percentage may often be shortened.

Thus, $33\frac{1}{3}\% = .33\frac{1}{3} = \frac{1}{3}$. Hence, if we wish to find $33\frac{1}{3}\%$ of a number, instead of multiplying the given number by $33\frac{1}{3}$, we may take $\frac{1}{3}$ of the number. Similarly,

$6\frac{1}{4}\% = \frac{1}{16}$ $16\frac{2}{3}\% = \frac{1}{6}$ $33\frac{1}{3}\% = \frac{1}{3}$ $62\frac{1}{2}\% = \frac{5}{8}$
$8\frac{1}{3}\% = \frac{1}{12}$ $20\% = \frac{1}{5}$ $37\frac{1}{2}\% = \frac{3}{8}$ $66\frac{2}{3}\% = \frac{2}{3}$
$12\frac{1}{2}\% = \frac{1}{8}$ $25\% = \frac{1}{4}$ $50\% = \frac{1}{2}$ $87\frac{1}{2}\% = \frac{7}{8}$

Ex. A sailing vessel is valued at $24,000 and a man owns $12\frac{1}{2}\%$ of the vessel. What is the value of his share?

$12\frac{1}{2}\%$ of $24,000 = $\frac{1}{8}$ of $24,000 = $3000 *Ans.*

EXERCISE 201

Find in the shortest way:

1. 25% of 44 books
2. $16\frac{2}{3}\%$ of 96 lamps
3. $33\frac{1}{3}\%$ of 750 boys
4. $37\frac{1}{2}\%$ of 640 lb.
5. 50% of 36 miles
6. $6\frac{1}{4}\%$ of $3870
7. $87\frac{1}{2}\%$ of $540.72
8. $66\frac{2}{3}\%$ of 240 bbl.
9. $8\frac{1}{3}\%$ of 540 doz.
10. $62\frac{1}{2}\%$ of $640.48
11. Make up and work three similar examples.

12. If $66\frac{2}{3}\%$ of a certain kind of ore is iron, how much pig iron can be made from 2400 lb. of this ore? From 4800 lb.? 12 tons?

13. A boy in one week sells $40 worth of vegetables for a gardener, receiving for his work $12\frac{1}{2}\%$ of the receipts. How much money does the boy receive?

14. On squared paper mark off a square, each side of which is 10 spaces long. Shade $12\frac{1}{2}\%$ of this square. On a similar square show the meaning of $33\frac{1}{3}\%$. Of $66\frac{2}{3}\%$. On another, the meaning of $37\frac{1}{2}\%$.

15. Work again Exercise 198 (p. 342).

EXERCISE 202

ORAL WORK

Find:

1. $33\frac{1}{3}\%$ of 18. Of 36. 63 90 600
2. $66\frac{2}{3}\%$ of 18. Of 24. 36 300 1200
3. $12\frac{1}{2}\%$ of 8. Of 24. 800 2400 720
4. 25% of 40. Of 64. $80.40 3000 ft. 50 in.
5. 50% of $36. Of 1 ft. 1 yd. 2 lb. 6 oz.

State in lower units:

6. 33⅓% of 1 ft. Of 1 yd. 1 cu. yd. 2 ft.
7. 25% of 1 peck. Of 1 gal. 1 lb.
8. 66⅔% of 1 ft. Of 1 yd. 2 yd. 2 ft. 1 cu. yd. 1 sq. yd.
9. 50% of 1 qt. Of 1 gal. 1 pk. 1 bu. 1 lb. 1 hour.
10. How many examples in Exercise 201 can you work at sight?
11. Practice the rapid addition of columns of figures.

163. Special Case. When the rate % is very small or very large, special care should be exercised.

Thus, ⅛% = ⅛ of 1% = .00⅛ = .00125. Also 225% = 2.25 when expressed decimally.

Ex. How much is ⅛% of $1200?

$1200 *Base*
.00⅛ *Rate*
$1.50 *Percentage*

EXERCISE 203

Find:

1. ½% of $800
2. ¾% of $68
3. ⅔% of $720
4. ⅞% of $3500
5. 215% of $650
6. 640% of $1200
7. 120% of $5000
8. 275% of 3000 men
9. What is ⅛% of $42,000?

10. A city had a population of 8750 some years ago, but its population is now 240 per cent of that number. What is the present population?

11. If I own $400 worth of property and pay a tax of ⅜%, how much money do I pay?

12. Find 300% of \$600. Find 3% of \$600. Find $\frac{1}{3}$%.

13. How many of the examples in this Exercise can you work at sight?

14. Practice the oral subtraction of dates (see p. 229) and of three-figured sums of money (dollars and cents).

164. II. To Find what Per Cent One Number is of Another (*i. e.* given b and p, to find r).

Ex. In a school containing 360 pupils, 198 are girls. What per cent of the pupils are the girls?

To find how many hundredths 198 is of 360, we must divide 198 by 360.

$$\begin{array}{r@{}l} & .55 \text{ or } 55\% \ \textit{Ans.} \\ 360) & \overline{198.00} \\ & \underline{180\,0} \\ & 18\,00 \\ & \underline{18\,00} \end{array}$$

Hence, to obtain the rate per cent,

Divide the percentage by the base.

In symbols, $r = \frac{p}{b}$.

EXERCISE 204

1. On squared paper mark off a rectangle 20 spaces long and 10 spaces wide. How many small squares does it contain? Shade 16 of the small squares. If 16 squares out of 200 are shaded, how many out of each 100 are shaded? What per cent is 16 of 200?

2. If you should shade 18 squares out of 300, how many would this be in each 100? 18 is what per cent of 300?

3. If you should shade 2 squares in every 25, how many would you shade in 50? In 75? In 100? Hence, 2 is what per cent of 25?

4. 3 out of every 20 is how many out of 40? Out of 60? Out of 80? Out of 100? Hence, 3 is what % of 20?

5. 7 is what per cent of 50? 9 is what per cent of 15? Of 17?

6. 6 is what per cent of 25? What per cent of 25 is 7? What per cent of 25 is 9?

7. $\frac{8}{25} = \frac{?}{100}$ $\frac{8}{25}$ = what per cent? $\frac{7}{25}$ = what %?

8. $\frac{11}{50} = \frac{?}{100}$ $\frac{11}{50}$ = what per cent? What % does each of the following equal?

$$\frac{8}{25} \quad \frac{9}{50} \quad \frac{7}{20} \quad \frac{8}{50} \quad \frac{1}{5} \quad \frac{1}{2} \quad \frac{3}{4} \quad \frac{19}{50} \quad \frac{17}{25}$$

9. Make up and work an example similar to Ex. 1. Also one similar to Ex. 3.

10. Mary found 50 eggs one day and received 12 of them for finding them. What per cent did she receive?

11. Walter picked 20 quarts of cherries and received 7 quarts for picking them. What per cent did he receive?

12. Mrs. Walters sold 42 chickens out of 300. What per cent of her chickens did she sell?

13. If a gold coin is 9 parts gold and 1 part other metals, what per cent of the coin is gold?

14. Of a farm of 200 acres, 80 acres are planted with corn, 40 with potatoes, 32 with oats, and the remainder is pasture land. What per cent of the farm is planted with corn? With oats? What per cent is pasture?

15. A man who has $1000 owes $3500. What per cent of his debts can he pay?

16. I bought a farm for $4800 and spent $1200 in improving it. I then sold it for $7200. What per cent did I gain on all that I invested?

17. Of three baseball nines, one won 8 games out of 20; another 9 games out of 21; and a third 7 games out of 16. What is the rate per cent of games won by each ball nine?

18. A player who was 217 times at the bat, made 42 base hits. What was his batting average (*i. e.* rate per cent of hits)?

19. A man had \$2500 at the beginning of a year and \$2775 at the end. What per cent did he gain?

20. The area of Rhode Island is what per cent of the area of New York? (See p. 386.)

21. The area of New York is what per cent of that of Texas?

22. Harriet is offered her choice between 3 out of every 20 eggs which she finds, or 7 out of every 50. Which is the better offer? What per cent is each offer?

23. Which is the larger rate of income: \$48 received from \$800, or \$63 from \$900?

24. One man loses 24 sheep out of 250; another 32 out of 300. Which man has the greater rate of loss?

25. Make up and work an example similar to Ex. 24.

EXERCISE 205

ORAL WORK

Express in per cent:

1. $\frac{1}{2}$ $\frac{1}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{2}{3}$ $\frac{3}{8}$ $\frac{1}{6}$ $\frac{1}{8}$ $\frac{5}{12}$ $\frac{5}{8}$ $\frac{9}{10}$ $\frac{2}{5}$

2. $\frac{1}{7}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{7}{10}$ $\frac{5}{7}$ $\frac{4}{9}$ $\frac{7}{100}$ $\frac{19}{40}$ $\frac{7}{11}$ $\frac{11}{10}$ $\frac{12}{5}$ $\frac{25}{6}$

3. What per cent of 8 is 4? Is 2? Is 1? Is 5?

4. What per cent of 12 is 6? Is 4? Is 8? Is 2?

5. What per cent of 24 is 3? Is 6? 8? 9? 12? 15? 16?

6. 12 is what per cent of 18? Of 24? 30? 48?

7. 15 is what per cent of 25? Of 30? 35? 50? 75?

8. 18 is what per cent of 27? Of 42? 54? 48? 180?

9. What per cent of a foot is 3 in.? 4 in.? 6 in.? 9 in.?

10. What per cent of a pound is 4 oz.? 8 oz.? 12 oz.? 1 oz.?

11. Make up and answer a similar example concerning parts of a bushel. Of a gallon.

12. 7 per score is what per cent? 9 per score? 5 per score?

13. Martha received $\frac{1}{8}$ of the eggs she found. What per cent did she receive?

14. Walter earned $20 and saved $7 of it. What per cent did he save?

15. If you sleep 8 hours out of 24, what per cent of the time do you sleep?

16. Which is easier to decide: the relative size of $\frac{12}{15}$ and of $\frac{18}{20}$, or the relative size of 7% and 8%?

17. What, then, is one of the uses of percentage? Can you state any other use of percentage?

18. How many examples in Exercise 204 can you work at sight?

19. Practice the oral addition of numbers like the following, as the teacher dictates them:

$2.15	$3.08	$5.30	$3.75	$4.27
1.25	1.12	2.20	1.25	1.08

165. III. Given the Rate and Percentage, to Find the Base (*i. e.* given r and p, to find b).

Ex. 1. In a certain school 40% of the pupils are boys. If there are 720 boys, how many pupils are in the school?

40% of the pupils = 720

Hence, 1% " " " = $\frac{1}{40}$ of 720 = 18

and 100%, or all the pupils = 1800 *Ans.*

Ex. 2. In a given school 48 pupils, or $33\frac{1}{3}\%$ of the whole number of pupils, are girls. How many pupils are in the school?

$33\frac{1}{3}\% = .33\frac{1}{3} = \frac{1}{3}$

Hence, $\frac{1}{3}$ of the pupils = 48

Hence, $\frac{3}{3}$, or all the pupils = 3×48 = 144 *Ans.*

EXERCISE 206

1. $36 is 20% of what sum of money?

2. $36 is 40% of what sum of money?

3. $36 is 2% of what sum of money?

4. $36 is 33⅓% of what sum of money?

5. A farmer sold 28 cattle, which was 4% of the number he had. How many did he have?

6. The farmer's wife sold 72 chickens, which was 18% of what she had. How many did she have?

7. In a school 20 pupils were absent. This was 5% of all the pupils enrolled in the school. How many pupils were enrolled?

8. A newsboy sold 40 papers, which was 80% of the number he started with. How many did he start with?

9. 12 is 33⅓% of what number? 66⅔% of what number? 6%? 2%?

10. After losing 25% of his marbles, a boy had 54 marbles left. How many had he at first?

11. 380 pupils are present at school and 5% are absent. How many pupils belong to the school?

12. Make up and work an example similar to Ex. 1. To Ex. 5.

13. Also to Ex. 9. To Ex. 10.

14. How many of the examples in this Exercise can you work at sight?

15. Work again Exercise 39 (p. 76), or similar examples dictated by the teacher.

APPLICATIONS OF PERCENTAGE

166. Commission and Brokerage. It is frequently an advantage to buy or sell goods through an agent.

Thus, a farmer may receive a higher price for potatoes by selling them through an agent in a city, than by selling them himself in his own neighborhood.

An agent who buys or sells general merchandise is called a **commission merchant.** In return for his services, he receives a percentage of the amount of the purchase or sale.

The percentage paid a commission merchant for his work is called **commission.**

The **base** of a commission is the amount of money paid out or received for the goods by the commission merchant.

The **net proceeds** is the sum left after the commission and other expenses have been paid.

Ex. An agent sold $3500 worth of goods and received a commission of 5%. What was his commission?

$$\begin{array}{rl} \$3500 & \textit{Base} \\ .05 & \textit{Rate} \\ \hline \$175.00 & \textit{Commission} \end{array}$$

EXERCISE 207

1. A commission merchant sold a car of lime for $80 and received 3% commission. What was his commission? How much money did he send to his employer?

2. In order to help pay for his education, James sold books during the summer. He sold $280 worth and received 40% commission. How much did he make?

3. An agent sold 420 tons of coal at $6.25 per ton, for a commission of 2%. What were the proceeds? What were the net proceeds, if $36.40 was paid for cartage?

4. A real estate agent sold a house for $8000 and sent the owner $7760. How many dollars did he keep as commission? What rate per cent did he charge for commission?

5. Henry keeps pigeons and in a certain month he has 2 dozen pairs of squabs to sell. He can sell the squabs near home for 15 cents each. A commission merchant in the city will sell them for him at 25 cents each, charging 5% commission, and the expressage will be 75 cents. Which is the better way for Henry to sell his squabs?

6. Henry's father has 300 bushels of wheat to sell. The miller near home will pay him 92 cents a bushel for it. A commission merchant in the city will sell it for 98 cents a bushel, and will charge 3% commission. If the expense of hauling the wheat to the city is $6.20, which way of selling is more profitable to Henry's father?

7. Martha raises chickens and in one month she has 520 eggs to sell. She can sell them near home at 18 cents a dozen, but a commission merchant in the city will sell them for 21 cents a dozen and charge 5% commission. If the expressage on the eggs is 45¢, in which way will she get more for the eggs?

8. Make up and work an example showing what is sometimes gained by selling goods through an agent.

9. An agent paid $1600 for an automobile and sold it for $2000. What per cent of profit did he make?

10. A collector charges 2½% for his services. What is his commission for collecting $2000?

11. A commercial traveler receives a salary of $25 a week and a commission of 2% on all the sales he makes. If in a year he sells $75,000 worth of goods, what is his income for the year?

12. Point out the base, rate, and percentage in Exs. 1–7.

13. Practice oral addition of small mixed numbers.

167. Commercial Discounts. It is the custom of manufacturers and dealers in merchandise to have a fixed or catalogue price of goods, and often to make deductions from the catalogue price.

The **list price** is the price printed in the catalogue.

A **commercial discount** is a deduction from the list price of goods.

The **net price** is the price after the discount has been deducted.

168. Computation of Successive Discounts. In making several successive discounts,

Deduct the first discount from the list price;
Then compute the next discount on the remainder;
Proceed in this way until all the discounts have been made.

Ex. Find the net amount of a bill of $250 for steam heating apparatus on which there were discounts of 40% and 10%, with 2% off for cash.

	40% of $250 = $100	$250 − $100 = $150
Then	10% of $150 = $15	$150 − $15 = $135
Also	2% of $135 = $2.70	$135 − $2.70 = $132.30 *Ans.*

EXERCISE 208

1. A librarian purchases a list of books amounting to $123.80, and is allowed 30% deduction and 4% discount for cash. Find the net cost of the books.

Find the net price for each of the following:

2. $420 with discounts of 25% and 10%.

3. $272.40 with discounts of 30% and 5%.

4. $500 with 40%, 10%, and 2% off.

5. $600 with $\frac{1}{3}$ and $\frac{1}{10}$ off.

6. 200 doz. hoes at $2.75 per dozen with 20% and 5% off.

7. 300 bbl. of flour @ $6.50 with 10% and 2% off.

8. Mr. X is putting steam heat into his house. At one store he can buy the fixtures for $500 with discounts of 30%, 10%, and 5%. At another he can get them for $600 with discounts of 40%, 15%, and 5%. Find the difference in cost at the two stores.

9. Make up and work an example similar to Ex. 8.

10. Find the net amount of the following bill if cash is paid:

PHILADELPHIA, PA., Sept. 2, 1912.

Mr. Albert Maurice,
37 Jackson St., Philadelphia.

Bought of Arnold, Sykes, and Co.
Wholesale Hardware Dealers
87 Markham St.

Terms 3% Cash.

1912						
Sept. 2	2 doz. Axes	$3.50	$7			
	2 doz. Hatchets	1.90	3	80		
	1½ doz. Rip Saws	4.20	6	30		
	1 Keg Nails		3	60	$20	70

Make out a bill for each of the following, supplying date and place. Find the net amount, if cash is paid.

11. Mr. W. B. Rogers bought of Mitchell, Ames, and Co.: 6 doz. cans of corn @ $1.40; 10 doz. cans of tomatoes @ $1.10; and 1000 lb. of sugar @ 6½¢; 3% cash.

12. E. R. Tilton bought of Phillips Bros.: 500 chairs @ 60¢; 50 rockers @ $1.75; 150 tables @ $1.25. Trade discounts 30% and 15%. 3% cash in 10 days.

13. W. R. Prentice bought of Janeway and Co., New York: 6 pieces of muslin, 40 yd. each, @ 8½¢; 7 pieces of calico, 44 yd.

each, @ $7\frac{1}{2}$¢; 8 pieces of broadcloth, 27 yd. each, @ $1.24. Trade discount $33\frac{1}{3}$%. 3% cash in 10 days.

14. Practice the oral subtraction of small mixed numbers.

EXERCISE 209

Review

1. The office expenses in a certain business were $7500 per year. By introducing a card system of keeping accounts, these expenses were diminished 20%. What amount was saved?

2. If you spend 4 hr. 48 min. in school in one day, what per cent of 24 hours do you spend there?

3. Of 1200 eggs put into an incubator, 50% hatched. Of the chickens that were hatched, 10% died. How many chickens were obtained from the 1200 eggs?

4. A boy who was getting $5 a week in wages, spent $50 for tuition at a night school. Afterward, because of his increased skill, his wages were increased to $9 per week. What would he gain in wages in a year of 50 working weeks by this means? This was what per cent of the money invested in an education?

5. It is estimated that there are 299,000 Indians in the United States. What cities in the United States have a population which differs from this number by less than 10%? (Consult the list in your Geography.)

6. In the year 1908 the Chicago National League baseball team won 99 games. This was $64\frac{3}{10}$ per cent of the games played. How many games did the team lose?

7. The largest per cent of games won in any year by any National League team was $79\frac{4}{5}$%, won by Chicago in 1880. If 17 games were lost, how many were won?

8. The time from 1776 to 1898 is what per cent of the interval from 1492 to 1776?

9. The height of the Washington Monument is 555 ft., of the Singer Building (N. Y.) 612 ft., of the Metropolitan Life Building (N. Y.) 700 ft., and of the Eiffel Tower 984 ft. By what per cent is the height of the Washington Monument exceeded by the height of each of the other structures?

10. The number of cubic inches in a gallon is what per cent of the number in a bushel? (See pp. 194 and 197.)

11. If p = \$12 and b = \$240, find r.

Find x in each of the following:

	b	r	p
12.	120	.30	x
13.	200	x	40
14.	250	.05½	x

	b	r	p
15.	8 qt. 2 pt.	.03	x
16.	4 yd.	x	1 yd. 2 ft.
17.	2 T. 4 cwt.	x	1200 lb.

18. A real estate agent sells a house for \$4000 and, after deducting his commission, sends the former owner of the house \$3880. What rate per cent was the agent's commission?

EXERCISE 210

ORAL REVIEW

1. In an examination a boy worked 5 out of 8 examples. Is this more or less than 60%, the passing mark?

2. Answer the question of Ex. 1 for 5 out of 9 examples. For 4 out of 7 examples.

3. Which pays better: an investment of \$400 which yields \$20 a year, or one of \$700 which yields \$35?

4. Find 10% of 200. 10 is what per cent of 200?

5. Find 4% of \$80. \$4 is what per cent of \$80?

6. $40 is what per cent of $200? Find 40% of $200.

7. The number of children of school age in a certain town is 2510. This is $33\frac{1}{3}$% of the whole population. How many of the population are not of school age?

SUG. Since $33\frac{1}{3}$% is $\frac{1}{3}$ and $66\frac{2}{3}$% is $\frac{2}{3}$ of the population, compare $\frac{1}{3}$ and $\frac{2}{3}$.

8. If $37\frac{1}{2}$% of a given number is 96, what is $12\frac{1}{2}$% of the number?

SUG. Compare $\frac{3}{8}$ and $\frac{1}{8}$.

9. If $12\frac{1}{2}$% of a given number is 36, what is 25% of the number? $37\frac{1}{2}$%? $62\frac{1}{2}$%?

10. If $62\frac{1}{2}$% of a given number is 45, what is $12\frac{1}{2}$% of the number? 25%?

11. Find $12\frac{1}{2}$% of 48.

12. 48 is $12\frac{1}{2}$% of what number?

13. How many examples in Exercise 209 can you work at sight?

14. Practice the oral multiplication and division of small decimal fractions.

CHAPTER XIV

INTEREST

EXERCISE 211

1. If I pay $300 for the use of a house for a year, what is the $300 called?

2. Instead of renting a house, Mr. A borrowed money and built a house to live in. Ought Mr. A to pay for the use of money thus borrowed?

3. If I pay $300 rent for the use of a house for 1 year, what ought I to pay for its use for 2 years?

4. If Mr. A pays $300 for the use of borrowed money for 1 year, what ought he to pay for its use for 2 years? For 3 years? For 3 years 6 months?

5. A young man borrowed $1000, which he used in getting an education. What do we call the money he pays for the use of the $1000?

6. If he pays $60 interest for the use of the $1000 for 1 year, what ought he to pay for its use for 2 years? For 3 years? For 4 years 3 months?

7. After he had finished his course of study, the yearly wages of the young man increased from $500 to $1800. Did he gain by borrowing the money and paying interest on the loan till he could repay the loan?

Can you name some other ways in which borrowing money and paying interest on it have probably been profitable to the young man?

8. What dangers are connected with borrowing money?

9. Why do we now make definitions and rules?

169. Interest is money paid for the use of money.

Interest is usually reckoned as a certain per cent of the money borrowed. This per cent is paid each year, and for parts of a year in proportion.

Hence, interest is a special kind of percentage.

Interest differs from percentage in that the element of time is to be carefully considered in connection with every problem.

The **principal** is the sum of money on which interest is paid.

The **rate** is the per cent of the principal paid for the use of the principal for one year.

The **time** is the number of years for which interest is paid.

The **interest** is the sum of money paid for the use of the principal for the given time.

The **amount** is the sum obtained by adding the interest to the principal.

170. Uses of Interest. By lending money and receiving interest on the loan, persons are able to get a larger return from work which they have done in the past.

By borrowing money and paying interest for the loan, persons are often able to get a larger return from the work which they will do in the future. (See Exs. 5, 6, 7, p. 362.)

171. Interest for an Exact Number of Years.

Ex. What is the interest on $350.75 for 3 years at 5%?

$$\begin{array}{rl} \$350.75 & \textit{Principal} \\ .05 & \textit{Rate} \\ \hline \$17.5375 & \textit{Interest for 1 yr.} \\ 3 & \textit{Time} \\ \hline \$52.6125, \text{ or } \$52.61 & \textit{Interest for 3 yr.} \end{array}$$

In computing interest, 5 or more mills in the final result count as 1 cent; less than 5 mills is usually rejected. During the computation, however, the mills are retained.

172. Interest for Years and Months.

In computing interest for a number of years and months,

Express the number of months as the fraction of a year;
Find the interest for 1 *year and multiply it by the number of years.*

Ex. Find the interest on $350 for 2 years 3 months at 5%.

2 yr. 3 mo. = $2\frac{1}{4}$ yr.
$350 *Principal*
.05 *Rate*
$17.50 *Interest for 1 yr.*
$2\frac{1}{4}$
$437\frac{1}{2}$
3500
$\$39.37\frac{1}{2}$, or $39.38 *Interest for 2 yr. 3 mo.*

EXERCISE 212

1. Find the interest on $380 for 2 years at 5%. At 6%. 7%.
2. Find the interest on $1060.50 for 3 years at 5%. At 7%.
3. Find the interest on $472.75 for 4 years at 6%. At $4\frac{1}{2}\%$.
4. Find the interest on $372.50 at 5% for 3 years. For 2 years. 7 years.

Find the interest on

5. $300 for 5 yr. 3 mo. at 6%. 7%. 4%.
6. $1240 for 2 yr. 6 mo. at 3%. $4\frac{1}{2}\%$. 6%.
7. $587.50 for 7 years at 6%. 5%. 7%.
8. $273.94 for 5 years at 3%. 6%. $3\frac{1}{2}\%$.
9. $870 for 4 yr. 3 mo. at 4%. 5%. 2%.

10. \$1360 for 10 yr. 7 mo. at 6%. 7%. 4%.

11. \$325.40 for 3 yr. 4 mo. at 3%. 5%. $2\frac{1}{2}$%.

12. \$93.75 for 7 yr. 8 mo. at 7%. 8%. $4\frac{1}{2}$%.

EXERCISE 213

ORAL WORK

Find the interest on

1. \$200 at 6% for 1 year. For 2 years. 3 years.
2. \$300 at 5% for 2 years. For 1 year. 1 yr. 6 mo.
3. \$1000 for 1 year at 6%. At 5%. For 2 years at 3%.
4. \$80 at 5% for 1 year. For 2 years. 3 years.
5. \$50 at 6% for 2 years. For 3 years. 2 yr. 6 mo.
6. \$1 at 6% for 1 year. For 2 years. 2 yr. 6 mo.
7. \$200 at 6% for 6 months. For 3 months. 2 months.
8. How many examples in Exercise 162 (p. 281) can you now work at sight? In Exercise 166 (p. 285)?

173. One method of computing **interest for years, months, and days** is to

Find the interest for the given number of years, then for the months, then for the days, and take the sum.

In computing interest, it is customary to regard the years as containing 12 months, and 1 month as containing 30 days.

Ex. Find the interest on \$312 for 2 yr. 7 mo. 15 da. at 5%.

$$
\begin{array}{lllll}
 & & \$312 & \textit{Principal} & \\
 & & .05 & \textit{Rate} & \\
\cline{3-3}
\text{Int. for 1 yr.} & = & \$15.60 & & \\
\text{Int. for 1 mo.} & = & \frac{1}{12} \text{ of } \$15.60 & = & \$1.30 \\
\text{Int. for 2 yr.} & = & 2 \times \$15.60 & = & \$31.20 \\
\text{Int. for 7 mo.} & = & \frac{7}{21} \times \$15.60 & = & 9.10 \\
\text{Int. for 15 da.} & = & \frac{1}{2} \times \$1.30 & = & .65 \\
\hline
\text{Int. for 2 yr. 7 mo. 15 da.} & & & = & \$40.95 \ \textit{Ans.}
\end{array}
$$

EXERCISE 214

Find the interest on

1. \$500 for 2 yr. 3 mo. 10 da. at 6%. At 5%.
2. \$360 for 3 yr. 4 mo. 20 da. at 6%. At 4%.
3. \$48 for 7 yr. 6 mo. 15 da. at 4%. At 7%.
4. \$560 for 5 yr. 8 mo. 24 da. at 5%. At 4½%.

When we wish to find the interest on a certain principal from one date to another date, it is necessary first to find the difference in time between the dates.

5. Find the interest on \$5600 from July 4, 1910, to Dec. 4, 1920, at 6%.

6. What is the interest on \$2500 from Oct. 25, 1915, to Mar. 10, 1930, at 5%?

7. A note dated Dec. 5, 1906, for \$4200, drawing interest at 4%, was paid June 13, 1912. What was the amount of principal and interest?

8. An inventor borrowed \$5000 at 6% to enable him to put an invention on the market. At the end of 3 yr. 2 mo. 6 da., he sold his invention for \$16,000. How much did he make by the transaction?

9. How many examples in Exercise 185 (p. 321) can you now work at sight?

174. Interest for an Exact Number of Days. When the time does not exceed a few months, it is customary to determine the fraction of a year by counting up the exact number of days and dividing by 360 or 365.

The United States government divides the exact number of days by 365 and thus computes what is called *exact interest.* Banks usually divide by 360. Ascertain the custom of your local banks in this respect.

Ex. Find the interest on $350 from Aug. 1 to Oct. 3 at 5%.

To ascertain the time, we have, no. days left in August = 30
no. days left in Sept. = 30
no. days left in Oct. = 3
Time in days 63

$$\text{Interest} = \frac{\overset{175}{\cancel{\$350}} \times \cancel{5}}{100} \times \frac{\overset{7}{\cancel{63}}}{\underset{\underset{\cancel{8}\ 4}{\cancel{40}}}{\cancel{360}}} = \frac{1225}{100 \times 4} = \$3.06 \textit{ Ans.}$$

Why is it best not to cancel any factor of the 100?

EXERCISE 215

In solving the following examples, use 360 days as a year, unless otherwise directed.

Find the interest on

1. $250 for 48 days at 6%.
2. $312 for 63 days at 5%.
3. $215.50 for 23 days at 6%.
4. $450 for 59 days at $3\frac{1}{2}$%.
5. $1200 for 33 days at 6%.
6. $190.50 from Apr. 19 to July 4 at $4\frac{1}{2}$%.
7. $3000 from Feb. 12 to Apr. 30 at 3%. (Use 1 yr. = 365 da.)
8. $2890 from June 21 to Sept. 22 at 4%.
9. $750 from May 12 to Oct. 20, at 6%.
10. $9672.50 from July 4 to July 14 at 5%.
11. $87.90 from Sept. 22 to Dec. 21 at 9%. (Use 1 yr. = 365 da.)

12. Mr. X, by borrowing $3000 from Feb. 22 to Mar. 20, was able to buy 1000 crates of oranges at $2.90 per crate and sell them at $3.50 per crate. If he paid interest at the rate of 6%, how much did he make by the transaction?

13. Mr. A, by borrowing $6000 on May 1 at 6%, was able to buy a house for $4000 and to spend $2000 on it for repairs. On Sept. 10 he sold the house for $7500. What did he make by the transaction?

14. Work again Exercise 200 (p. 347).

175. Interest in Savings Banks. Interest on deposits as computed in savings banks presents the following peculiarities:

1. The rates of interest are low, usually from 2% to 4%.
2. Interest is not computed on the fractional part of a dollar.

Thus, if you have $21.62 on deposit in a savings bank, interest is computed on $21, not on $21.62.

3. In crediting interest, a fractional part of a cent is rejected.

Thus, $7.958 cents is reckoned as $7.95.

4. Interest is usually payable twice a year, on January 1 and on July 1.
5. If interest is not drawn when due, it is added to the amount on deposit.

Ex. Walter Harper had $80 in the savings bank on Jan. 1, 1912. On Mar. 15 he deposited $12 more, and on May 1, $10 more. What was the balance to his credit on July 1, if the bank paid 4% interest?

Balance Jan. 1, 1912	$80.00
Interest on $80 for 6 mo. at 4%	2.40
Deposit Mar. 15	12.00
Interest on $12 from Mar. 15 to July 1 (3½ mo.)	.14
Deposit May 1	10.00
Interest on $10 from May 1 to July 1 (2 mo.)	.06
Balance July 1, 1912	$104.60

EXERCISE 216

1. On Jan. 1, 1912, Harry had $32 in the savings bank. During the winter by shoveling snow and tending furnaces he made $12, which he deposited on Mar. 1. What did he have to his credit on July 1, 1912, if the bank paid 3% interest?

2. During the summer by picking berries he made $22.50, which he deposited on Sept. 10. What did he have on deposit on Jan. 1, 1913?

3. Hazel had $32.50 in the savings bank on Jan. 1, 1912. During the winter by embroidering and selling handkerchiefs she made $17, which she deposited on Mar. 15. What did she have to her credit on July 1, 1912, if the bank paid 4% interest?

4. During the summer by canning and preserving fruit she made $18, which she deposited on Sept. 1. What did she have to her credit on Jan. 1, 1913? What would be left if on Jan. 1, 1913, she drew out $8.50?

5. Mark had $32.50 in the savings bank on Jan. 1, 1912. If he deposited $6.75 on Mar. 15, $12 on July 15, and $8 on Oct. 1, how much did he have to his credit on Jan. 1, 1913, if the bank paid $4\frac{1}{2}$% interest?

6. Our school had $482.50 in the savings bank on July 1, 1912. It deposited $172.30 on Sept. 15. What would it have to its credit on Jan. 1, 1913, if the bank paid 4% interest?

7. On Jan. 1 Richard had $42.50 in a savings bank paying 4 % interest. He deposited $14 on May 15. Anna had $38.75 in the same bank on Jan. 1, and deposited $18.50 on Feb. 15. Did Richard or Anna have the larger sum in the bank on July 1?

8. Philip received 25 cents a week for spending money. By saving some of this and earning other money, he was able to put

$2 in the savings bank on April 1; $2 on July 1; and $2 on Sept. 1. If the bank paid 3% interest, how much would he have in the bank on the following Jan. 1?

9. Make up and work an example similar to Ex. 8.

10. Practice the oral estimate of the number of acres in a tract of land as in Exs. 19–22 (p. 322).

11. Work again Exercises 204 and 210 (pp. 350, 360).

EXERCISE 217

GENERAL REVIEW

1. Work again Exs. 1–10 of Exercise 117 (pp. 204–205).

2. In 596,938.4275, what is the place value of each figure? The value of the left-hand 5 is how many times as great as that of the right-hand 5?

3. If the right-hand 5 in Ex. 2 is represented by a line $\frac{1}{4}$ inch long, how long a line would represent the left-hand 5, if the length were in proportion to the relative value of this 5?

4. In 596938.4275, how many periods are there? How many items of work does the 7 do? The 8?

5. Work again Exs. 11–20 of Exercise 117 (pp. 205–207).

6. Add 200 pt. and 5 gal. 2 qt. (1) Express the result in a form easy to visualize. (2) Express it in a form easy to multiply by 5.

Find the missing number x in each of the following:

7. $\$72.53 + \$187.37 + \$916.32 + x = \$2000$

8. 2 ft. 5 in. + 5 yd. + 1 ft. 7 in. + 4 yd. 2 in. + x = 10 yd. 2 ft. 3 in.

9. $2\frac{1}{2} + 3\frac{1}{8} + 1\frac{1}{16} + 3\frac{1}{2} + x = 50$

10. Simplify $2\frac{7}{8} + \frac{3}{100} + \frac{4}{25} - 2\frac{1}{2} + .06 + 5\%$

11. On a certain farm the wheat crop for the last 5 years has been 326 bu., 384 bu., 272 bu., 300 bu., and 313 bu. What was the average crop for this period?

This year an improved variety of wheat was used for seed and the crop was 410 bu. What per cent is this above the average for the last 5 years?

Multiply:

12. \$506.72 by 3027

13. $20\frac{7}{8}$ by $5\frac{2}{8}$

14. 2 bu. 1 pk. 3 qt. by 5

15. 2515 by .0018

16. .0018 by 2515 3.1416 by $\frac{22}{7}$

17. Find the interest on \$317.50 for 2 yr. 6 mo. 12 da. at 4%.

18. If the divisor is 243, the remainder 89, and the quotient 357, what is the dividend?

19. If an irrigating ditch is to have a fall of $\frac{1}{8}$ in. per yard, what will be the fall in 300 ft.?

20. Work again Exs. 21–30 in Exercise 117 (p. 207).

21. Simplify $\dfrac{.32 + .58}{20 - 12}$

22. Simplify $\dfrac{1.06 \times .32}{.24 \times 1.8}$

Compute in the shortest way:

23. $340 \times 20\frac{5}{8} - 340 \times 15\frac{1}{2}$

24. $\$725 \times 1.06 - \$725 \times 1.03$

25. $\frac{22}{7} \times 13^2 - \frac{22}{7} \times 12^2$

26. $3.1416 \times 27^2 - 3.1416 \times 15^2$

27. If each person occupies a space 2 ft. square, how many persons can stand in a field 200 ft. by 300 ft.?

28. How many boxes, each 2′ × 20″ × 10″, can be filled from a bin 20′ × 6′ 6″ × $4\frac{1}{2}$″?

29. The ear of corn which in a recent year received the prize as the most perfect ear, sold for $150. If this ear contained 1042 grains, how many cents did each grain sell for?

30. A field planted with tested seed corn yielded a crop at the rate of 96 bu. per acre. An adjoining field planted with untested corn yielded 57 bu. per acre. What would be the difference in profits on 100 acres if corn is worth 70¢ a bushel?

31. Work again Exs. 31–40 of Exercise 117 (pp. 207–208).

32. In $152.3875, remove the 3. What change is made in the value of 8? Of 2? Is there a change in the value of any other figures? How many items of work does the 3 do?

33. If the summer half of the year extends from Mar. 20 to Sept. 22, how many more days are there in the summer half than in the winter half of the year? What per cent longer is the summer half?

34. Find the area of a rectangle $\frac{7}{8}$ in. wide and 2 ft. long.

35. Work again Exs. 41–50 in Exercise 117 (pp. 208–209).

36. One fourth of my money is invested in real estate, one fourth in bonds, and the remainder, $3800, is in the bank. What am I worth all together?

37. If a trolley car leaves the city hall at noon and every 40 minutes afterward, at what time will the next car leave after 3.05 P. M.?

38. Find the weight of a piece of iron 12′ × 2′ × 3″, if 1 cu. ft. of iron weighs 480 lb.

39. A pumping engine makes 3 strokes per minute and lifts 19.3 cu. ft. per stroke. How many gallons does it lift in 1 day?

40. Find the difference between two thousand eight ten thousandths and two thousand and eight ten thousandths.

41. What will be the cost of the poles for a telephone line $4\frac{1}{2}$ mi. long, if the poles are to be 150 ft. apart and cost 70 cents each?

42. By the use of selected cotton seed, the yield of cotton on a certain farm was increased from $\frac{3}{5}$ bale per acre to $1\frac{1}{2}$ bales. What was the increase on 32 acres worth at 12¢ a pound, if a bale contains 500 lb.?

43. At $45 each, how many cattle will be worth the same sum as 144 acres of land at $85 an acre?

44. On squared paper show the meaning of 8%. Of $\frac{1}{8}$%. Of 80%.

45. A train is traveling at the rate of a mile a minute. How many feet is this per second?

46. Mr. Ames has a lot which is 50′ × 120′, and on this lot is a house 20′ × 30′. The house is 20′ from the front of the lot and 10′ from one side of it. Draw a diagram which he can send to a person who is thinking of purchasing the house and lot.

47. What is the Golden Rule of arithmetic? Make up an example showing its value.

48. On squared paper mark off a square 10 × 10, and on this show that $\frac{2}{3} \times \frac{3}{4}$ (of 100) $= \frac{1}{2}$ (of 100).

49. The office charges in a certain business were $6570 per year. By introducing a card system, the charges were reduced 20%. How much was saved per year?

50. The area of the United States proper is three million twenty-five thousand six hundred square miles, and that of Alaska and other outside possessions is seven hundred sixteen thousand five hundred fifty-five square miles. Can you tell the sum of these from the numbers as expressed in words? Now use figures and find the sum.

51. Write three decimal fractions which are easier to write than the equivalent common fractions. Also write three common fractions which are easier to work with than the equivalent decimal fractions.

52. If 6 lb. of bluestone are used in making 50 gal. of Bordeaux mixture for spraying fruit trees, how many pounds are needed to make 1000 gal.?

53. If a motor boat can go 30 miles in 58 minutes, how far will it go in 1 hour?

54. In a certain city in 4 years after the establishment of special courts for juvenile offenders, the number of such offenders diminished from 2761 to 1543. What was the per cent of decrease? (What is the percentage in this example? What the base?)

55. If the exact length of a year is 365.24223 days and the year is taken as 365¼ days, what is the error? In 400 years, would this error amount to more or less than 3 days?

56. Express the year of your birth in the Roman notation. What principles of the Roman notation have you used?

57. Will a measure 7″ × 7″ × 5″ contain more or less than a pint?

58. Find the difference between a 3-foot cube and 3 cubic feet.

59. A certain steamship is approximately 500 ft. long, 40 ft. wide, and 30 ft. deep, and ¾ of its space is available for freight.

If 40 cu. ft. will accommodate one ton of freight, how many tons will the steamship carry?

60. Find the states which differ from New York in area by less than 15%.

61. Find the states which differ from your own state in area by less than 20%.

62. Anna has a flower bed 8 ft. long and 4 ft. wide. If the length and width are each increased by 25%, by how many square feet is the area increased? By what per cent is the area increased? Show the increase on a diagram.

63. What will be the changes in the flower bed of Ex. 62, if the original length and width are each increased 100%?

64. Make up and work an example similar to Ex. 62.

65. From an 8-gallon can of milk, Mr. Swift has sold to various customers 1 gal. 1 pt., 3 qt. 1 pt., and 1 gal. 2 qt. 1 pt. How can he tell how much he has left, without measuring it? Find out how much he has left.

66. Add 3 to both the numerator and the denominator of $\frac{9}{12}$. Is the value of the fraction increased or diminished? How much? Instead of adding, subtract 3 from each and answer the same questions.

67. Make up and solve an example which shows reduction ascending. Another which shows reduction descending.

68. An acre of potatoes whose vines were sprayed to prevent the blight yielded 220 bu. An adjoining unsprayed acre yielded 130 bu. If the spraying cost $2.20 and potatoes were worth 45¢ a bushel, how much was gained by spraying the potatoes?

69. Mr. X lost $\frac{2}{5}$ of his money by a real-estate investment and then had $1600 left. How much did he lose?

70. In July, 1908, relay boy runners carried a message from New York to Chicago in 119 hr. and 22 min. How many days was this? If the distance was 980 miles, what was the average rate per hour? The time per mile?

71. A vein of coal 240 yd. long, 126 yd. wide, and 4 ft. 6 in. thick will contain how many tons, if 40 cu. ft. make a ton?

72. A certain lot is 40′ × 100′. On the lot are a house 20′ × 40′ and a barn 30′ × 15′. What per cent of the lot is covered with buildings?

73. In a foundling asylum in New York City, by the use of pasteurized milk the death rate of small children was reduced from 44.36 per cent to 19.8 per cent. Out of 700 children, how many lives were saved by the use of pasteurized milk? (Point out the long way and the short way of working this example and use the short way.)

74. When the minuend and the difference are given, how do you find the subtrahend? Show your process by an example.

75. When the difference and the subtrahend are given, how do you find the minuend? Illustrate by an example.

76. If India is approximately a triangle whose base is 1800 miles and whose altitude is 1200 miles, what is its area?

Simplify:

77. $\dfrac{5\frac{2}{5} \times 3\frac{1}{3}}{7\frac{7}{8}}$

78. $\dfrac{\frac{5}{8} \times \frac{3}{4}}{4 - 1\frac{3}{16}}$

79. $\dfrac{2\frac{1}{7} - 1\frac{1}{2}}{4\frac{1}{2} - 1\frac{5}{7}}$

80. $\dfrac{2\frac{1}{4} + 1\frac{1}{2} \div \frac{2}{3}}{8\frac{1}{2} - \frac{5}{6} \times \frac{3}{4}}$

81. A honey bee is approximately $\frac{1}{2}$ in. long, $\frac{1}{4}$ in. wide, and $\frac{1}{8}$ in. thick. How many bees are there in a hive which is 18 in. long, 15 in. wide, 15 in. deep, if half the hive is filled with bees?

82. Make up an example to show the use of percentage in telling which of two investments is the more profitable.

83. On Jan. 1, Walter was 4 ft. 8 in. tall and one year later he was 4 ft. 11 in. tall. What was the per cent of his increase in height? In the same time his weight increased from 75 lb. to 86 lb. By what per cent did his weight increase?

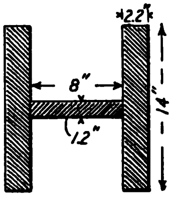
84. If the area of Belgium is 11,373 square miles and the population is 7,500,000, what is the population per square mile?

85. If the United States were populated as thickly as Belgium, what would its population be?

86. A house is 40′ × 20′ and during a storm 2 inches of rain falls on it. If this rainfall all drains into a cistern 10 ft. long and 10 ft. wide, what will be the rise in the cistern?

87. The section of a girder has the form shown in the diagram.

Each of the two end parts is 14″ × 2.2″, and the cross part is 8″ × 1.2″. Find the total area of the cross section.



88. If the United States were divided into 48 equal states, what would be the area of each state?

89. A dam costing $4,000,000 irrigates 250,000 acres of desert land. If the increase in the value of the irrigated land is $120 per acre, the increase in the value of the land exceeds the cost of the dam by how much?

90. If one wolf on the average destroys $1000 worth of cattle, and the United States forest rangers destroy 1500 wolves in one year, what is the value in money of this part of their work?

91. Make a list of all the exact divisors of 3.1416 which are less than 50.

92. A cubic yard of water weighs how much less than a ton?

93. If we increase by 10% each dimension of a bin measuring 10′ × 10′ × 20′, by what per cent is its volume increased?

94. Which is traveling with greater speed: a motor boat that goes 18 miles in 30 minutes, or one that goes 25 miles in 42 minutes?

95. If Palestine is 145 miles long and 45 miles wide, what decimal part of the area of New York State is it?

96. Write seventy-three million six hundred ninety thousand and sixty-eight thousandths in the shortest way.

97. In the shortest way, simplify 2153 + 2153 + 2153 + 2153 + 2153 + 2153.

98. Find in the shortest way how many times $784 can be subtracted from $100,000.

99. Compute $\frac{840 \times 316 \times 210}{720 \times 48 \times 96}$ in the shortest way.

100. Compute 823 × $173 − 623 × $173 in the shortest way.

101. Compute in the shortest way 840 × $.12½. Also 840 ÷ $.12½.

102. Does every process in arithmetic have some advantage or special use connected with it? Give two examples.

EXERCISE 218

GENERAL ORAL REVIEW

1. Work again Exercise 118 (p. 215).

2. Find the value of the following, grouping by 10's, 11's, and 20's as far as possible:

(*1*) 3 + 7 + 5 + 5 + 8 + 12
(*2*) 4 + 7 + 6 + 4 + 7 + 8
(*3*) 11 + 7 + 3 + 5 + 15 + 9
(*4*) 6 + 5 + 4 + 8 + 6 + 2 + 3
(*5*) 7 + 8 + 13 + 2 + 5 + 14

3. Harvard University was founded in 1643. How many years was this after the Pilgrims landed at Plymouth?

4. The following are dates of important inventions: steam engine, 1773; cotton gin, 1793; Fulton's steamboat, 1807; Morse telegraph, 1835; sewing machine, 1846; telephone, 1876.

Give the interval of years between each of these dates and each date following it.

5. With 48 states in the United States, how many United States senators are there?

6. In the electoral college of the United States in 1908 there were 483 votes. How many votes were needed to make a majority?

7. If the area of the United States is taken as 3,000,000 sq. mi. and that of New York as 50,000 sq. mi., how many states of the size of New York could be made out of the total area of the country?

8. If one person in 5 is a voter, how many voters are there in a population of 90,000,000?

9. Write the following list of fractions on the blackboard:

$$\begin{array}{ccccccccc}
\frac{1}{2} & \frac{1}{7} & \frac{1}{12} & \frac{2}{11} & \frac{3}{10} & \frac{4}{11} & \frac{5}{11} & \frac{7}{9} & \frac{8}{11} \\
\frac{1}{3} & \frac{1}{8} & \frac{2}{3} & \frac{3}{4} & \frac{3}{11} & \frac{5}{6} & \frac{5}{12} & \frac{7}{10} & \frac{9}{10} \\
\frac{1}{4} & \frac{1}{9} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} & \frac{5}{7} & \frac{6}{7} & \frac{7}{11} & \frac{9}{11} \\
\frac{1}{5} & \frac{1}{10} & \frac{2}{7} & \frac{3}{7} & \frac{4}{7} & \frac{5}{8} & \frac{6}{11} & \frac{7}{12} & \frac{10}{11} \\
\frac{1}{6} & \frac{1}{11} & \frac{2}{9} & \frac{3}{8} & \frac{4}{9} & \frac{5}{9} & \frac{7}{8} & \frac{8}{9} & \frac{11}{12}
\end{array}$$

Give the product of each pair of the above fractions, as the teacher points.

10. Divide each fraction by every other, as the teacher points.

11. Divide each fraction by 2, 3, 4, 5, 6, and 7.

12. Give the sum of each pair of fractions. Give the difference of each pair.

13. Does a box 20″ × 10″ × 10″ contain more or less than a bushel?

14. Is a field 200′ × 210′ more or less than an acre?

15. How many thousands are there in the product of 310 by 105?

16. Write the following arrangement of numbers on the blackboard. As the teacher points, give the sum of each pair. The difference. The product. The quotient.

.04	2	.001	.9	.07	.3	.2	.06
.003	.01	.1	5	.015	.08	1.2	.05
.8	.008	.02	.009	.006	.6	.09	.4

17. Write the following numbers on the blackboard. As the teacher points, give 5% of each of these numbers:

$100 $1 $200 $250 $1000 200 inches

18. Give 10% of the numbers in Ex. 17. 3% 1% $\frac{1}{10}$% 25%

19. Which is greater: 6% of $300, or 5% of $250?

20. What is the value of a bale of cotton containing 480 lb., at 10¢ a pound? At 12¢? At 15¢?

21. One mule can pull 58 tons in a canal boat. How many mules would be needed to pull the same amount on a macadam road, if 1 mule can pull 2 tons on such a road?

22. Recite the prime numbers between 1 and 100. Time yourself at this.

23. Add:	**24.** Subtract:	**25.** Multiply:	**26.** Divide:
1 qt. 1 pt. 2 qt. 1 pt.	3 qt. 1 qt. 1 pt.	1 ft. 2 in. 3	4)12 ft. 8 in.
1 ft. 6 in. 2 ft. 9 in.	4 bu. 2 pk. 1 bu. 3 pk.	1 qt. 1 pt. 3	3)4 pk. 4 qt.

27. By holding your hands apart in various positions, show how large 1 sq. yd. is. 1 cu. yd.

28. How long will a 40-gal. barrel of spraying material last, if $\frac{1}{2}$ gal. is sprayed per minute?

29. 24 days is what per cent of the month of June?

30. In dividing a fraction by 2, why do we sometimes multiply the denominator by 2?

31. Will a measure 7″ × 8″ × 5″ contain more or less than a peck?

32. 3400 acres of irrigated land is worth how much, at $150 per acre? How many farms of 200 acres each will this land make? What will each farm cost?

33. What important fact in arithmetic is associated with each of the following numbers?

231 1728 2150.42 5280 62.5 1760 320.

34. Divide in the shortest way: $20 by $\$.12\frac{1}{2}$. By $.25. By $\$.33\frac{1}{3}$. By $\$.62\frac{1}{2}$. By $\$.66\frac{2}{3}$.

35. Divide $24 by the numbers in Ex. 34. Also divide $36 by them.

36. At $12\frac{1}{2}$¢ per yard, how many yards of muslin can be bought for $2?

37. Explain, by an example, the difference between a multiple, a common multiple, and a least common multiple.

38. What is meant by a common measure? Give an example.

39. If the corn crop of the United States is 4 times as large as the wheat crop, and a bushel of corn has $\frac{2}{3}$ the value of a bushel of wheat, compare the value of the two crops.

40. If 1 pt. of water weighs 1 lb., 64 pt. would weigh how much? How do you connect this with the fact that 1 bu. of potatoes weighs 60 lb.? Also that 1 bu. of corn weighs 56 lb.?

41. If you double the numerator of a fraction, what effect does this have on the value of the fraction? What effect, if you double the denominator?

42. See if you can add the following in 2 minutes:

(1)	(2)	(3)	(4)	(5)
$13.76	$10.03	$51.72	$17.80	$14.72
2.19	3.96	91.36	40.06	61.93
15.87	42.17	8.40	50.92	17.42
6.03	18.23	17.43	37.62	16.17
9.12	17.05	16.09	18.72	93.05
4.18	7.42	18.27	15.42	17.62

43. Work again Exercise 77 (p. 128).

44. Work Exercise 97 (p. 167).

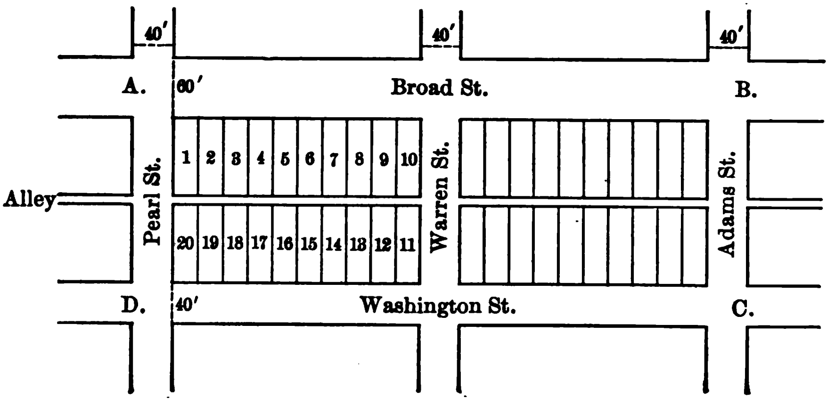
45. Work Exercise 108 (p. 191).

46. Work Exercises 200, 202, and 210 (pp. 347, 348, 360).

EXERCISE 219

INDUSTRY REVIEW: MAKING A HOME

1. Mr. Hart wishes to buy a lot in order to build a house on it. On the diagram each of the lots marked 1, 2, 3, 4, etc. is 30 ft. wide in front and 120 ft. deep. Lots fronting on Broad St. cost \$300 each and the corner lots $\frac{1}{4}$ more. The lots on Washington St. cost \$200 each and the corner lots cost $\frac{1}{4}$ more. What would be the cost of lot 5? Of lot 10? Of lots 7 and 8 together?



2. Mr. Hart bought lots 11 and 12 together. What did these cost him?

3. Later he built a concrete walk 8 ft. wide in front of his lots on both Warren and Washington streets. What did this walk cost him at 80¢ per square yard?

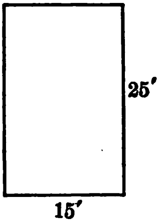
4. If he had to pay 20¢ per running foot for the curbing, what did this cost him?

5. Mr. Hart's building lot is what common fraction of an acre? What decimal fraction of an acre?

6. At the prices stated in Ex. 1, what was received all together for lots 1–20?

7. What would the real-estate dealer have received for lots 1–20, if he had sold the corner lots for $8 per running foot and the other lots for $7 per running foot?

8. Would he receive more by selling them as in Ex. 1 or as in Ex. 7? How much more?



9. Mr. Hart's house had a cellar of the shape and size shown in the diagram. If the cellar was dug 5′ 6″ deep, what was the cost of having it dug at 40¢ a cubic yard?

10. What is the scale of this diagram?

11. The wall around the cellar is to be 18 in. thick and is to rise 2 ft. 6 in. above the ground. How many cubic feet will it contain (no allowance for corners)?

12. How many bricks will be needed to build the wall, if 22 bricks are used per cubic foot? What will these cost at $18 per M?

13. What would it cost Mr. Hart to have the cellar floor covered with concrete 4 in. deep at $4.50 per cubic yard?

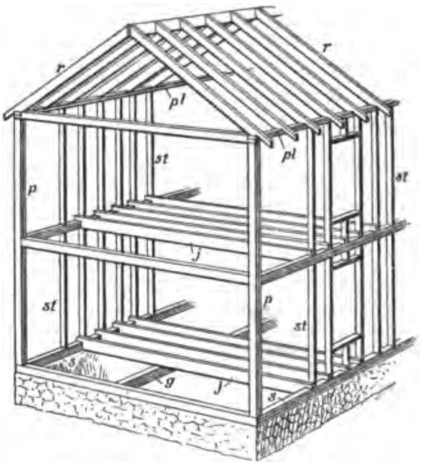
14. Can you find the cost of building such a cellar wall and concrete floor in your neighborhood?

15. The following is part of the bill for lumber which Mr. Hart bought for his house:

2 sills, each 16′ × 4″ × 6″
4 posts, each 18′ × 4″ × 6″
2 plates, each 24′ × 4″ × 4″
56 pieces of studding, each 18′ × 2″ × 4″
17 joists, each 16′ × 3″ × 7″
26 rafters, each 15′ × 2″ × 5″

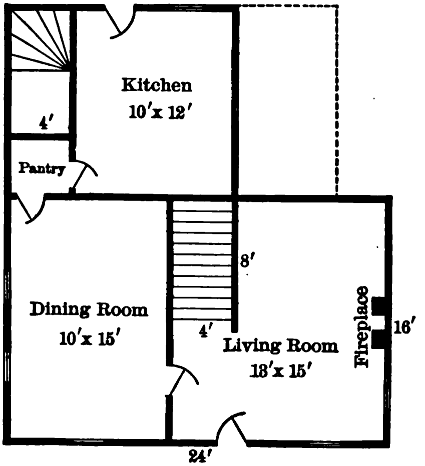
What does this lumber cost at $32 per M? As the teacher names sills, joists, etc., point at one of each on the diagram.

16. The rest of his lumber cost $342.50. The bill for hardware used by the carpenters was $37.50. Mr. Hart paid his carpenters $312.75 for their labor. What had his house cost, when the carpenters had finished and before the plasterers began work?



g = girder *j* = joist *p* = post *pl* = plate
r = rafter *s* = sill *st* = studding

17. If the ceiling of the dining room is 9 ft. high, how much will it cost to plaster the walls and ceiling at 30¢ a square yard, no allowance being made for windows or doors?



18. Draw a diagram like that on p. 325, showing the side walls and ceiling of the dining room.

19. Make up an example about papering the dining room.

20. Mr. Hart paid $150 for painting, $180 for plastering, $92.50 for papering, and $47 for grading his lot and making walks. What did these items cost all together?

21. What did his house and lot cost in all?

22. After using $1515.60 which he had in the savings bank when he began building, how much would Mr. Hart still owe on his house?

23. If he can save $12.50 out of his salary each month, how long will it take him to pay what he owes, no allowance being made for interest?

24. Using the prices current in your neighborhood, show how you would buy meat or fish for a family of 5 persons during one week. Make out a bill for this and pay it by check.

25. Similarly make out a bill for purchases of eggs and milk for one month.

26. Similarly make up examples concerning purchases of groceries, vegetables, fruits, dry-goods, and marketing for a Christmas dinner.

27. If you had $5 to spend for Christmas gifts for your friends, how would you spend the money?

28. If you were going away to a boarding school and had $75 to spend for clothing, how would you spend it?

IMPORTANT NUMERICAL FACTS

(Material for Examples)

AREAS AND POPULATION (YEAR 1910)

UNITED STATES PROPER	AREA IN SQ. MI.	POPULATION	UNITED STATES PROPER	AREA IN SQ. MI.	POPULATION
Alabama	52,250	2,138,093	**New York**	**49,170**	**9,113,279**
Arizona	113,020	204,354	North Carolina . . .	52,250	2,206,287
Arkansas	53,850	1,574,449	North Dakota	70,795	577,056
California	158,360	2,377,549	Ohio	41,060	4,767,121
Colorado	103,925	799,024	Oklahoma	70,057	1,657,155
Connecticut . . .	4,990	1,114,756	Oregon	96,030	672,765
Delaware	2,050	202,322	Pennsylvania	45,215	7,665,111
Dist. of Col. . . .	70	331,069	**Rhode Island**	**1,250**	**542,610**
Florida	58,680	751,139	South Carolina . . .	30,570	1,515,400
Georgia	59,475	2,609,121	South Dakota	77,650	583,888
Idaho	84,800	325,594	Tennessee	42,050	2,184,789
Illinois	56,650	5,638,591	**Texas**	**265,780**	**3,896,542**
Indiana	36,350	2,700,876	Utah	84,970	373,351
Iowa	56,025	2,224,771	Vermont	9,565	355,956
Kansas	82,080	1,690,949	Virginia	42,450	2,061,612
Kentucky	40,400	2,289,905	Washington	69,180	1,141,990
Louisiana	48,720	1,656,388	West Virginia	24,780	1,221,119
Maine	33,040	742,371	Wisconsin	56,040	2,333,860
Maryland	12,210	1,295,346	Wyoming	97,890	145,965
Massachusetts . .	8,315	3,366,416			
Michigan	58,915	2,810,173	OUTLYING POSSESSIONS		
Minnesota . . .	83,365	2,075,708			
Mississippi . . .	46,810	1,797,114	Alaska	590,884	64,356
Missouri	69,415	3,293,335	Guam (Ladrones) . .	210	. . . .
Montana	146,080	376,053	Hawaii	6,449	191,909
Nebraska	77,510	1,192,214	Panama Canal Strip .	474	. . . .
Nevada	110,700	81,875	Philippine Islands . .	115,026	. . . .
New Hampshire .	9,305	430,572	Porto Rico	3,435	1,118,012
New Jersey . . .	**7,815**	**2,537,167**	Samoan Islands . . .	77	. . . .
New Mexico . . .	122,580	327,301	. . . .	. . . .	. . . .

Area of North America	6,446,000 sq. mi.
Area of Great Britain and Ireland . .	121,391 sq. mi.
Area of France	207,054 sq. mi.
Population of New York	4,766,883
Population of London	6,580,616
Population of the earth	1,520,000,000 (approx.)

ASTRONOMICAL FACTS

	DIAMETER IN MILES	DISTANCE IN MILES
Sun	866,400	92,800,000
Moon	2,162	240,000
Mars	4,230	(from sun) 141,500,000

Diameter of the earth	7,918 miles
Circumference of the earth	25,000 miles

HEIGHTS OF MOUNTAINS

	FEET		FEET
Mt. Washington	6,290	Mt. Mitchell	6,711
Pike's Peak	14,147	Mt. Whitney	14,501
Mt. McKinley	20,464	Mt. Blanc	15,744
Mt. Everest	29,002	Acongua	23,802

DATES (A. D. UNLESS OTHERWISE STATED)

	YEAR
Rome founded	B. C. 753
Battle of Marathon	B. C. 490
Fall of Rome	476
Battle of Hastings	1066
Printing with movable type	1438
Fall of Constantinople	1453
Discovery of America	1492
Jamestown settled	1607
Landing at Plymouth	1620
Declaration of Independence	1776
Washington inaugurated	1789
First steam railroad in the United States . . .	1826
Telegraph invented	1844
Telephone invented	1876

Heights (or Lengths) of Structures

	Feet
Washington Monument	555
Singer Building (N. Y.)	612
Metropolitan Life Building (N. Y.)	700
Eiffel Tower	984
Lusitania	790
Olympic	882
Span of Brooklyn Bridge	$1595\frac{1}{2}$
Simplon Tunnel	$12\frac{1}{4}$ mi.
Panama Canal	49 mi.
Suez Canal	100 mi.

Records (Year 1910)

100 yd. dash	$9\frac{3}{5}$ seconds
Quarter mile run	47 seconds
1 mile run	4 min. $15\frac{2}{5}$ sec.
Running high jump	6 ft. $5\frac{5}{8}$ in.
Running broad jump	24 ft. $7\frac{1}{4}$ in.
Pole vault	12 ft. $10\frac{7}{8}$ in.
100 yd. swim	$55\frac{2}{5}$ seconds
1 mile swim	23 min. $16\frac{4}{5}$ sec.
1 mile skate	2 min. 36 sec.
1 mile in automobile	$27\frac{1}{3}$ seconds
Transatlantic voyage (from N. Y.)	4 da. 10 hr. 51 min.
Typewriting in one hour from new copy	6135 words
Shorthand reporting in one minute	187 words
Crop of corn on 1 acre	$255\frac{3}{4}$ bushels
Crop of cotton on 1 acre	4 bales
Milk yield, 1 cow, 1 year	27,432 pounds
Butter yield, 1 cow, 1 year	$1164\frac{3}{5}$ pounds
Eggs laid by 1 hen, 1 year	268

Lengths of Rivers

	Miles		Miles
Hudson	280	Mississippi	3160
Ohio	950	Rhine	850
Colorado	1360	Amazon	3300
Missouri	3100	Nile	3400

Distances

	Miles
From New York to Boston	234
From New York to Albany	142
From New York to Buffalo	440
From New York to Chicago	912
From New York to Denver	1930
From New York to San Francisco	3250
From New York to Philadelphia	90
From New York to Washington	228
From New York to New Orleans	1372
From New York to Jacksonville	1077
From New York to Havana	1410
From New York to London	3375
San Francisco to Manila	4850

Resources (Crops, etc., Year 1910)

Coal in U. S.	2,500,000,000,000 tons (approx.)
Iron ore in U. S.	15,000,000,000 tons
Water-power of Niagara	7,000,000 horse power
Water-power of U. S.	200,000,000 horse power
Wealth of U. S.	$120,000,000,000
Corn crop of U. S.	3,000,000,000 bushels
Wheat crop of U. S.	700,000,000 bushels
Cotton crop of U. S.	13,000,000 bales

Velocities

Wind (average)	18 mi. per hr.
Sound	1,090 ft. per sec.
Rifle bullet	2,500 ft. per sec. (average)
Message on submarine cable	2,480 mi. per sec.
Light	186,000 mi. per sec.

Rainfall (Mean Annual)

	Inches		Inches
Phœnix (Arizona)	7.9	New York	44.8
Denver	14	New Orleans	57.4
Chicago	34	Cherrapongee (Asia)	610.0

Weights

Boy 12 years old (average)	75 lb.
Man 30 years old (average)	150 lb.
Horse (average)	1000 lb.
Elephant (average)	$2\frac{1}{2}$ tons
Whale	60 tons
1 cu. ft. air	$1\frac{1}{4}$ oz.
1 cu. ft. water	62.5 lb.
1 cu. ft. cork	.24 × 62.5 lb.
1 cu. ft. maple	.75 × 62.5 lb.
1 cu. ft. clay	1.2 × 62.5 lb.
1 cu. ft. stone (average)	2.5 × 62.5 lb.
1 cu. ft. aluminium	2.6 × 62.5 lb.
1 cu. ft. iron (wrought)	7.8 × 62.5 lb.
1 cu. ft. lead	11.3 × 62.5 lb.
1 cu. ft. gold	19.3 × 62.5 lb.

Miscellaneous

Normal temperature of human body	98.7°
Boiling point of water	212°
Fusion point of iron	2,800° (approx.)
Number of teeth in human body	32
Height of earth's atmosphere	50 miles
Dimensions of double tennis court	78′ × 36′
Dimensions of baseball diamond	90′ × 90′
Dimensions of football field	330′ × 160′

On blank pages at the end of the book, the teacher may have the pupils write other numerical facts which are important in the locality in which the school is situated; as distances to important cities, lengths of the nearest large rivers, heights of mountains or high buildings, and important numerical facts relating to local industries. Thus, in agricultural regions, the pupil may record and use facts like the following: 3 gal. milk make 1 lb. butter (av.); the shrinkage of corn on the cob in a crib is 3% per month.

Numerical data for interesting examples will also be found in the almanacs published by the great metropolitan dailies, as by the *New York World* or *New York Tribune.*

DRILLS AND GAMES

Special drills and games are important in arithmetic as a means of arousing interest and of cultivating rapidity and accuracy of work. A number of drills and games have been described in the preceding text (see pp. 33, 111, 161, 314); those which follow may be used by the teacher according to the needs of the class and the opportunities which arise from day to day.

1. Drill with Numbered Cards. One of the simplest forms of written drills with cards is obtained by the use of a set of cards numbered 1, 2, 3, etc., each card containing one of these numbers and nothing else. For instance, let ten examples in addition numbered from 1 to 10 be written on the blackboard. Let the teacher also place in a box ten numbered cards. Each of the pupils to be drilled draws a card from the box and works the blackboard example which has the same number as the card drawn. After a pupil has worked an example, he draws another card from the box, and so on.

The examples used may come under any of the leading topics of arithmetic, as subtraction, division, addition of fractions, percentage, etc.; or they may be mixed examples in two or more topics.

2. Drill with Cards Containing Examples (called Blackboard Cards). Another important drill is that with cards on each of which an example has been written. These examples are numbered for the convenience of the teacher, who has in her hand a numbered list of answers to the examples. The cards are placed in a box; each pupil draws a card, goes to the blackboard, and works the example on the card. The teacher checks the work from her list of answers. If the work is correct, the pupil credits himself with one example and draws another example from the box.

These cards may be used as extra work for pupils who work rapidly; or they may be used in a competitive drill, the winner being the pupil who first works, say, five examples correctly. These blackboard cards may also be used in the Tally Game and Relay Race (see p. 393).

3. Abbreviation Cards for Oral Drill. A useful oral drill may be obtained by the use of a set of cards each of which contains an abbreviation used in compound numbers, as *lb.*, *qt.*, *mi.*, *da.* The pupils in turn draw these cards from a box. After drawing a card, the pupil states the meaning of the abbreviation on the card and recites the table or tables in which the given word is used.

The cards may each contain an abbreviation for a month of the year and the pupil may be asked to give the number of the month drawn; or the number being given on the card, the pupil may be asked to name the corresponding month.

4. **Cards with Fractions for Oral Drill.** Cards placed in a box may each contain a small common fraction, and the pupil, on drawing a card, may be asked to give as many fractions as he can which are equivalent to the fraction drawn. Or he may be asked to name the decimal fraction or the per cent which is equivalent to the given common fraction. Again, the cards may each contain a decimal fraction or a per cent, and the pupil may be asked to give the equivalent common fraction.

Mixed oral drills may be devised by combining different kinds of cards as described in Notes 3 and 4.

5. **Flash Cards for Oral Drill.** Flash cards are cards on each of which a simple example has been written, as an example in the addition or subtraction of two-figured numbers. The cards are shown one by one, for an instant, and then quickly withdrawn, and the pupil is asked to give the answer to the example on the card (or a complete oral solution of the example).

This drill gives pupils valuable training in concentration of attention and rapidity of perception.

6. **Written Drill with Fraction Cards.** The drill with fraction cards described in Note 4 may be converted into a written drill by having pupils, who have each drawn a card, go to the blackboard in pairs and add, subtract, multiply, or divide the fractions. If both pupils in a pair get the correct result, the pupil finishing first wins.

7. **Time Drill; Beating One's Record.** The best kind of game is that in which a class or a pupil keeps a record of the time required to do a given piece of work or set of examples, and keeps practicing in order to be able to beat each preceding record. A record may be kept on the blackboard of the time required by a class in a review of all the fundamental addition and subtraction facts, or of the multiplication or division facts, and efforts may be made repeatedly to improve the record. Similarly a pupil may keep a record of the time he has required in working the examples on a given page or part of a page, and of his successive improved records in working the same examples.

8. Beating an Assigned Time. The drill described in Note 7 may be varied by assigning to the class a set of examples not worked before, and asking how many of the members of the class can work these examples in less than three minutes or some other specified time.

9. Time Races; The Tally Game. A time race is a race in working examples rapidly and correctly, and conducted between two halves of a class, between two schoolrooms or two grades, or between teams chosen to represent two of these. Time races may be arranged in various ways. One of the most useful and interesting of these is the Tally Game.

At the outset of this game the following tally sheet is placed on the blackboard:

1 — 20
2 — 19
3 — 18
4 — 17
5 — 16
6 — 15
7 — 14
8 — 13
9 — 12
10 — 11
11 — 10
12 — 9
13 — 8
14 — 7
15 — 6
16 — 5
17 — 4
18 — 3
19 — 2
20 — 1

The class is divided into two sides, or two teams are chosen. The contestants work at their seats. A number of problems (say four) are dictated to the class or written on the blackboard. At the word "Go," all pupils begin to work. The first pupil to finish all the assigned examples places his paper on the teacher's desk and is given the number 1. Each pupil in turn is given his number in the order of finishing the work. Tally is kept by multiplying the number of correct examples on a pupil's paper by the number opposite his place number on the tally sheet. For example, if the pupil who finishes third in order has two examples right, he has 2 × 18, or 36, to his credit. The score is kept, by sides, on the blackboard by the teacher or a pupil. That side wins which makes the highest aggregate score.

10. Relay Races. Another interesting time race is the relay race. In this race the two sides or teams which compete are placed in two rows of seats. On the front desk of each of these two rows are placed as many problems (of equal difficulty and on separate cards) as there are pupils in the row. At the word "Go," the first pupil in each row, taking a card, goes to the blackboard and works the example on the card; he then takes another card from the front desk, gives the card to the second pupil in his row, and returns to his seat. The second pupil goes through the same process, and gives the third card to the third pupil in his row, and so on.

The remainder of the class, divided into two equal parts, watches for and reports mistakes in the work on the blackboard.

The row finishing first wins the race unless there are mistakes in the work of that row. In this case, the row having no mistakes wins.

Time races are useful not only in cultivating rapidity and accuracy in the work of pupils, but in making evident the processes in which the class or particular pupils need further drill.

11. The Game of Drawing to Scale. Exercises in drawing to scale may be converted into a game, as follows: Divide the class into two sides or groups, and let each pupil draw a rectangle to some convenient scale, marking on the diagram the dimensions of the rectangle, but not the scale of the drawing. Let the pupils in the two groups exchange drawings, and let each pupil determine the scale of the drawing received. Each pupil, by making a correct drawing, may score one point; also by detecting the scale of a drawing received, or an error in the same, he may score another point. The side scoring most points wins the game.

The game may be varied by having pupils write on their diagrams the scale of the drawing instead of the dimensions of the rectangle, and then after exchanging the drawings they may determine the dimensions.

12. The Game of Bills and Accounts. In the study of bills, the work may be enlivened by a competitive game like the following: The class is divided into two equal groups; each pupil makes out a bill for four articles bought at any well-known store; and pupil by pupil the groups exchange bills. Each pupil tests the bill received by him, and if he finds it correct, receipts it. Each pupil may score a point by making out a correct bill; also he may score a point by detecting an error in a bill made out by another pupil. The side scoring the most points wins the game.

It is constantly to be borne in mind that the best kind of game is one which is not too competitive in its nature and in which each pupil can achieve a measure of success without discouraging or humiliating other pupils. In this respect the game of beating one's record, or aiding the class to beat its record, ranks highest.

13. The Forty-five Combinations. In the oral time drills, emphasis should be placed on the mastery of the forty-five combinations as follows:

TABLE OF 45 COMBINATIONS IN ADDITION

	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2		4	5	6	7	8	9	10	11
3			6	7	8	9	10	11	12
4				8	9	10	11	12	13
5					10	11	12	13	14
6						12	13	14	15
7							14	15	16
8								16	17
9									18

TABLE OF 45 COMBINATIONS IN MULTIPLICATION

	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2		4	6	8	10	12	14	16	18
3			9	12	15	18	21	24	27
4				16	20	24	28	32	36
5					25	30	35	40	45
6						36	42	48	54
7							49	56	63
8								64	72
9									81

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ANSWERS

PART I

Page 22

1. $ 16,906
2. 11,691 in.
3. 15,806 ft.
4. $ 117.29
5. $ 1097.50
6. 23,744
7. $ 33,410
8. 149,829 lb.
9. $ 22,964.01
10. $ 17,462.38

Page 23

11. 386,698
12. 134,244
13. 120,082
14. 349,891
15. $ 536.56
16. $ 49,895
17. 19,393 bu.
18. $ 30,259.60
19. 22,384 tons
20. 785 men
21. 3123 ft.

Page 24

22. 1,703,469 sq. mi.
23. $ 15,090
24. 3,959,517
25. 66,465 sq. mi.
26. 104,250 sq. mi.
27. 371,570 sq. mi.; 323,570 sq. mi.
28. 553,325 sq. mi.; 472,240 sq. mi.; 426,765 sq. mi.

Page 25

1. 17,533 tickets,
6. $ 279,075
7. $ 595,522
8. $ 295,306
9. $ 1,606,895

Page 28

1. 342
2. $ 1.31
3. 324
4. 178
5. 2212

Page 29

6. 3084
7. 512
8. 2678
9. 19,800
10. $ 60.55
11. $ 21.38
12. 14,855
13. 2240 yd.
14. 2135 ft.
15. $ 12.13
16. $ 188.93
17. 60.63 in.
18. 183,626
19. $ 4473.28
20. 34,448 in.
21. 188,089
22. $ 1108
24. 768
25. 105 ft.
26. 1010 ft.
27. 789 bu.
28. 78.12 mi.

Page 30

29. $ 35.
30. 31,310 mi.
31. 716,555 sq. mi.
32. 40,855; 257,465; 216,610 sq. mi.
34. 1825
35. 2600
36. 1306
37. 4098
38. 1108
39. 4531
40. 14,520
41. 24,622
42. 125,028
43. 64,303
44. 224,066
45. 103,875

Page 33

1. $ 223

Page 34

2. 2654 bu.
3. 910 mi.
4. 79,712
5. $ 33,000
6. $ 617.48

Page 35

7. 370; 138; 198 mi.
9. 90¢
10. 37,785 sq. mi.
11. 48,000 sq. mi.
12. 45,475 sq. mi.

Page 39

1. 1334 ft.
2. 1692 in.
3. $ 9144
4. 26,837
5. 20,124
6. 134,784
7. 1,050,003
8. 1,174,621
9. 2,946,882
10. 2,622,964
11. 2,672,105
12. 3,878,043
13. 6,284,208
14. $ 47,117.75
15. $ 9975.66
16. 58,548
17. 257,706
18. 3,191,718
19. 3,791,328
20. 7,270,822
21. 5,970,375
22. 3,251,660
23. 6,292,924
24. 24,331,452

Page 40

25. 79,966,364
26. 22,220,927
27. 42,546,240
28. 35,497,655
29. 24,631,008
30. 3,007,368
31. 30,718,260
32. 15,609,750
33. 6,397,560
34. 62,355,319
35. 84,749,652
36. 535,602,537
37. $ 51,624
38. 212,072; 376,536; 1,406,600 tons
39. 195,360; 1,362,240 ft.
41. 2603 ft.
42. $ 6075
43. 6540; 130,800; 13,080 ft.
44. $360
45. 9810; 16,350; 32,700; 235,440; 1,242,600; 1,504,200
46. 175,200; 700,800; 7,884,000; 4,029,600; 46,428,000; 62,196,000
47. $ 15.78

Page 41

48. $ 62.01
49. $ 61
50. $ 6483
51. $ 321

Page 43

1. 6096
2. 10,926
3. 101,574
4. $ 9990.72
5. $ 15.24
6. 45,815 in.
7. 10,344 qt.

Page 44

8. $ 37.80
9. $ 50.88
10. 15,456 bu.
11. $ 14,896; $ 40,964; $ 223,440

1. $ 5300
2. 10,400 in.
3. 6800 ft.
4. $ 4800
5. 190,600
6. $ 40,600
7. 458,600
8. 80,600
9. 22,300
10. 382,100
11. 181,000
12. 405,300
13. $ 7500

Page 45

14. $ 12,500; $ 15,800; $ 22,500
15. $ 5600; $ 17,500; $ 850
16. $8000; $ 13,000; $ 5750
17. 17,920 sq. mi.
18. 16,280 sq. mi.
19. $ 4300
20. $11,448

Page 47

1. $33.18
2. $ 19.62
3. $ 14.64
4. $ 25.68

Page 48

5. Yes; R = $ 3.28
6. $1\frac{2}{3}$ doz.
7. 2700 eggs; $63
8. $ 68; $ 85; $ 15.75
10. 4910 lb.
11. 26 cows
12. 9817 lb.
13. 238 lb.

Page 50

1. $ 192
2. 45 in.
3. 89 qt. R = 4
4. 54
5. 1099 yd.
6. $ 1613
7. 1121 in.
8. 1351
9. $ 41.15
10. 3841
11. 11,532
12. 13,395
13. 9450
14. $ 97.39
15. 7173
16. 7095

Page 51

17. 179
18. 175
19. $ 112
20. 241 ft.
21. $ 11.21
22. $ 20.87
23. 12,657
24. 1356. R = 1
25. 493. R = 5
26. 3824
27. 15,290. R = 3
28. 20,588. R = 2
29. 8731. R = 1
30. 9124. R = 6
31. 8369. R = 6
32. 5623
33. 1533
34. 8240. R = 4
35. $ 655
36. $ 275
37. 1752; 3504; 17,520 yd.
38. $ 2275
39. $11,077\frac{1}{2}$ sq. mi.
40. 142; 4571
41. 56 da.; 8 wk.
42. 53

Page 53

1. 116 in.
2. 35. R = 3
3. 129
4. $ 120. R = $ 12
5. 217. R = 48
6. 413. R = 39
7. 141. R = 14
8. 1038. R = 37
9. 1234. R = 5
10. 965. R = 97
11. 361
12. 409
13. 117. R = 54
14. 1448. R = 6
15. 836. R = 29
16. 3425. R = 7
17. 2614. R = 35
18. 1000. R = 9

Page 54

19. $ 3115
20. $ 315. R = $14
21. 374. R = 9
22. 146. R = 9
23. 199 in. R = 7 in.
24. 175. R = 28
25. 84 ft. R = 5 ft.
26. 149. R = 36
27. 837. R = 20
28. 1607. R = 35
29. 962. R = 46
30. 1006. R = 83
31. $ $221\frac{85}{89}$
32. 38 bu.
33. $75
34. $ $.52\frac{45}{83}$
35. $66\frac{24}{27}$ bu.
36. $34\frac{12}{13}$ mi.
37. 68; 187+ beeves
38. $63031\frac{42}{43}$ sq. mi.

Page 55

1. $ 42
2. $ 58
3. 83
4. 628
5. 106. R = 321
6. 1073. R = 213
7. 591
8. 97
9. 804
10. 204
11. 407
12. 509
13. 753
14. 654
15. 609. R = 64
16. 892
17. 75. R = 517
18. 9. R = 238
19. 804
20. 509
21. 6032
22. 26. R = 388
23. 245. R = 116
24. 133. R = 137
25. 963. R = 85
26. 720. R = 306
27. 3011. R = 292
28. 3947. R = 237
29. 6671. R = 250
30. 7318. R = 228

Page 56

31. 7+; 43+ gal.
32. 73 shares
33. $5\frac{75}{817}$
34. $13\frac{528}{744}$
37. 120

Page 57

1. $ 357. R = $ 48
2. 576. R = 72
3. 1030. R = 37
4. 1492. R = 25
5. 39 R = 48
6. 38 R = 539
7. 839. R = 77
8. 255. R = 32
9. 5635. R = 15
10. 460. R = 23
11. 36
12. 25. R = 17
19. 11; $11\frac{8}{410}$; $22\frac{88}{410}$ hr.
13. 317. R = 89
14. 521
15. 754. R = 3800
16. 288 hr. R = 7 min.
17. $11\frac{10}{90}$ trips
18. $17\frac{17}{80}$ bu.

Page 58

20. $ 121
21. $11\frac{80}{280}$

Page 60

1. 4
2. 1059
3. 395
4. 232
5. 543
6. 1789

Page 62

16. $ 74.52
17. 1704 lb.

Page 63

1. 72,184
2. $ 87
3. 2,016,755; 2,506,655; 6,515,670
4. 4 times
5. 10 times
6. 3218
7. 478
8. 20
9. 1085
10. $ 1500

Page 64

12. XXVII. CCCI. CLVI.
13. 141 poles
14. $ 5600
15. 4120 in.
16. 60
17. 30
18. 1500
19. 6
20. $ 27; $ 189
21. $ 65
22. $ 3.92
23. $ 98.70
24. 1235 marbles
25. 9+ yr.
26. $ 1400

Page 65

27. 540 cu. ft.
28. Lost \$1711

Page 66

1. 71¢
2. \$ 1
3. 80¢
4. 79¢
5. 51¢
6. 36¢
7. 34¢
8. \$ 1.05
9. 60¢
10. \$ 1.49

Page 67

12. 71¢
13. 21¢
15. 50¢; 60¢; 90¢; 50¢
16. \$ 19.60

Page 74

1. 3
2. \$ 3
3. 20
4. 375
5. $187\frac{1}{2}$
6. 10
7. $67\frac{1}{2}$
8. 9
9. 15
10. 13
11. $22\frac{4}{5}$
12. $12\frac{41}{64}$
13. 112
14. 120
15. $13\frac{5}{7}$
16. $5\frac{1}{7}$
17. 16
18. $4\frac{2}{3}$
19. 22; 704 bu.
20. 10 tons
21. \$ 1575
22. 61,625 da.
23. 6 hr.
24. 4 doz.

Page 75

27. 18
28. 6
29. 10
30. 3
31. 9
32. 6
34. \$ 1.56

Page 78

1. 6
2. 3
3. 2
4. 12
5. 10
6. 9
7. 15
8. 12
9. 25
13. 2 gal.

Page 79

14. 6 in.
15. \$ 5
16. 16 boys
17. 6 gal. 9 times

Page 82

1. 36
2. 63
3. 360
4. 792
5. 72
6. 315
7. 462
8. 3120
9. 400
10. 360
11. 840
12. 2520
18. 150 min. or 2 hr. 30 min.
19. 120 min. or 2 hr.
20. 12 o'clock
22. 24

Page 83

2. 4; 96
3. 5; 420
4. 18
5. 6 o'clock

Page 84

6. 711
7. 101,101
8. 2 in.; a G. C. D.

Page 85

1. $ 2.50
2. $ 3
3. $ 1.30
4. $ .45
5. $ 2
6. $ 1.50
7. $ .60
8. $ 1.90
9. $ 11
10. $ 66.35
11. Boy $ 2.52 more
13. $ 7.20; $ 106.85; $ 99.65

Page 91

1. $\frac{11}{2}$ yd.
2. $\frac{121}{4}$ sq. ft.
3. $\frac{1461}{4}$ da.
4. $\frac{93}{7}$

Page 92

5. $\frac{9}{4}$, $\frac{22}{7}$, $\frac{17}{8}$, $\frac{87}{9}$, $\frac{32}{5}$, $\frac{93}{8}$, $\frac{81}{5}$, $\frac{90}{7}$, $\frac{71}{6}$, $\frac{107}{8}$
6. $\frac{48}{7}$, $\frac{42}{5}$, $\frac{102}{11}$, $\frac{55}{12}$, $\frac{88}{9}$, $\frac{60}{7}$, $\frac{135}{11}$, $\frac{107}{10}$, $\frac{77}{9}$, $\frac{85}{11}$
7. 37, 48, 59, 61
8. 67, 41, 109, 79, 91
9. $\frac{150}{13}$, $\frac{147}{16}$, $\frac{187}{14}$, $\frac{112}{5}$, $\frac{487}{15}$, $\frac{279}{21}$, $\frac{895}{11}$
10. $\frac{893}{12}$, $\frac{1501}{18}$, $\frac{1477}{15}$, $\frac{1226}{11}$, $\frac{3071}{42}$, $\frac{1087}{135}$, $\frac{4857}{101}$

Page 93

1. $4\frac{3}{4}$ in., $ $4\frac{3}{5}$, $6\frac{1}{2}$ gal., $4\frac{1}{4}$ ft., $3\frac{1}{3}$ ft.
2. $5\frac{3}{4}$, $3\frac{3}{4}$, $4\frac{3}{4}$, $4\frac{3}{5}$, $7\frac{2}{5}$, $8\frac{3}{5}$, $6\frac{2}{7}$, $8\frac{3}{7}$
3. $5\frac{1}{3}$, $5\frac{3}{5}$, $5\frac{4}{5}$, $9\frac{1}{3}$, $5\frac{3}{11}$, $9\frac{7}{8}$, $12\frac{2}{9}$, $12\frac{4}{7}$, $4\frac{5}{11}$
4. $8\frac{1}{12}$, $16\frac{3}{4}$, $16\frac{3}{5}$, $15\frac{7}{8}$, $6\frac{7}{11}$, $19\frac{4}{5}$, $42\frac{6}{7}$, $36\frac{4}{11}$, $55\frac{5}{9}$
6. $4\frac{7}{9}$, $12\frac{3}{7}$, $5\frac{1}{11}$, $9\frac{3}{5}$, $15\frac{2}{5}$, $11\frac{3}{11}$, $12\frac{2}{9}$, $21\frac{3}{7}$
7. $20\frac{7}{9}$, $39\frac{3}{5}$, $142\frac{1}{3}$, $175\frac{1}{5}$, $79\frac{7}{11}$, $77\frac{11}{12}$, $71\frac{1}{12}$

Page 95

1. $\frac{2}{3}$, $\frac{3}{2}$, $\frac{7}{9}$, $\frac{1}{3}$, $\frac{3}{5}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{5}{12}$, $\frac{1}{6}$, $\frac{1}{3}$
2. $\frac{1}{2}$, $\frac{3}{5}$, $\frac{1}{3}$, $\frac{9}{25}$, $\frac{12}{13}$, $\frac{9}{19}$, $\frac{11}{13}$, $\frac{13}{15}$, $\frac{12}{23}$
3. $\frac{9}{16}$, $\frac{7}{16}$
4. $\frac{5}{6}$
5. $\frac{2}{3}$, $\frac{1}{3}$
8. $\frac{3}{8}$
9. $\frac{5}{16}$

Page 96

11. $\frac{3}{4}$
12. $\frac{3}{4}$
13. $\frac{5}{18}$
14. $\frac{1}{2}$¢
15. $2\frac{1}{2}$¢

Page 98

1. $\frac{4}{6}$, $\frac{5}{6}$
2. $\frac{5}{10}$, $\frac{8}{10}$
3. $\frac{9}{12}$ in., $\frac{10}{12}$ in.
4. $\frac{9}{30}$ ft., $\frac{4}{30}$ ft.
5. $\frac{15}{24}$ ft., $\frac{4}{24}$ ft.
6. $\frac{8}{12}$ gal., $\frac{1}{12}$ gal.
7. $\frac{6}{24}$, $\frac{1}{24}$
8. $\frac{6}{21}$, $\frac{8}{21}$
9. $\frac{8}{12}$, $\frac{3}{12}$, $\frac{9}{12}$
10. $\frac{4}{12}$, $\frac{10}{12}$, $\frac{3}{12}$
11. $\frac{12}{18}$, $\frac{15}{18}$, $\frac{8}{18}$
12. $\frac{9}{12}$, $\frac{5}{12}$, $\frac{3}{12}$
13. $\frac{15}{24}$, $\frac{14}{24}$, $\frac{18}{24}$
14. $\frac{27}{48}$, $\frac{28}{48}$, $\frac{42}{48}$
15. $\frac{20}{30}$, $\frac{24}{30}$, $\frac{21}{30}$
16. $\frac{16}{18}$, $\frac{15}{18}$, $\frac{12}{18}$
17. $\frac{15}{30}$, $\frac{20}{30}$, $\frac{24}{30}$
18. $\frac{84}{210}$, $\frac{175}{210}$, $\frac{90}{210}$
19. $\frac{264}{330}$, $\frac{55}{330}$, $\frac{210}{330}$
20. $\frac{396}{924}$, $\frac{756}{924}$, $\frac{847}{924}$
21. $\frac{99}{792}$, $\frac{704}{792}$, $\frac{720}{792}$
22. $\frac{396}{693}$, $\frac{189}{693}$, $\frac{539}{693}$
23. $\frac{180}{780}$, $\frac{715}{780}$, $\frac{624}{780}$
24. $\frac{13}{16}$; $\frac{5}{8}$
25. $\frac{3}{8}$
26. $\frac{7}{25}$; $\frac{4}{25}$
28. $\frac{32}{48}$, $\frac{15}{48}$, $\frac{28}{48}$, $\frac{22}{48}$

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3. $\frac{9}{10}$
4. $1\frac{5}{18}$
5. $1\frac{5}{24}$
6. $1\frac{5}{12}$
7. $1\frac{17}{24}$
8. $1\frac{11}{12}$
9. $2\frac{1}{20}$
10. $1\frac{17}{18}$
11. $1\frac{17}{24}$
12. $1\frac{13}{28}$
13. $2\frac{11}{36}$
14. $2\frac{5}{72}$
15. $\frac{43}{48}$
16. $\frac{87}{100}$
17. $\frac{807}{1000}$
18. $2\frac{1}{2}$ in.
19. $\frac{5}{8}$ yd.
20. $2\frac{3}{8}$ in.
21. $\frac{1}{3}$
22. $\frac{7}{16}$

Page 101

29. Edwin. $\frac{1}{8}$ in. more.

Page 102

1. $3\frac{7}{8}$ in.

Page 103

2. (1) $3\frac{5}{8}$ in. (2) $5\frac{7}{8}$ in. (3) $4\frac{1}{4}$ in. (4) $6\frac{1}{8}$ in. (5) $6\frac{3}{8}$ in.

3. $1\frac{3}{4}$
4. $5\frac{3}{4}$
5. $35\frac{3}{4}$
6. $4\frac{1}{3}$
7. $13\frac{1}{3}$
8. $43\frac{1}{3}$
9. $7\frac{1}{10}$
10. $11\frac{1}{10}$
11. $476\frac{1}{10}$
12. $8\frac{1}{24}$
13. $18\frac{1}{24}$
14. $542\frac{1}{24}$
15. $7\frac{11}{12}$
16. $11\frac{5}{6}$
17. $8\frac{11}{24}$
18. $11\frac{17}{24}$
19. $15\frac{5}{18}$
20. $10\frac{11}{12}$
21. $18\frac{1}{24}$
22. $13\frac{23}{24}$
23. $22\frac{17}{36}$
24. $12\frac{5}{16}$
25. $10\frac{83}{100}$
26. $10\frac{423}{1000}$
27. 4 ft. $11\frac{1}{4}$ in.
28. $8\frac{5}{12}$ in.
29. $6\frac{3}{4}$ tons
30. $10\frac{1}{4}$ in.
31. $16\frac{7}{8}$ in.

Page 104

33. 58 yd. 34. $34.80 35. 8 in.

1. $\frac{1}{4}$ in. 2. (1) $1\frac{1}{4}$ in. (2) $\frac{3}{8}$ in. (3) $\frac{3}{8}$ in. (4) $\frac{1}{10}$ in. (5) $\frac{3}{10}$ in. (6) $\frac{1}{2}$ in.

Page 105

4. $ $\frac{2}{5}$
5. $\frac{1}{2}$ ft.
6. $\frac{1}{3}$ gal.
7. $\frac{1}{4}$ in.
8. $\frac{1}{6}$
9. $\frac{19}{24}$
10. $\frac{4}{15}$
11. $\frac{1}{6}$
12. $\frac{2}{35}$
13. $\frac{5}{42}$
14. $\frac{47}{72}$
15. $\frac{13}{81}$
16. $\frac{13}{48}$
17. $\frac{11}{80}$
18. $1\frac{13}{80}$
19. $\frac{13}{18}$
20. $\frac{35}{156}$
21. $\frac{7}{18}$
22. $1\frac{1}{4}$
23. $2\frac{1}{8}$

24. $3\frac{1}{3}$ 29. $\frac{1}{24}$ in. 30. $\frac{9}{64}$

Page 106

1. $1\frac{1}{4}$ in. 2. (1) $1\frac{1}{4}$ in. (2) $2\frac{3}{8}$ in. (3) $\frac{3}{4}$ in. (4) $\frac{1}{4}$ in. (5) $1\frac{5}{8}$ in.

Page 107

3. $1\frac{1}{4}$
4. $2\frac{1}{4}$
5. $12\frac{1}{4}$
6. $1\frac{1}{2}$
7. $3\frac{1}{2}$
8. $12\frac{1}{4}$
9. $1\frac{7}{10}$
10. $2\frac{7}{10}$
11. $8\frac{7}{10}$
12. $2\frac{7}{24}$
13. $5\frac{7}{24}$
14. $170\frac{7}{24}$
15. $2\frac{1}{2}$
16. $4\frac{5}{18}$
17. $2\frac{11}{24}$
18. $2\frac{4}{9}$
19. $4\frac{9}{10}$
20. $2\frac{14}{15}$
21. $6\frac{37}{42}$
22. $1\frac{5}{8}$
23. $4\frac{25}{36}$
24. $2\frac{51}{100}$
25. $1\frac{3}{8}$ in.
26. $14\frac{1}{4}$ lb.
27. $1\frac{3}{8}$ in.
28. $1\frac{3}{4}$ in.
29. $24\frac{1}{4}$ in.
30. $\frac{7}{8}$ in.
31. $2\frac{5}{8}$ in.

Page 108

33. $1\frac{5}{8}$ yd.
34. $3\frac{1}{3}$ sec.
35. $25\frac{3}{4}$ gal.
36. $123\frac{3}{4}$ A

1. $\frac{19}{24}$
2. $\frac{13}{24}$
3. $36\frac{1}{4}$
4. $12\frac{11}{18}$
5. $\frac{3}{13}$, $\frac{1}{117}$
6. $\frac{3}{8}$
7. 20 gal.

Page 109

8. 19 tons
9. $\frac{53}{144}$
10. $1\frac{9}{14}$
11. $\frac{4}{45}$; $\frac{8}{45}$

Page 110

1. $6\frac{2}{3}$
2. $16\frac{1}{4}$
3. $6\frac{1}{4}$
4. 8
5. $11\frac{1}{3}$
6. $14\frac{3}{5}$
7. $26\frac{2}{3}$
8. $23\frac{2}{3}$
9. $68\frac{1}{3}$
10. $12\frac{1}{2}$ in.
11. $4\frac{1}{2}$ lb.
12. 6 gal.
13. $3\frac{1}{3}$ ft.
14. 6¢.; 10¢.
15. 2¢.
16. $2\frac{5}{8}$ in.; $\frac{3}{8}$ in.; $4\frac{3}{8}$ in.; $\frac{5}{8}$ in.

Page 111

18. 24¢.
19. $\frac{3}{4}$ mi.; 45 mi.
21. 75¢.
22. 134

Page 113

1. $\frac{1}{8}$ in.
2. $\frac{1}{4}$ in.
4. $\frac{21}{40}$
5. $\frac{13}{77}$
6. $\frac{1}{6}$
7. $\frac{4}{9}$
8. $\frac{3}{5}$
9. $\frac{1}{12}$
10. $\frac{8}{9}$
11. $\frac{1}{6}$
12. $1\frac{1}{20}$
13. $\frac{15}{77}$
14. $\frac{209}{780}$
15. $1\frac{1}{5}$
16. $\frac{1}{2}$
17. $\frac{1}{3}$
18. 1
19. $\frac{1}{14}$
20. $\frac{1}{3}$
21. $\frac{55}{168}$
22. $\frac{1}{6}$; $\frac{1}{6}$
23. $\frac{1}{2}$
24. $\frac{7}{32}$; $\frac{21}{32}$

Page 115

1. $\frac{4}{5}$; $\frac{3}{5}$; $\frac{5}{64}$
2. $\frac{2}{17}$
3. $\frac{3}{32}$
4. $\frac{3}{145}$
5. $\frac{2}{135}$
6. $\frac{2}{99}$
7. $\frac{3}{355}$
8. $\frac{3}{20}$
9. $\frac{4}{35}$
10. $\frac{1}{12}$
11. $\frac{10}{33}$
12. $\frac{15}{188}$
13. $\frac{9}{1100}$
14. $\frac{1}{21}$
15. $\frac{1}{44}$
16. $\frac{1}{57}$
17. $\frac{1}{110}$
18. $\frac{5}{32}$
19. $\frac{2}{3125}$
20. $\frac{2}{9}$
21. $\frac{4}{5}$ in.; $1\frac{3}{5}$ in.
22. $\frac{3}{7}$

Page 117

1. 3
2. 3; 3; 2.
3. 3
4. 4
5. 3
6. $2\frac{1}{2}$
7. $1\frac{1}{4}$
8. $2\frac{1}{4}$
9. 3
10. 4
11. 6.
12. 2.
13. $1\frac{1}{5}$
14. $1\frac{1}{14}$
15. 8.
16. 4; 8; 16
17. 4.
18. 16
19. 36
20. $10\frac{2}{3}$
21. 18
22. $14\frac{2}{5}$
23. 16
24. 11 pieces
25. 9 pieces

Page 118

26. $2\frac{2}{3}$; $2\frac{2}{7}$
27. 2
29. 4

Page 119

12. $2\frac{1}{4}$ 13. $ 480; $\frac{14}{27}$; $\frac{8}{9}$; $2\frac{1}{6}$; 1; $\frac{7}{36}$; 7 14. $ 7.50 15. $ 20

Page 120

2. [× 25] (1) 15,300 (2) $ 13,500 (3) 2400 ft. (4) 13,200 ft.
[× 50] (1) 30,600 (2) $ 27,000 (3) 4800 ft. (4) 26,400 ft.
5. (1) 30,700 (2) $ 34,400 (3) 24,300 ft. (4) 209,400 yd.
7. 41,800
8. (1) 61,200 (2) $ 69,000 (3) 62,333$\frac{1}{3}$ ft. (4) 218,800 yd.
10. 600
11. (1) 5300 (2) $ 10,800 (3) 12,700 ft. (4) 9000 ft.

Page 121

13. 1800
16. 3000
14. (1) 13,500 (2) $ 27,300 (3) 38,700 ft. (4) 27,600 yd.
17. (1) 23,000 (2) $ 46,000 (3) 114,500 ft. (4) 233,500 yd.
1. 2800; 8400; 14,000; 19,600; 5600
2. 800; 1600; 300; 900; 2100
3. 24,300; 21,600; 40,500; 43,200; 8100
4. 13,500; 36,000; 40,500; 72,000; 27,000
5. 15,625; 83,333$\frac{1}{3}$; 31,250; 46,875; 41,666$\frac{2}{3}$
6. 81,600; 91,800; 122,400; 61,200; 163,200
7. 75,000; 200,000; 525,000; 225,000; 300,000

Page 122

8. $ 12.83; $ 9.63
9. $ 157.33; $ 147.50
10. $ 234.38; $ 281.25
11. $ 377.33; $ 495.25
12. $ 164.25; $ 146
13. $ 16; $ 21
14. $ 308

Page 123

1. 40¢
2. $ 900
3. 30¢
4. $ 1050
5. 80¢
6. $ 120
7. $ 5600
8. $ 14,400
9. 1040 lb.
10. 18 mi.
11. 35 bu.
12. 24; 21; 108

Page 124

14. 300 A. 15. 63 games 16. 7 eggs 17. 320 A. 18. 1295 bu.; 370 bu.

Page 125

1. 24¢ 2. $ 490 3. 16¢ 4. $ 10,000 5. $ 1000 6. 49 games 7. 36 da.

Page 126

1. $14\frac{2}{3}$
2. $14\frac{181}{380}$
3. $4\frac{7}{80}$
4. $17\frac{1}{9}$
5. $\frac{8}{33}$
6. $1\frac{5}{42}$
7. $1\frac{9}{20}$
8. $\frac{7}{80}$
9. $\frac{13}{42}$
10. $2\frac{23}{24}$
11. $2\frac{9}{20}$
12. $1\frac{15}{16}$
13. $1\frac{5}{6}$
14. $\frac{13}{24}$
15. $8\frac{1}{2}$
16. $5\frac{69}{80}$
17. 3
18. $\frac{5}{8}$
19. $3\frac{5}{9}$
20. $131\frac{1}{4}$
21. 290
22. $884\frac{4}{5}$
23. $14\frac{2}{3}$
24. $1\frac{1}{2}$
25. $3\frac{179}{488}$
26. $ 5
27. 120
28. 30
29. 22,000
31. 20 da.
32. $1\frac{3}{4}$ pt.
33. First

Page 127

34. 180 oranges
35. $1\frac{1}{10}$ sec.
36. $53\frac{5}{6}$ sec.
37. 32¢
38. 84 papers
39. $ 9360
40. 8; 20; 60
41. $ 2.79
42. $ 170; $ 340

Page 128

43. $\frac{2}{3}$; $\frac{2}{5}$; $\frac{4}{27}$
44. Lose $ 1

Page 130

2. (1) 6¢ (2) 7¢ (3) 17¢ (4) 14¢ (5) 2¢ (6) 7¢ (7) 3¢ (8) 4¢ (9) 7¢ (10) 24¢
3. 10¢

Page 131

10. $ 43.75; 1¢; 9¢
11. 10 oz.
12. 20 oz.; 8 oz.

Page 132

13. 2; 4; 8; 8; 2; same
14. Express is 64¢ cheaper
15. 75,785; $84\frac{836}{903}$

Page 141

2. $\left\{\begin{matrix} .8; \\ .65; \end{matrix}\right. \quad \left\{\begin{matrix} .6; \\ .58; \end{matrix}\right. \quad \left\{\begin{matrix} .516; \\ .555. \end{matrix}\right.$

Page 142

3. 21.88 ft.
4. 21.07 gal.
5. 24.11 T.
6. 46.1
7. 408.3
8. 382.66
9. 1248.0974
10. 16.059
12. 1.606
14. 4.79 in.
15. 4 ft. 11.79 in.
16. 5.94 in.
17. 828.65 ft.
18. 7.18 in.

Page 143

2. $\left\{\begin{matrix} .5; \\ .23; \end{matrix}\right. \quad \left\{\begin{matrix} .3; \\ .32; \end{matrix}\right. \quad \left\{\begin{matrix} .172; \\ .342. \end{matrix}\right.$

Page 144

3. 12.5 ft.
4. 118.8 ft.
5. 5.97 ft.
6. 7.282 in.
7. 14 gal.
8. 1.33
9. 6.65
10. 10.07
11. .057
12. .717
13. 2.772
14. 4.61
15. 5.244
16. 6.322
17. 17.103
18. 11.615
20. 1.47 in.
21. 1.7 in.
22. .7 sec.
23. .267 lb.
24. $ .05; $ .005; $ .045

Page 145

5. .28 ft.
6. .68 in.
7. 1.08 cu. in.
8. $ 8.491
9. 16.015 ft.
10. 101.2 yd.
11. $ 136.24
12. 3180.45
13. 385.424

Page 146

16. 8.6 in.
17. 22.05 lb.
18. $ 3.125
19. .276
21. .7 in.; .42 in.; .56 in.
22. $ 40.92

Page 147

2. .03 .04 .08
.06 .04 .15
.2 .24 .56

Page 148

3. .128 ft.
4. 8.25 ft.
5. .324 in.
6. .0544 yd.
7. .345
8. .3704
9. .981
10. .10455
11. .0076
12. .01146
13. .30272
14. 3.70524
17. 1.29 in.
18. $ 4.15
19. $ .63
20. 53.96 ft.
21. 113.74 sq. yd.

Page 150

5. 2.5 ft.
6. $ 2.12
7. 12.02 yd.
8. 17.46 in.
9. .016
10. 1.211
11. 3.15
12. .016
13. .032
14. .11
15. 1.14
16. 1.96
17. .2
18. .216
19. .0538
20. 1.36
21. 7.02
22. 3.85
25. .5; .05; .005
26. $ 1.55; $ 1.80
27. $ 2.28; $ .33$\frac{1}{3}$
28. .222 ft.

Page 151

29. 31.25 lb.; 15.625 lb.; 12.5 lb.
30. .12 in.; .06 in.; .03 in.; .024 in.
32. 2.333+
33. 2.4; 1.714+; 1.25; 2.076+; .685+
34. 2.511+
35. 4.329+ cu. ft.

Page 152

5. 25
6. 21.2
7. 120.2 yd.
8. 130.2 in.
9. 1.6
10. 12.11
11. 3150
12. 1.6
13. 1.1
14. 114
15. 1960
16. 20
17. 2.16
18. 13.6
19. 5265

Page 153

20. 24.5; 4.9; 49
23. 20; 2000; .5; .05; .005
24. 400 ft.
25. 120 pieces
26. 416 mi.
28. 68 bottles
29. 16 cu. ft.
30. 13
31. 8
32. 2.95

Page 154

2. $ 200; $ 60
3. 200 chickens
4. $ 1500

Page 155

6. $ 1400; $ 6400; $ 20,000
7. $ 2000

Page 156

2. $\frac{1}{4}$, $\frac{2}{5}$, $\frac{1}{25}$, $\frac{4}{25}$, $\frac{18}{25}$, $\frac{21}{25}$, $\frac{9}{25}$
3. $\frac{9}{200}$, $\frac{1}{200}$, $\frac{1}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{21}{40}$, $\frac{17}{200}$
4. $2\frac{3}{20}$, $3\frac{3}{4}$, $10\frac{1}{20}$, $8\frac{1}{8}$, $3\frac{3}{8}$
5. 3 mo.; 9 mo.; 6 mo.
6. 4 oz.; 12 oz.; $3\frac{1}{5}$ oz.; $9\frac{3}{5}$ oz.

Page 157

2. .8, .625, .15, .28, .312+, .147+, .190+, .95, .984+, .343+
3. .015, .023+, .008, .048, .012
4. 1.25, 7.125, 8.875, 9.7, 5.375, 29.187+, 8.675
5. .75, .5, .666+, .25
6. .5, .2, .4, .066+, .6, .1
7. .75, .5, .25, .166+, .416+, .666+

Page 159

1. $\frac{9}{10}$ 2. $\frac{3}{20}$; $\frac{17}{20}$ 3. Latter 4. First, 2%; second, 3%
7. $10; $16; $6; $9 9. $15; $35; $37.50; $38.75; $42

Page 160

2. $6; $3; $30; $16; $12
3. $85.33; $80; $48; $42.67; $21.33
4. $21.50; $14.33; $28.67; $10.75; $16.13

Page 161

5. $6.20; $21.70; $16.53; $12.40; $1.55 9. $45; $40; $52.50
7. $18; $24; $16 10. $84
8. $14.38; $11.50; $20.13 11. $3 12. $30.63

Page 163

1. $16.50

Page 164

2. $9.46 3. $4.44 4. $12.63 5. $64.10 6. $1.16 7. $2.01

Page 165

8. $2.05 9. $2.36 12. $8.75

Page 166

1. .75 4. 5.16 8. .625, .333+, .25, .666+, .75, .25, .16
2. $111.11 6. 590.4 9. 6
3. 5.06; 18.06; 5.0785 7. .375

Page 167

11. $5.78 14. $12.50 17. $708.32
12. 73 bu. 15. $52.50 18. $329.38
13. Lime, 650 lb.; sand, 1450 lb. 16. $4.20 19. 16
20. $22; $18; $26

Page 169

1. 22 hr. 20 min. 3. 40.83 mi. 4. $78
5. $12.60 7. $92.45

Page 170

8. 12 mi. less 10. $1\frac{3}{5}$¢; $14\frac{7}{12}$¢; $10\frac{5}{12}$
9. 49.42 T.; 161 cars; 6440 ft. 12. $\frac{1}{300}$¢; $1\frac{118}{300}$¢
13. 185,181; $21\frac{1587}{8007}$

Page 175

1. 36 oz.
2. 6125 lb.
3. 860 lb.
4. 14,300 lb.
5. 4500 lb.
6. 4900 lb.
7. 6 lb. 4 oz.
8. 62 lb. 8 oz.
9. 7 cwt. 50 lb.
10. 1 T. 17 cwt. 60 lb.
11. 5 T.
12. 6 cwt. 25 lb.

Page 176

16. (1) 1.5 (2) 2.5 (3) 1.775 (4) 6.25
17. $ 21; $ 17.50; $ 24.50
18. 20 da.; 20¢
19. $\frac{7}{8}$ lb. = 14 oz.
20. $\frac{5}{8}$ lb. = 10 oz.
21. 437.5; 3500 gr.
22. 80 weights; 2 weights
23. 50 T. 61 lb.
24. $ 8.29
25. 1000 oz.

Page 177

26. $\frac{3}{8}$ T.; .375
27. 35$\frac{5}{7}$ bu.
28. 1200 lb.
29. $ 43

Page 178

1. 33 in.
4. 1060 rd.
2. 11 ft. 132 in.
5. 3820 yd.
3. 42 ft.
6. 31 in.

Page 179

8. 6 ft. 8 in.
9. 8 ft. 4 in.
10. 4 yd. 2 ft.
11. 1 mi. 280 rd.
12. 6 mi. 80 rd.
13. 1 mi. 43 rd. 3$\frac{1}{2}$ yd.
15. 440; 880 yd.
16. 3520 ft.; 4400 ft.; 3960 ft.; 1173$\frac{1}{3}$ yd.; 1466$\frac{2}{3}$ yd.; 1320 yd.
17. 22,598.4 ft.; 13,728 ft.; 7532.8 yd.; 4576 yd.
18. 2346 ties; $ 1407.60
19. 72 shingles

Page 180

24. 363$\frac{1}{3}$ yd.
27. 11.363+ mi.
25. 88 ft.; 29$\frac{1}{3}$ yd.; less than $\frac{1}{12}$ as great
30. 18 sec.

Page 183

4. 32 sq. ft.
5. 488 sq. in.
6. 6912 sq. in.
7. 13,827 sq. in.
8. 1455$\frac{1}{2}$ sq. in.
9. 21 sq. ft.
11. 10 sq. yd. 6 sq. ft.
12. 4 sq. ft. 124 sq. in.
13. 6 sq. ft. 136 sq. in.
14. 3 A. 20 sq. rd.
15. 12 A. 80 sq. rd.
16. 8 sq. ft. 98 sq. in.

Page 184

18. $ 39.60; $ 8.64
19. 432 sq. ft.; 48 sq. yd.; $ 57.60; 84 ft.; 28 yd.
20. 2.066 A.
21. $ 316.80
22. $\frac{8}{242}$ A. more
23. More
24. 1440 A.; 2560 A.; 16,000 A.; 64,000 A.
26. $ 7840.80

Page 185

27. 7; 8; 11; 8
28. 324 bricks
29. 2106; 6156 sq. ft.
30. 4800 sq. yd.; 380 yd.; 4800 sq. yd.; 280 yd.
33. .186 − A.
34. 8 ft.

Page 187

1. 1512 sq. yd. 2. 36 sq. yd. 3. 132 sq. ft. 4. 324 sq. in.
5. $24\frac{1}{3}$ sq. yd. 6. $461\frac{1}{4}$ sq. ft. 7. 30 ft. 8. 15 ft.

Page 188

1. 285 sq. ft. 2. 368 sq. ft. 4. 11,400 sq. ft.

Page 189

6. 256 sq. ft.; 320 sq. ft.; 576 sq. ft.; 320 sq. ft.; 896 sq. ft.
7. 1st exceeds 2d by 3600 sq. in.

Page 191

1. 8640; 13,824; 27,648; 43,200; 15,552 cu. in. 2. $20\frac{55}{216}$; $45\frac{5}{216}$; 459 cu. ft.
3. 31,256 cu. in. 4. 46,656; 186,624 cu. in.
5. 352,812 cu. in. 6. 1367 cu. ft.

Page 192

9. 80; 100; 75; 125; 150 13. 10 cu. ft. 14. 64 cu. yd.
15. 64 cu. ft. 16. 33.66 cu. ft. 17. 900 cu. in. 18. 4 ft. 19. 4 ft.

Page 193

20. 10 ft. 23. 60 T. 29. 4 cu. yd.
21. $ 43.20; $ 20 24. Second holds 200 cu. ft. more
22. 9 T. 27. 29 pupils 30. 62.5 lb.

Page 194

31. 40,000 lb.

Page 195

1. 205 qt. 5. 38 pt. 9. 2 bu. 2 pk.
2. 35 qt. 6. 39 pt. 10. 8 bu. 3 pk.
3. 12 pk. 7. 1 bu. 3 pk. 11. 2 bu. 1 pk. 3 qt. 1 pt.
4. 112 qt. 8. 1 bu. 3 pk. 3 qt. 12. 6 bu. 1 pk.
14. Bushel, 3 times; $\frac{1}{2}$ peck, once 15. 24; 32 da.
16. 6451.26; 5376.05; 12,364.915 cu. in.
17. $ 1.50; $ 1 18. 18¢ 19. 1600 ears 20. $35\frac{5}{7}$ bu.; $89\frac{3}{7}$ bu.

Page 196

21. 4608 lb. (if 1 bu = $1\frac{1}{4}$ cu. ft.) 22. 7 bu. $3\frac{1}{2}$ pk.
23. 7 qt.; 2 qt. 24. 25 qt. 1 pt.; $ .55; $ 2.65.

Page 197

1. 7 pt. 2. 58 qt. 3. 22 pt. 4. 45 pt.

Page 198

6. 3 gal.
7. 6 gal. 2 qt.
8. 18 gal. 3 qt. 1 pt.
9. 12 gal. 2 qt.
10. 25 gal.
11. 125 gal.
13. Gallon, 25 times
15. 10 children
16. 45¢; 75¢
17. 12 gal. 2 qt.; 18 gal. 3 qt.; 25 gal.
18. 6 qt.
19. $27\frac{5}{12}$ gal.

Page 199

20. $52\frac{1}{4}$ qt.; $ 2.09
21. $53\frac{1}{3}$ da.
22. 40 gal.
23. 1848; 302.42 cu. in.

Page 201

3. 2607 sec.
4. 555 min.
5. 8100 sec.
6. 7215 sec.
7. 5 min.
8. 20 min.
9. 16 hr. 40 min.
10. 50 min.
12. 6210 da.
13. 75; 113; 50 da.
14. 5 yr. 2 mo.

Page 202

15. 69¢; 76¢
16. 27¢; 57¢
18. $1\frac{10}{29}$ sec.
19. 2 min. $32\frac{1}{2}$ sec.
20. 3 hr. 15 min.; $1\frac{58}{143}$ min.
22. 2 yr. 3 mo.

Page 203

1. 3 yd. 1 ft. 6 in.
2. 1 bu. 3 pk. 7 qt.
3. 15 lb. 10 oz.
4. 7 min. 30 sec.
5. 3 sq. ft. 18 in.
6. 15 gal. 3 qt.
7. 3 sq. yd. 8 sq. ft.
8. 1 cu. ft. 1272 cu. in.
9. 3 yd. 7 in.
10. 10 qt.
11. 17 ft.
12. 13 pk.
13. 24 da.
14. 48 sq. ft.
15. 53 pt.
16. 11 pt.
17. 136 cu. ft.
18. 87 in.

Page 204

19. $ 6.69
20. Bushel larger by 302.42 cu. in.
21. $ 3.15
22. $19\frac{2}{15}$ A.
23. 170 da.
24. $ 316.80
25. $ 48.72
26. $ 1.60
27. $3\frac{29}{32}$ bu.

Page 205

6. 216,610 sq. mi.
9. $ 3396.27
10. $950.92
11. $ 9382.92

Page 206

12.		
	Coal,	$ 2253.04
	Lumber,	1682.76
	Lime,	557.88
	Fertilizer,	974.31
	Seeds,	252.62
	Total,	$ 5720.61

M.	$ 1041.97
T.	596.05
W.	685.84
Th.	763.54
F.	2294.91
S.	338.30
Total,	$ 5720.61

13. Friday; reading
14. M. 419; av. 83.8
T. 388; av. 77.6
W. 404; av. 80.8
Th. 402; av. 80.4
F. 434; av. 86.8
General average, 81.88

Reading, 453; av. 90.6
Spelling, 357; av. 71.4
Arithmetic, 391; av. 78.2
Geography, 411; av. 82.2
Penmanship, 435; av. 87
General average, 81.88

Page 207

16. $1\frac{17}{24}$
17. $\frac{17}{24}$
18. $\frac{1}{100}$
19. $\frac{23}{36}$
20. $13\frac{17}{24}$
21. $7\frac{7}{12}$
22. $4\frac{5}{6}$
23. $41\frac{11}{12}$
24. $232\frac{19}{24}$
25. 76.143
26. 2.018 in.
27. .331
29. $2391.11
30. 26 T. 600 lb.
31. 1 ft. $2\frac{7}{8}$ in.

Page 208

32. (*e*) × (*a*)
(1) 1908.205
(2) 1406.592
(3) 2834.28
(4) 6360.40
(5) 53,061.3
(6) 6617.59
(7) 24,218.04

(*e*) × (*b*)
(1) 704.568
(2) 1879.7184
(3) 1102.22
(4) 20,480.488
(5) 8936.64
(6) 1401.372
(7) 12,887.457

(*c*) ÷ (*a*)
(1) .241+
(2) 1.124+
(3) .24
(4) .287
(5) .533+
(6) .123+
(7) .509+

33. (a) 102.
(b) 60.8
(c) 45.622
(d) 3,700.94
(e) 5,908.557
(f) 32,789.34
Total, 42,607.259

(1) 1,595.945
(2) 2,144.932
(3) 2,200.76
(4) 5,579.165
(5) 12,137.847
(6) 8,381.295
(7) 10,567.315
Total, 42,607.259

35. $\frac{11}{48}$ 36. (1) 1020 (2) 4362 (3) 5968 in. (4) 2596.8 (5) 8 ft. 6 in.
37. 19 qt.
38. 257 in.
39. 125 oz.
40. 7712 min.
41. 579 pt.
42. 101 yd.

Page 209

43. $11.04 44. $464 45. $19.20 46. 15 prime numbers
47. 231 = 11 × 7 × 3
5280 = 11 × 5 × 3 × 2 × 2 × 2 × 2 × 2
1760 = 11 × 5 × 2 × 2 × 2 × 2 × 2
144 = 3 × 3 × 2 × 2 × 2 × 2
1728 = 3 × 3 × 3 × 2 × 2 × 2 × 2 × 2 × 2
48. 5280 = 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 15, 16, 20, 22, 24
1760 = 1, 2, 4, 5, 8, 10, 11, 16, 20, 22
1728 = 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24
51. (1) 40,400 (2) 9100 (3) 32 (4) 1456 53. $30\frac{1}{4}$

Page 210

54. \$ 7.50
55. \$ 48; \$ 144; \$ 112
56. 88 yd.
57. 60 bu.
58. 12 sq. ft.
59. 63 sq. ft.
60. 4 ft.
61. 12 yd.
63. 160 cu. in.
64. 9360 cu. in.
65. 4 in.
66. 5 ft.
67. 4 ft.
68. 13$\frac{5}{28}$ bu.; \$10.57

Page 211

69. Sulphur, 225 lb.; charcoal, 337$\frac{1}{2}$ lb.; niter, 1687$\frac{1}{2}$ lb.
70. \$ 15.84
71. 8 mo.
72. 3 hr. 58.8+ min.
73. N. J. 2.17+ times as large
74. 132 ft.; bullet 18$\frac{2}{11}$ times as fast
75. No
76. \$ 3.98; \$ 3.25; \$ 3.65
77. 17,526 voters; 953,377 voters

Page 212

78. 23$\frac{8}{11}$ gal.
79. \$ 68.44; \$ 2737.60
83. Telegram costs 20¢ more
84. 600
85. 274
86. 12
87. 1832

Page 213

88. (1) \$ 2550 (2) 45 (3) 5687 (4) 35$\frac{20}{23}$
89. 9$\frac{11}{21}$¢; less
90. .257
91. .266
93. 23$\frac{1}{8}$ yd.
94. $\frac{5}{8}$ bu.; .625 bu.

Page 214

96. 200 mi.
97. 413 bu.
98. 12,692$\frac{4}{13}$ times
99. 15,000 cu. ft.; gas \$ 2.75 cheaper
101. 2112 steps; 17 min. 36 sec.
102. \$ 1152
103. \$ 6400
104. 1961
105. \$ 84; \$ 88

Page 220

1. 29$\frac{1}{8}$¢
2. $\frac{5}{21}$; less
3. Making it 46¢ less

Page 221

4. Making it 6¢ less
5. Lemonade 33¢ cheaper
6. 54¢
7. \$ 2.95
8. 77¢
9. 55¢
11. 84%

Page 222

12. 18
13. 2.4 in.
14. .848+ mi. 16 min. 57.6 sec.
15. 5:43 p.m.

PART II

Page 227

1. 48   3. 48   5. $ 228.45   7. 364 pupils
2. 54   4. 338   6. $ 487.56   8. 6060 lb.; 505 lb.; $ 787.80
10. 2,488,322 sq. mi.; 88,868$\frac{9}{14}$ sq. mi.   11. 43,290 sq. mi.; 6184$\frac{2}{7}$ sq. mi.

Page 228

5. 5451   8. $ 405.13   11. $ 54,113.67
6. $ 224.35   9. $ 1050.91   12. $ 6,517,376.63
7. $ 217.04   10. $ 2162.84   13. $ 1,456,237.48

Page 229

14. 105 ft.; 1115 ft.; 8972 ft.   15. 2,197,656 sq. mi.
16. 3,420,400 sq. mi.   17. $ 7,036,885

Page 230

1. 211   2. 65   3. 266   4. 4.66   5. 369.16   6. 502 mi.

Page 231

7. $ 670.23   9. 418.79   10. 464.80   11. 434.92   12. 519.43   13. 2285.28

Page 232

1. $ 442.95   2. $ 41.54
3. Wash., $ 135.56; Ad., $ 126.36; Jeff., $ 139.84; Mad., $ 124.45; Mun., $ 136.96. Total, $ 663.17. Jan., $ 195.83; Feb., $ 110.10; Mar., $ 81.29; Apr., $ 103.98; May, $ 81.18; June, $ 90.79. Total, $ 663.17.

Page 234

1. 24,491,776   3. $ 2,286,168.36
2. 657,103,326   4. 78,612,807
5. 3,868,722; $ 20,771.46; $ 160,039.44; $ 8,023,555.08; $ 100,570.74; 32,064,000
6. 15,791,490; 34,752,861; 34,362,900; 15,455,583; 231,761,583; 4,633,200
7. 41,775,360 ft.   8. 369,911,520 sq. ft.
9. 8,425,728 cu. in.; 67,868,928 cu. in.
10. $ 175.24   12. $ 647.19   14. 115,908 oz.
11. $ 7669.76   13. 163,152 in.   15. 286,296 qt.
16. $ 358.88; $ 557.55

Page 235

1. $ 289
2. $ $297\frac{1}{2}$
3. $647\frac{1}{2}$ ft.
4. $656\frac{1}{4}$ in.
5. $2878\frac{3}{4}$
6. $ 2124
7. $2138\frac{3}{4}$
8. 624
9. $1014\frac{3}{4}$
10. $533\frac{1}{4}$
11. $ 2025
12. $ $30\frac{3}{8}$
13. $ $162\frac{1}{2}$
14. $ 32
15. $ $192\frac{5}{8}$
16. $ 4680
17. $ 9432

Page 236

18. $ 800; $ 1600; $ 300; $ 21
19. $ 7.50; $ 60; $ 30; $ 45
20. $2962\frac{1}{2}$; 7900; 5925; 11,850; 15,800
23. 320; 270; 400; 300
24. 336; 738; 1824; 1206
25. $ 240.75; $ 285.33
26. $ 121.50; $ 207; $ 168

Page 239

1. 117
2. 567
3. 243
4. 75
5. 84
6. 97
7. 485
8. 3360. R = 3637
9. 66,894
10. 3896
11. 76,805
12. 305
13. 507
14. 708
15. 27
16. 63. R = 46
17. 308
18. 530. R = 27
19. 44. R = 1200
20. 3270
21. 205 bbl.
22. 58 schoolhouses
23. $39\frac{480}{1250}$; $212\frac{780}{1250}$

Page 240

24. 72 cu. ft. R = 584 cu. in.
25. 4 cu. ft. R = 80 lb.
26. 11 mi. R = 1920 ft.
27. 13 mi. R = 120 yd.
28. 54 round trips. R = 1504 mi.
29. $6\frac{4}{11}$ times
30. 4307
31. $ 33,047.19; 23+ lives
32. 79; $12\frac{58}{79}$ mi.

Page 242

5. $ 2613
6. 752
7. 9360

Page 243

9. $57\frac{3}{5}$ T.
10. 12 T.
11. $ 202.40
12. $ 98.30
13. $ 421.02
14. $ 18.98
15. $ 250
16. $ 684
17. 14
18. 22
19. $120\frac{3}{4}$
20. 275
21. $122\frac{1}{4}$
22. $122\frac{4}{5}$
23. $20\frac{5}{11}$ mi.

Page 244

2. May, $ 1.80; Ada, $ .90
3. Man, 240; boy, 80 fish

Page 245

4. 120; 60
5. 135 and 45; 144 and 36; 150 and 30
6. 120 papers
7. Helen, 96; Florence, 24 post cards
8. State, $ 12,000; county, $ 6000
9. David, 1 qt.; William, 1 qt.; Harry, 2 qt.

1. By 7. (1) $3060\frac{3}{7}$ (2) $442\frac{6}{7}$ (3) $754\frac{2}{7}$ (4) $134,680\frac{5}{7}$
By 8. (1) $2677\frac{7}{8}$ (2) $387\frac{1}{2}$ (3) 660 (4) $117,845\frac{5}{8}$
By 9. (1) $2380\frac{1}{3}$ (2) $344\frac{4}{9}$ (3) $586\frac{2}{3}$ (4) $104,751\frac{2}{3}$
By 5. (1) $4284\frac{3}{5}$ (2) 620 (3) 1056 (4) 188,553
By 6. (1) $3570\frac{1}{2}$ (2) $516\frac{2}{3}$ (3) 880 (4) $157,127\frac{1}{2}$

Page 246

3. 10,000 times; 69 yd. 1 ft. 4 in. 5. \$ 623.93 7. \$ 4.25
4. \$ 3308 6. 10,000 sq. ft. 8. \$ 36
9. \$ 1400 10. \$ 233.50

Page 247

11. $203\frac{3}{8}$ lb. 13. 1636 ft. 1. $1\frac{1}{4}$ A.

Page 248

2. $\frac{5}{8}$ A. 3. B, $49\frac{3}{8}$ A.; C = 50 A.; F = $16\frac{7}{8}$; wood lot, $13\frac{1}{8}$ A.
4. 2 mi. 78 rd. 5. \$ 1713.84 6. \$ 1365 7. \$ 3078.84

Page 249

8. \$ 1650; profits \$1428.84 9. \$ 276.90 11. \$ 89.25
12. \$ 9.30 13. \$ 15.56 14. \$ 6.26; \$ 3.13 15. \$ 850

Page 250

16. \$ 210

Page 254

2. (1) 3 (2) 20 (3) 10 (4) 49
8. (1) 4 (2) 3 (3) 4 (4) 3 (5) 3 (6) 6

Page 255

8. (1) 4; (2) 3; (3) 4; (4) 3; (5) 3; (6) 6 4. 6 qt.

Page 256

5. 2 qt.; 4 qt. 9. $1\frac{87}{120}$

Page 258

8. 1 9. $\frac{1}{24}$ 10. $\frac{21}{100}$ 11. $8\frac{1}{4}$ 12. $\frac{41}{240}$ 13. $5\frac{3}{16}$

Page 259

22. $\frac{1}{4}$ 23. $\frac{1}{2}$ 24. $\frac{16}{87}$ 25. $\frac{15}{32}$

1. 3
2. $7\frac{1}{2}$
3. 2
4. $3\frac{1}{3}$
5. 24
6. $7\frac{1}{5}$
7. $13\frac{1}{2}$
8. $22\frac{2}{3}$
9. $46\frac{3}{4}$
10. $5\frac{1}{2}$
11. $33\frac{3}{4}$
12. $8\frac{4}{5}$
13. $\frac{2}{11}$
14. $195\frac{1}{2}$
15. 23
16. $24\frac{3}{8}$ sq. ft.

Page 260

17. 76¢ 18. $19\frac{1}{3}$ sq. ft. 19. $9\frac{9}{32}$ sq. yd. 20. $ 4.20 21. $42\frac{1}{2}$ lb.
22. $120\frac{5}{16}$ cu. ft. 23. $ 1.36 24. 2 ft. $6\frac{4}{5}$ in. 25. $ 46.88

Page 261

1. 20; 10 3. $\frac{15}{14}$ 6. 512 8. $\frac{1}{6}$
2. 90; 30 4. 4 7. $1\frac{1}{6}$ 9. $\frac{5}{7}$
10. $3\frac{1}{3}$ 11. $\frac{24}{35}$ 12. $1\frac{1}{6}$

Page 262

13. $3\frac{6}{7}$ 18. $\frac{26}{35}$ 23. 12 bags 28. 12 mo.
14. $\frac{6}{25}$ 19. $\frac{48}{65}$ 24. 10 strips 29. 28 slices
15. $\frac{7}{32}$ 20. $2\frac{2}{5}$ 25. $6\frac{6}{11}$ yd. 30. 40 pieces
16. $\frac{8}{21}$ 21. $100\frac{1}{3}$ 26. 10 shares 31. 16 weeks
17. $\frac{11}{16}$ 22. $\frac{24}{35}$ 27. 6 turns

Page 264

1. $\frac{2}{3}$ 4. $2\frac{1}{4}$ 7. $23\frac{5}{6}$ 10. $7\frac{11}{28}$ 13. $\frac{9}{11}$
2. $\frac{21}{26}$ 5. 6 8. $10\frac{19}{20}$ 11. $\frac{5}{24}$ 14. $\frac{1}{9}$
3. $1\frac{1}{3}$ 6. $4\frac{2}{3}$ 9. $17\frac{1}{4}$ 12. $1\frac{1}{4}$ 15. $7\frac{1}{2}$
16. 6 19. $181\frac{9}{11}$ rd. 22. $26\frac{2}{3}$ da.
17. 8 20. 228 rows 23. 8 ruffles
18. 12 baskets; 96 baskets 21. $1\frac{5}{6}$ in. 24. 6 yr.

Page 265

25. 42 boxes 26. $17\frac{1}{7}$ 27. $1\frac{3}{5}$ in. 28. $5\frac{13}{36}$ 29. $31\frac{1}{4}$ ft. 30. $5\frac{27}{48}$

Page 266

1. $\frac{3}{4}$ 4. $\frac{10}{19}$ 7. $\frac{15}{38}$ 10. $2\frac{5}{17}$ 12. $ 7.50
2. $1\frac{3}{5}$ 5. $\frac{5}{27}$ 8. $2\frac{6}{47}$ 11. $ 12.75 13. $ 20.43
3. $\frac{12}{35}$ 6. $2\frac{6}{11}$ 9. $1\frac{9}{16}$

Page 267

2. $ 150 5. 80¢; $ 1 8. $\frac{1}{12}$; $\frac{1}{3}$
3. $168\frac{3}{4}$ lb. 6. 3 mi.; $10\frac{1}{2}$ mi. 9. Man, $\frac{3}{5}$; boy, $\frac{3}{10}$
4. 2 in.; 3 in. 7. $60\frac{5}{14}$ 10. County, $ 1200; state, $ 4800

Page 268

1. $\frac{1}{3}$ 4. $1\frac{5}{6}$; $3\frac{11}{60}$ 7. $\frac{1}{3}$; $\frac{3}{7}$; $\frac{1}{2}$ 10. $ 7.50
2. $\frac{19}{24}$ 5. $\frac{15}{14}$; $\frac{1}{14}$ 8. $2\frac{15}{16}$ in. 11. $3\frac{57}{72}$ T.
3. $6\frac{2}{3}$; $7\frac{1}{2}$; 20; 4 6. $\frac{1}{4}$; $\frac{1}{80}$; $\frac{1}{16}$ 9. $5\frac{25}{286}$ 12. $ 4375

Page 269

13. 45 lb.; $ 8.10; $ 162 14. $4477\frac{1}{2}$ sq. mi. 15. $42\frac{3}{5}$ mi.
16. First 17. 4000 lives
1. $ 64 2. $243\frac{17}{21}$ doz. 3. $26\frac{1}{4}$¢

Page 270

4. 60¢
5. $1.24
6. $2.66
7. 44¢
8. $377.28
9. $4097.28

Page 271

10. $1.79
11. ⅙¢
12. $2.70; $32.40

Page 275

3. $1049.19
4. $3715.46
5. $308.9140
6. $81.12029
7. .0577
8. 4.1204
9. .1655
10. 39.2807
11. .875
12. 2148.692
13. $270.01
14. $653.33
15. 34.794
16. 295.57
17. 1703.10

18. (1) 293.90
(2) 18.077
(3) 15.1174
(4) 3.31541
(5) 2.96901
Total, 333.37882

By row
71.21778
57.99121
43.96746
86.17072
37.78155
36.2501
Total, 333.37882

Page 276

19. 4.6475 A.
20. $.0504, $.0049, $.0455; $.0066, $.057, $.0115
21. .267 lb.; 2.67 lb.; 26.7 lb.; .0267 lb.
22. 2898.38
23. 1057.056

Page 277

3. 333.576 in.
4. .9881 ft.
5. .1488 A.
6. 4.80624 lb.
7. 6.6 sec.
8. .003870756
13. 2.75; .00275; 2752.75; .0275; 27,500
14. 1500.27; .150027; 150,027; 15.0027; 1.50027; 15,002.7
15. $23.6425
16. 54.7497 lb.
17. $2.196
18. 57.5 lb.
19. $2.08

Page 278

20. $546.05
21. $7.80
22. $90.63
23. $5.95
24. $23.19

3. 61.5
4. 2000
5. 34.928+
6. 750
7. 1.36
8. 39.27

Page 279

14. .154+
15. .0007+
16. .025+
17. 165.217+
18. .014+
19. .0002+
20. .023+
21. 8181.818+
22. 3470.297+
24. 212 books
25. 4.953+; 14.534+; 29.069+
26. 803.564+ bu.
27. 420.168 marks
28. $17,532.32
29. 8.323+ ft.
30. 2432+ qt.
31. 3.662+

Page 280

4. $\frac{1}{8}$
5. $\frac{11}{40}$
6. $\frac{3}{8}$
7. $\frac{3}{500}$
8. $\frac{3}{5000}$
9. $\frac{3}{2000}$
10. $6\frac{33}{80}$
11. $\frac{3}{80}$
14. .25; .125; .0625; .4375; .32; .85
15. .593+; 1.3125; 4.72; 3.3375
16. .1666+; .2857+; .5555+; .6363+
17. .3888+; .8823+; 3.1111+; 12.8333+

Page 281

20. .4375 lb.
21. 4.1
22. 2.23
1. 6; 18; 12; 24; 30; 42
2. 800; 900; 600; 1500; 2100; 150

Page 282

3. 64; 60; 32; 12; 8
5. $ 6.75
6. $ 18
7. $ 23.33
8. $ 62.67
9. $ 59.50
10. $ 11.88
11. $ 1.50
12. $ 62.25
13. $ 15.33
14. $ 51.25
15. $ 11.25
16. $ 29.25
17. 10,800; 9600; 14,400; 5760; $4114\frac{2}{7}$; 57,600
18. 7680; 11,520; 2560; 1440; 3840
19. $115\frac{1}{5}$; 108; 144; 96; 288
20. 438
21. 565
22. 375
23. 46
24. 20; 60 yd.
25. $16\frac{1}{2}$; 36 lb.
26. 18; 27 collars
27. 16 gal.
28. 6 baseballs
29. 14 qt.
30. 160 bu.
31. 6 doz.
32. 24 qt.

Page 283

33. 16 penknives
34. $ 3.75
35. 240 yd.
36. $ 2.
37. 512 lb.
38. $ 50
39. 128 bu.
40. 30 cans
41. 10; 14 collars
42. 18; 9; 30 qt.
43. $ 8

Page 284

6. $ 907.50
7. $ 312.50

Page 285

8. (1) 336 (2) 21
9. $ 456.31+
2. $ 2000

Page 286

4. 40¢
6. $ 4020
7. $ 300
8. $ 13,000
9. $ 19,901.78
11. 5.4 in.
12. .642+ in.
13. 70 chickens
14. 90 A.
15. 40 games
17. $ 630

Page 287

18. 2.16; 1.08
19. .198; .033
20. $ 16.32; $ 8.16
21. .3
22. $ 14.25
23. 600 lb.

Page 291

1. Balance, $11.51

Page 292

3. .00024; .1
4. .015
5. .003875
6. (1) 2.45 (2) 101.87 (3) .7
(4) 41.6 (5) 16 (6) 1.26

Page 293

8. 24 yd.
9. Mr. Walters, $ 3900; partner, $ 3000
10. $ 115
11. 46.8 oz.; 2.92+ lb.
12. Mary, $ 7; Anna, $ 5
13. 5.492+ mi.
14. .185
15. $ 557.88
16. 42.665 mi.
17. 14,134+ ft.

Page 294

1. $ 1245
2. $ 3801.50
3. $ 1840.05

Page 295

4. 150 trees
5. 1,875,872 sq. mi.; 38.15+ states
6. 17.23− states
9. 391,667− A.; $\frac{1}{80}$ (approx.)

Page 300

1. 50s
2. 57d
3. 69d
4. 1080d
5. 8s 4d
6. £1 15s
7. £4 3s 4d
8. £5
9. 8s 4d
10. £1 10d
11. $ 48.67
12. $ 63.26
13. $ 25.79
14. $ 33.09
15. $ 28.23
16. $ 16.79
17. £100
18. £205.49
19. £120
20. £3082.297+

Page 301

21. $ 16.55
22. 49¢

Page 302

2. 207′
3. 2851′
4. 635″
5. 22,800″
6. 15,150″
7. 43,225″
8. 3° 30′
9. 6′ 55″

Page 303

12. 36° 120° 108° 67½°
270° 135° 105° 60°
13. 15° 30° 75° 22½°
14. 5° 11¼° 3¾°
1. 22½° 67½° 45°
2. 90° 157½° 157½°

Page 304

3. 45°
4. 112½° 135° 90°
6. N. E.; S. W.

Page 305

1. 84; 144; 120
2. 5; 144; 25
3. 40; 200
4. 480; 360; 960
5. 70; 80; 132
6. 1920; 240
7. 9.6
8. .25; 25%
9. Latter

Page 306

1. 322 in.
2. 182 qt.
3. 1185 hr.
4. 78 sq. ft.

Page 307

5. 76,478″
6. 10,646 cu. in
7. 6780 lb.
8. 341 ft.
9. 4884 sq. rd.
10. 35,118 min.
11. 105 pt.
12. 35 cu. ft.
13. 1685½ sq. ft.
14. 5457 sq. ft.
15. 8 T. 15 cwt. 60 lb.
16. 7 cu. ft. 740 cu. in.
17. 182 gal. 3 qt. 1 pt.
18. 10 hr. 45 min. 15 sec.
19. 6 sq. yd. 7 sq. ft. 104 sq. in.
20. 10 mo. 24 da. 1 hr.
21. 71 bu. 2 pk. 5 qt. 1 pt.
22. 1 A. 29 sq. yd.
23. 1 cwt. 88 lb. 1 oz.
24. 263 gal. 1 qt.
25. 75 bu. 1 pk. 1 qt. 1 pt.
26. 29 rd. 4 yd. 11 in.
27. 7 reams
28. 1 mi. 156 rd. 3 yd. 2 ft.
29. 1825 min.; 1 da. 6 hr. 25 min.
30. 240 hr.
31. 78 T. 8 cwt.
32. 5
33. 22 ft. 6 in.

Page 309

1. 14 bu. 2 pk.
2. 13 gal. 3 qt.
3. 24 cu. yd. 11 cu. ft. 307 cu. in.
4. 6 T. 14 cwt. 45 lb.
5. 81 qt. 1 pt.
6. 42 lb. 1 oz.
7. 8:17 p. m.
8. 206 lb. 4 oz.
9. 187 lb.

Page 310

10. 5 ft. 11¼ in.
11. 16 ft. 3 in.; 16.25 ft.
12. 12 yd. 2 ft. 3 in.
13. £25 7s 3d; $72.35

Page 311

1. 6 yd. 7 in.
2. 10° 54′ 3″
3. 141° 17′ 43″
4. 72 A. 93 sq. rd. 21¼ sq. yd.
5. 14 bu. 6 qt.
6. 192 gal. 2 qt.
7. 128 yr. 2 mo. 9 da.
8. 154 yr. 3 mo. 28 da.
9. 6 yr. 6 mo.
10. 7 yr. 6 mo. 11 da.
11. 25 yr. 8 mo. 8 da.
12. 83 yr. 3 mo. 23 da.
13. 56 yr. 2 mo. 3 da.
14. 154 lb. 12 oz.
15. 14 yd. 1 ft. 11 in.
16. 10 yd. 1 ft. 7 in.

Page 312

17. 74 lb.
19. 3:49 a. m.
18. 2.9″
21. 15 lb. 12 oz.
22. 128 yr. 2 mo. 9 da.; 283 yr. 8 mo. 22 da.;
405 yr. 6 mo. 19 da.; 155 yr. 6 mo. 13 da.;
277 yr. 4 mo. 10 da.; 121 yr. 9 mo. 27 da.
23. 15 hr. 9 min.; 9 hr. 14 min.
25. 1 hr. 25 min.; 28 hr. 20 min.
24. 6 yr. 4 mo. 12 da.
26. 4 mo. 22 da.

Page 313

27. 6 bu. 2 pk.
1. 19 da. 20 hr. 30 min.
2. 39 bu. 2 pk. 6 qt.
3. 12 yd. 1 ft. 8 in.
4. 35 T. 4 cwt. 55 lb.
5. 45 sq. yd. 3 sq. ft. 56 sq. in.
6. 144 bu. 2 pk.; 144.5 bu.
7. 82 gal. 2 qt.; 82.5 gal.
8. 389.8125 cu. ft. or 389 cu. ft. 1404 cu. in.

Page 314

9. 49 sq. ft. 126 sq. in.; 49.875 sq. ft.
10. 360 hr.; $ 144
11. 210 yd. 2 ft.
12. 7 mi. 49 rd. 2 yd. 2 in.

Page 315

1. 15 gal. 1 qt. 1 pt.
2. 14 bu. 2 pk. 4 qt.
3. 12 yd. 2 ft. 9 in.
4. 10 wk. 5 da. 13$\frac{5}{6}$ hr.
5. 28 mi. 171$\frac{2}{3}$ rd.
6. 18
7. 18
8. 2° 15′
9. 2° 49′ 2$\frac{2}{3}$″
10. 4 min. 36$\frac{1}{4}$ sec.
11. 1 gal. 1 qt. .28 pt.; 1.285 gal.

Page 316

12. 274 posts
13. 3 lb. 12.4 oz.; 83¢
14. 1 mi. 258$\frac{14}{17}$ yd.
1. 4620 cu. in.
2. $ 150
3. $ 12
4. 2 mi. 208 rd. 2 yd. 2 ft.
5. $ 316.80

Page 317

6. 21 hr. 42 min. 55 sec.
8. $ 1.60
7. 537.605 cu. in.; 67.2+ cu. in.
9. Former; 50¢
11. .291+
12. .466+
13. .6
14. .783+
15. .875
16. .219+
18. 9 mo.
19. 1 mo. 24 da.
20. 7 mo. 6 da.
21. 21$\frac{3}{5}$ da.
22. 8 mo. 21 da.
23. 11 mo. 4$\frac{4}{5}$ da.
24. 5 ft. 10$\frac{1}{2}$ in.
25. 6 min. 53$\frac{19}{25}$ sec.
26. 25 yr. 11 mo. 4 da.

Page 318

27. 24 da. 5 hr. 35 min.
28. 1 hr. 46 min. 40 sec.

Page 321

1. $ 1920; $ 1530
2. $ 931.67
3. Former
4. $1\frac{262}{363}$ A.
5. $3\frac{87}{121}$ A.
6. $1\frac{89}{121}$ A.
7. $10\frac{40}{121}$ A.
8. 2
9. 1
10. 36
11. 50
12. 40
13. $3\frac{3}{4}$ A.
14. $4\frac{1}{14}$ A.
15. $2\frac{188}{700}$ A.
16. $18\frac{18}{49}$ A.
17. 16
18. Less

Page 322

19. Greater; less.
21. 40 A.
22. 120; 2240 A.
23. 720; 640; 100 A.
24. Less
26. 192 sq. ft.; 768 sq. ft.
27. 216 sq. ft.; $1\frac{1}{4}$
28. $21\frac{7}{8}$ sq. mi.
29. $312\frac{1}{2}$ sq. mi.

Page 323

1. 23,040 A.; 36 sq. mi.
2. 640; 1280; 2240; 80 A.

Page 324

3. 1280 rd.
4. $ 512
5. $ 3600

1. $ 36
2. $ 93.29
3. $ 56,320
4. $ 54.27
5. 96 shingles; 28 rows; 2688 shingles; $48.38

Page 325

6. 48,000 shingles

Page 326

1. $ 29.73
2. $ 7.36
3. $ 9.51
4. $ 34.96
5. $ 26.21
6. $ 35.68

Page 327

1. 7
2. 9
3. 6 strips; 44 yd.
4. 70 yd., $ 56; 90 yd., $ 108

Page 328

5. $59\frac{1}{2}$ yd.
6. 68 yd.

1. 24 rolls; $ 6
2. $ 5.17
3. $ 8.10

Page 330

1. 50 sq. ft.
2. 64 sq. in.
3. 288 sq. in.
4. 252 sq. in.
5. 240 sq. in.
6. $22\frac{1}{2}$ sq. ft.
7. $\frac{1}{4}$ sq. mi.
8. $\frac{9}{40}$ sq. mi.
9. 160 A.; 144 A.
10. 120 sq. ft.
11. $ 42.01

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12. 600 sq. in.
13. 18 sq. ft.
14. $116\frac{2}{3}$ sq. ft.
15. 60,000 sq. ft.
16. 24,900 sq. ft.
17. 414.6008 sq. ft.
18. 1.377+; .571+; .009+
19. $ 15.20
20. 4400 sq. mi.; 2,816,000 A.
21. 704,000 A.
23. N. J. $1\frac{1}{3}$ times as great

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2. 24; 30

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3. 5
4. 10; 15
5. $13\frac{1}{3}$
6. $10\frac{2}{3}$
7. $21\frac{1}{3}$
8. 30
9. $8\frac{1}{2}$
10. 17
11. $31\frac{1}{2}$
12. $21\frac{1}{4}$
14. \$ 297.60
15. \$ 1.28
16. \$ 8
17. \$ 96
18. \$ 79.34
19. \$ 109.52

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20. \$ 2999.23
21. 1000 ft.
22. $133\frac{1}{3}$
23. 20
25. 96¢; \$ 2.40; \$ 2.70
26. \$ 38,016

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1. .148+
2. 337.49+
3. 336
4. $76\frac{4}{5}$
5. 192
6. 360
7. Yes

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8. No.
9. 800
10. 96,000
1. $897\frac{51}{77}$
2. $1615\frac{61}{77}$
3. $59\frac{5}{7}$
4. $70\frac{10}{77}$
5. $667\frac{7}{11}$
6. $786\frac{6}{7}$

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7. 78,750
8. $65\frac{5}{8}$ hr.
9. 10.026 in.
10. 150 gal.
1. \$ 28.80
2. $52\frac{1}{5}$ loads

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3. 81 ft.
4. $850\frac{1}{2}$ cu. ft.
5. 18,711 bricks; \$168.40
6. $177\frac{1}{5}$ loads
7. \$ 113.64 –
1. $6\frac{3}{4}$ sq. ft.
2. 3 yd.
3. \$ 25.78; \$ 10
4. 20 T.
5. 10¢; one 10¢ stamp

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6. Pool; 144 sq. ft.
7. \$ 13.77
8. 6,750,000,000 lb.
9. 9.52 qt.
10. $14\frac{22}{23}$ sec.; $4\frac{203}{220}$ sec.
11. 6 ft. $4\frac{5}{8}$ in.
12. $4\frac{92}{109}$ sec.
13. $4\frac{17}{132}$ mi.
14. 2 da. 23 hr. $6\frac{2}{3}$ min.
15. \$ 260.13

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16. 205 T.
17. 111 cars
1. \$ 1445.78
2. 6806 T.; 4083+ T.
3. \$ 8574.30
4. \$ 2305.56

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5. $8\frac{1}{4}$ approx. working days
6. $2\frac{3}{4}$ approx. trainloads
7. 189,100 sq. mi.; 121,024,000 A.
8. 395,385,408,000 T.
10. 714+ T.; 35,714+ T.; 714,285+ T.
11. 42,500 cu. ft.; 30,357+ T.

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11. 20; 10; 15; 50
12. $ 6
13. $ 5

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2. 28 chickens
3. 126 eggs
4. $ 10; $ 15; $ 40; $ 42.50
8. 24; 340; 151.2; 16.8 in.
9. $ 444; $ 31.44; $ 379.2; 633.6 ft.
10. 177.6; 303.03 gal.; $ 11.62; 1577.31
11. 4800; 7360; $ 8000; $ 89.89
12. $ 940; 413.6 yd.; 1034 lb.; $ 8.76

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14. $ 45
15. $ 168
16. $ 2295
17. $ 1125
18. 54 marbles
19. 46.8 lb
21. $ 1612.50
22. Former

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1. 11
2. 16
3. 250
4. 240
5. 18
6. $ 241$\frac{7}{8}$
7. $ 473.13
8. 160
9. 45
10. $ 400.30
12. 1600 lb.; 3200 lb.; 8 T.
13. $ 5

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1. $ 4
2. 51¢
3. $ 4.80
4. $ 30.625
5. $ 1397.50
6. $ 7680
7. $ 6000
8. 8250 men
9. $ 52.50
10. 21,000
11. $ 1.50

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12. $ 1800; $ 18; $ 2
1. 8%
2. 6%
3. 8%

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4. 15%
5. 52$\frac{16}{17}$%
6. 24%; 28%; 36%
7. $\frac{12}{100}$; 12%; 28%
8. $\frac{22}{100}$; 22%; 32%; 18%; 35%; 6%; 20%; 50%; 75%; 38%; 68%
10. 24%
11. 35%
12. 14%
13. 90%
14. Corn, 40%; oats, 16%; pasture, 24%
15. 28$\frac{4}{7}$%
16. 20%

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17. 40%; 42$\frac{6}{7}$%; 43$\frac{3}{4}$%
18. .193
19. 11%
20. .025+
21. .147+
22. First, 15%; second, 14%
23. Latter
24. Latter

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1. $ 180
2. $ 90
3. $ 1800
4. $ 108
5. 700 cattle
6. 400 chickens
7. 400 pupils
8. 50 papers
9. 36; 18; 200; 600
10. 72 marbles
11. 400 pupils

Page 355

1. $ 2.40; $ 77.60
2. $ 112
3. $ 2572.50; $ 2536.10

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4. $ 240; 3% 7. Latter by 39¢ 10. $ 50
5. Latter by $ 3.45 9. 25% 11. $ 2800
6. Latter by $2.98

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1. $ 83.19 2. $ 283.50 3. $ 181.15 4. $ 264.60 5. $ 360

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6. $ 418 7. $ 1719.90 8. Latter $8.55 less 10. $ 20.08
11. $ 81.87 12. $ 331.86 13. $ 201.33

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1. $ 1500 2. 20% 3. 540 chickens 4. $ 200; 400%
5. 55 games 7. 67 games

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8. $42\frac{18}{71}$% 9. $10\frac{10}{37}$%; $26\frac{14}{111}$%; $77\frac{11}{37}$% 10. $10\frac{7}{10}$%
11. 5% 13. 20% 15. .54 pt. 17. $27\frac{3}{11}$+%
12. 36 14. 13.75 16. $41\frac{2}{3}$% 18. 3%

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1. $ 38; $ 45.60; $ 53.20 3. $ 113.46; $ 85.10
2. $ 159.08; $ 222.71 4. $ 55.88; $ 37.25; $ 130.38

5. $ 94.50 6. $ 93 7. $ 246.75 8. $ 41.09 9. $ 147.90
$ 110.25 $ 139.50 $ 205.63 $ 82.18 $ 184.88
$ 63 $ 186 $ 287.88 $ 47.94 $ 73.95

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10. $ 863.60 11. $ 32.54 12. $ 50.31
$ 1007.53 $ 54.23 $ 57.50
$ 575.73 $ 27.12 $ 32.34

Page 366

1. $ 68.33; $ 56.94 4. $ 160.53; $ 144.38 7. $ 5127.73
2. $ 73.20; $ 48.80 5. $ 3500 8. $ 10,045
3. $ 14.48; $ 25.34 6. $ 1796.88

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1. $ 2 3. 83¢ 5. $ 6.60 7. $ 18.99
2. $ 2.73 4. $ 2.58 6. $ 1.81 8. $ 29.86
9. $ 20.13 10. $ 13.43 11. $ 1.95

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12. $ 587
13. $ 1368

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1. $ 44.60
2. $ 67.98
3. $ 50.33
4. $ 69.57; $ 61.07
5. $ 61.30
6. $ 666.44
7. Richard, $57.41; Anna, $58.28
8. $ 6.09

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3. 250,000,000 in. or 3945 + mi.
6. (1) 30 gal. 2 qt. (2) 122 qt.
7. $ 823.78
8. 1 ft. 1 in.
9. $39\frac{98}{16}$

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10. .675
11. 319 bu.; $28\frac{188}{319}$%
12. $ 1,533,841.44
13. $109\frac{19}{32}$
14. 11 bu. 2 pk. 7 qt.
15. 4.527
16. 9.8736
17. $ 32.17
18. 86,840
19. $12\frac{1}{2}$ in.
21. .1125
22. .785 +
23. $1742\frac{1}{2}$
24. $ 21.75
25. $78\frac{4}{7}$
26. 1583.3664

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27. 15,000
28. $210\frac{3}{5}$
29. 14 + ¢
30. $ 2730
33. 7 da.; $3\frac{168}{175}$%
34. 21 sq. in.
36. $ 7600
37. 3:20 p. m.
38. 2880 lb.

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39. $623{,}695\frac{31}{77}$ gal.
40. 1999.8
41. $ 111.30
42. $ 1728
43. 272 cattle
45. 88 ft.
49. $ 1314

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50. 3,742,155 sq. mi.
52. 120 lb.
53. $31\frac{1}{29}$ mi.
54. $44\frac{816}{2761}$%
55. .108 da. more
57. More
58. 24 cu. ft.
59. 11,250 T.

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60. Ala., Ark., Iowa, La., Mis., N. Car., Pa., Tenn., Va., Wis.
62. 18 sq. ft.; $56\frac{1}{4}$%
63. 96 sq. ft.; 300%
65. 4 gal. 1 qt. 1 pt.
66. Increased $\frac{1}{20}$; decreased $\frac{1}{12}$
68. $ 38.30
69. $ 1066.67

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70. 4 da. 23 hr. 22 min.; 8.21 – mi.; $7\frac{151}{490}$ min.
71. 30,618 T.
72. $31\frac{1}{4}$%
73. 172 lives
76. 1,080,000 sq. mi.
77. $2\frac{3}{7}$
78. $\frac{1}{6}$
79. $\frac{8}{15}$
80. $\frac{4}{7}$
81. 129,600 bees

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83. $5\frac{5}{14}\%$; $14\frac{2}{3}\%$
84. 659.45 +
85. 1,995,231,920
86. 16 in.
87. 71.2 sq. in.
88. $63,033\frac{1}{3}$
89. $ 26,000,000
90. $ 1,500,000
92. 312.5 lb.

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93. $33\frac{1}{10}\%$
94. First
95. .132 +
97. 12,918
98. 127. R = $ 432
99. $16\frac{233}{1152}$
100. $ 34,600
101. $ 105; 6720

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1. $ 300; $ 375; $ 600
2. $ 450
3. $ 133.69
4. $ 39.20
5. $\frac{20}{121}$; .165

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6. $ 5250
7. $ 4350
8. $ 930 more in Ex. 1
9. $ 30.56
11. 960 cu. ft.
12. $ 380.16
13. $ 20.83
15. $ 55.84

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16. $ 1803.03
17. $ 20
20. $ 469.50
21. $ 2272.53
22. $ 756.93
23. 5 + yr.

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